

1. This sample document provides a template for writing a  $\text{\LaTeX}$  document suitable for homework assignments and projects in Math 520. Be sure to look at the .tex document and not just the PDF!! There is a wealth of information in the .tex document as well as a few proof-writing tips that cannot be seen in the PDF. You can use the .tex document as a template for assignments that you turn in. First be sure to put your name and the assignment above. The first few “problems” will explain how the document works. Then there will be some “problems” that illustrate various notations and formatting environments. Compare what is in the typeset version of this document to the file `latexsample.tex`. Note in particular that anything typed after a percent sign in the text file is treated as a comment and is ignored by the compiler. Comments in the text file refer both to  $\text{\LaTeX}$  and to hints about writing good solutions and proofs.
  - (a) To list things in numerical order, use the “enumerate” command. This can be nested as shown.
  - (b) To list things that are not in numerical order, you can use the “itemize” command.
    - If you don’t put anything after the item, it will look like this with a bullet.

**Problem 1.3.1** If you want to put something else, other than a number, letter or bullet, use the description command, and put what you want in square brackets.

**Problem 1.3.2** This is what you should use if you need numbers that aren’t in numerical order, such as homework or project exercises.
  - (c) Don’t forget to end each enumerate and itemize loop.
  - (d) If you don’t, you will get an error when you compile.
2. The very basics of  $\text{\LaTeX}$  (compare the typeset document and the text file):

Extra spaces in the text file do not appear in the typeset document.

Except for a double carriage-return, that makes a new line. Single carriage returns don’t do anything.

3. Mathematical expressions are typed between dollar signs like this:  $y = x^2 + 1$ . To make a centered equation on its own line, use double dollar signs, like this:

$$y = x^2 + 1.$$

Double dollar signs also make things larger, like integral signs and fractions. Inline text with only one dollar sign give you this:  $\int_0^1 x^2$  But double dollar signs give you this.

$$\int_0^1 x^2$$

Fractions can be very hard to read inline.  $\frac{1}{2}x^2 - \frac{3}{4}x + \frac{1}{8}$ . To get the same effect as the double dollar signs, but without the centering, use the “displaystyle” command.

$$\frac{1}{2}x^2 - \frac{3}{4}x + \frac{1}{8}$$

4. (a) Many  $\text{\LaTeX}$  commands and math symbols start with a backslash symbol. For example,  $\sin x$  and  $\{x \in \mathbb{R} \mid x \geq 0\}$ . Notice that the set-notation parentheses need to have backslashes before them (while regular parentheses do not). This is because in  $\text{\LaTeX}$ , those squiggly parentheses often have other uses. In order to print a backslash, you need to use the *command* backslash, rather than the symbol!

- (b) If you need to put something in italics you do it *like this*. Or maybe you need to have something in **boldface**. Or maybe ***both***.
- (c) Don't forget to end each of your environments. In other words, don't forget your `\end{enumerate}`, or you will get a compiling error. Also make sure to end dollar sign environments and parentheses.
5. Here is some random notation you might need:

$x_2, x_{25}, x^2, x^{25}, \pm 4, x \neq 17, x > 5, x < 5, x \geq 5, x \leq 5, \{1, 2, 3\}, \{x \mid \sqrt{x} > 2\}, \infty.$

$A \subset B, A \subseteq B, A \not\subset B, A \not\subseteq B, A \setminus B, A^c, A \cap B, A \cup B, x \in A, x \notin A, |A|, \mathcal{P}(A), \emptyset.$

$\frac{5}{1+x}, \frac{5}{1+x}, \bigcap_{i=1}^n S_i, \bigcap_{i=1}^n S_i, \bigcup_{i=1}^n S_i, \bigcup_{i=1}^n S_i, \sum_{k=1}^{10} a_k, \sum_{k=1}^{10} a_k, \prod_{k=1}^{10} a_k, \prod_{k=1}^{10} a_k.$

$\mathbb{R}, \mathbb{Q}, \mathbb{Z}, \mathbb{N}, \clubsuit, \diamond, \heartsuit, \spadesuit, \rightarrow, \leftarrow, \leftrightarrow, \longrightarrow, \longleftarrow, \longleftrightarrow, \Rightarrow, \Leftarrow, \Leftrightarrow, \implies, \impliedby, \iff, \mapsto, \mapsto.$

$\mathcal{P}, \mathcal{S}, \mathcal{F}, \forall, \exists, \vee, \wedge, \neg, \sim, \approx, \equiv, \times, *, \star, |, a|b, |x|, \|x\|, \lceil x \rceil, \lfloor x \rfloor, \{x \in \mathbb{Z} \mid x \text{ is prime}\}.$

$\gcd, \text{lcm}, \binom{n}{k}, \binom{n+1}{k}, a = \binom{n+1}{k}, <, \leq, >, \geq, f: [0, \infty) \rightarrow \mathbb{R}, f \circ g, \{, \}, \$, \%, \&, -, \#.$

$\triangle ABC, \circ, 90^\circ, \overline{AB}, \overrightarrow{AB}, \overleftarrow{AB}$

6. You write in tables by hand by using the `vspace` command to leave space. But in case you are interested in making a table, this is how to do it (Truth table from Math 245):

$P$	$Q$	$P \wedge Q$	$\neg(P \wedge Q)$	$\neg Q$	$\neg(P \wedge Q) \wedge \neg Q$
T	T	T	F	F	F
T	F	F	T	T	T
F	T	F	T	F	F
F	F	F	T	T	T

Since the truth-values for  $\neg Q$  and  $\neg(P \wedge Q) \wedge \neg Q$  are the same for all possible truth-values of  $P$  and  $Q$ , the two statements are logically equivalent.

7. In this problem will will prove a theorem two different ways, to illustrate two different formats for writing proofs with steps and reasons clearly written out.

- (a) Prove that  $n^3 + n$  is even for every integer  $n$ .

*Proof.* Suppose  $n$  is any integer. We will examine two cases:

If  $n$  is even, then  $n = 2k$  for some  $k \in \mathbb{Z}$ , and therefore:

$$\begin{aligned} n^3 + n &= (2k)^3 + (2k) && \text{(since } n = 2k\text{)} \\ &= 8k^3 + 2k \\ &= 2(4k^3 + k). && \text{(factor out a 2)} \end{aligned}$$

Since  $4k^3 + k \in \mathbb{Z}$ , this means  $n^3 + n$  is divisible by 2 and therefore is even.

On the other hand, suppose  $n$  is odd. Then  $n = 2k + 1$  for some  $k \in \mathbb{Z}$ , and thus:

$$\begin{aligned} n^3 + n &= (2k + 1)^3 + (2k + 1) && \text{(since } n = 2k + 1\text{)} \\ &= (8k^3 + 12k^2 + 6k + 1) + (2k + 1) && \text{(multiply out)} \\ &= 8k^3 + 12k^2 + 8k + 2 \\ &= 2(4k^3 + 6k^2 + 4k + 1). && \text{(factor out a 2)} \end{aligned}$$

Once again,  $n^3 + n$  is a multiple of 2 and therefore is even. □

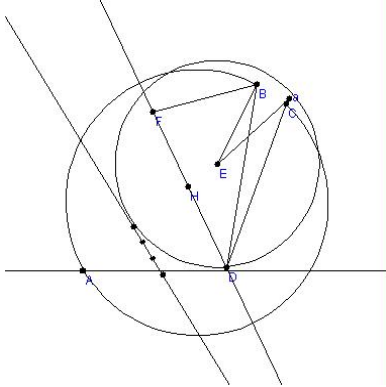
(b) Prove that  $n^3 + n$  is even for every integer  $n$ .

*Proof.* Note that  $n^3 + n = n(n^2 + 1)$ . It suffices to show that for any  $n \in \mathbb{Z}$ , either  $n$  is even or  $n^2 + 1$  is even. (Since then the product of  $n$  and  $n^2 + 1$  will have to be even.) For any integer  $n$  we have:

$$\begin{aligned} n \text{ is even} &\iff n^2 \text{ is even} && \text{(Theorem 2.1.9)} \\ &\iff n^2 + 1 \text{ is odd.} \end{aligned}$$

Therefore if  $n$  is even,  $n^2 + 1$  is odd; and if  $n$  is odd, then  $n^2 + 1$  is even. (The second implication is the contrapositive of the backwards part of the “if-and-only-if” statement: If  $n^2 + 1$  is odd, then  $n$  is even.) In any case, one of  $n$  or  $n^2 + 1$  must be even, and therefore  $n^3 + n = n(n^2 + 1)$  must be even.  $\square$

8. A **GREAT** reference for  $\text{\LaTeX}$  is *LaTeX User's Guide and Reference Manual* by Leslie Lamport. There are many, many other things you can do with  $\text{\LaTeX}$  including diagrams and importing pictures. I am not an expert on this, but play around with it, and you can create some amazing things. To import a picture into your document, use the “includegraphics” command. Make sure that the file you want to import is in the same directory as the document that you are compiling. Here is a diagram from Geometry Explorer.



You will need to create your own image to show up here.

Thanks again to Laura Taalman for assistance with this document.