



Stock valuation

A reading prepared by
Pamela Peterson Drake

OUTLINE

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1. Valuation of common stock

"[A] stock is worth the present value of all the dividends ever to be paid upon it, no more, no less. The purchase of a stock represents the exchange of present goods for future goods..."

John Burr Williams, *The Theory of Investment Value*, p. 80.

When you buy a share of common stock, it is reasonable to figure that what you pay for it should reflect what you expect to receive from it -- return on your investment. What you receive are cash dividends in the future. How can we relate that return to what a share of common stock is worth?

The value of a share of stock should be equal to the present value of all the future cash flows you expect to receive from that share of stock:¹

$$\text{Price of a share of stock} = \frac{\text{First period's dividends}}{(1 + \text{discount rate})^1} + \frac{\text{Second period's dividends}}{(1 + \text{discount rate})^2} + \frac{\text{Third period's dividends}}{(1 + \text{discount rate})^3} + \dots$$

Because common stock never matures, today's value is the present value of an infinite stream of cash flows. Another complication is that common stock dividends are not fixed, as in the case of preferred stock.² Not knowing the amount of the dividends -- or even if there will be future dividends -- makes it difficult to determine the value of common stock.

¹ The cash flows that are valued are the cash dividends.

² Preferred stock's dividends are generally fixed in amount, yet there remains uncertainty as to whether the dividends will be paid in the future because they are paid at the discretion of the company's board of directors.

Let D_t represent the dividend per share of common stock expected next period, P_0 represent the price of a share of stock today, and r_e the required rate of return on common stock. The **required rate of return** is the return shareholders demand to compensate them for the time value of money tied up in their investment and the uncertainty of the future cash flows from these investments. The required rate of return is the opportunity cost of the owners' capital.

The current price of a share of common stock, P_0 , is:

$$P_0 = \frac{D_1}{(1+r_e)^1} + \frac{D_2}{(1+r_e)^2} + \frac{D_3}{(1+r_e)^3} + \dots + \frac{D_\infty}{(1+r_e)^\infty}$$

which we can write using summation notation,

$$P_0 = \sum_{t=1}^{\infty} \frac{D_t}{(1+r_e)^t}.$$

So what are we to do? Well, we can grapple with the valuation of common stock by looking at its current dividend and making assumptions about any future dividends it may pay.

A. The dividend valuation model

If dividends are constant forever, the value of a share of stock is the present value of the dividends per share per period, in perpetuity. The summation of a constant amount (that is, if $D_1 = D_2 = \dots = D_\infty = D$) discounted from perpetuity simplifies to:

$$P_0 = \frac{D}{r_e}.$$

This is generally the case for a preferred stock and is the case for some common stocks. If the current dividend is \$2 per share and the required rate of return is 10 percent, the value of a share of stock is:

$$P_0 = \frac{\$2}{0.10} = \$20.$$

Stated another way, if you pay \$20 per share and dividends remain constant at \$2 per share, you will earn a 10 percent return per year on your investment every year. A problem in valuing common stock, however, is that the amount of cash dividends often changes through time.

If dividends grow at a constant rate, the value of a share of stock is the present value of a growing cash flow. Let D_0 indicate this period's dividend. If dividends grow at a constant rate, g , forever, the present value of the common stock is the present value of all future dividends is:

$$P_0 = \frac{D_0(1+g)}{(1+r_e)^1} + \frac{D_0(1+g)^2}{(1+r_e)^2} + \frac{D_0(1+g)^3}{(1+r_e)^3} + \dots + \frac{D_0(1+g)^\infty}{(1+r_e)^\infty}$$

Pulling today's dividend, D_0 , from each term,

$$P_0 = D_0 \left[\frac{(1+g)}{(1+r_e)^1} + \frac{(1+g)^2}{(1+r_e)^2} + \frac{(1+g)^3}{(1+r_e)^3} + \dots + \frac{(1+g)^\infty}{(1+r_e)^\infty} \right]$$

Using summation notation:

Cash dividend v. Stock dividends

Cash dividends are distributions of cash to shareholders in proportion to the ownership interest. We generally report cash dividends on the basis of the **dividend per share (DPS)** (i.e., dividends divided by earnings available to shareholders).

Stock dividends, which are the issuance of additional shares of stock in proportion to existing holdings, does not involve cash and does not affect the value of a share of stock directly. There is some evidence that stock dividends may provide a signal about the future prospects of the company, but in general stock dividends should not affect the current value of the stock other than reducing the stock's value to reflect the additional shares outstanding (that is, the shareholder "pie" is now cut into more pieces after the stock dividend, though no cash has changed hands).

$$P_0 = D_0 \left[\sum_{t=1}^{\infty} \frac{(1+g)^t}{(1+r_e)^t} \right] \text{ which approaches } D_0 \left[\frac{(1+g)}{(r_e - g)} \right].$$

If we represent the next period's dividend, D_1 , in terms of this period's dividend, D_0 , compounded one period at the rate g ,

$$P_0 = \frac{D_0(1+g)}{(r_e - g)} = \frac{D_1}{(r_e - g)}$$

This equation is referred to as the **dividend valuation model** (DVM) or the **Gordon model**.³

Consider a firm expected to pay a constant dividend of \$2 per share, forever. If this dividend is capitalized at 10 percent, the value of a share is \$20:

$$P_0 = \frac{\$2}{0.10} = \$20$$

If, on the other hand, the dividends are expected to be \$2 in the next period and grow at a rate of 6 percent per year, forever, the value of a share of stock is \$50:

$$P_0 = \frac{\$2}{(0.10 - 0.06)} = \$50$$

Does this make sense? Yes: if dividends are expected to grow in the future, the stock is worth more than if the dividends are expected to remain the same. The stock's price will actually grow at the same rate as the dividend.

If today's value of a share is \$50, what are we saying about the value of the stock next year? If we move everything up one period, D_1 is no longer \$2, but \$2 grown one period at 6 percent, or \$2.12. Therefore, we expect the price of the stock at the end of one year, P_1 , to be \$53:

$$P_1 = \frac{\$2.12}{(0.10 - 0.06)} = \$53$$

At the end of two years, the price will be even larger:

$$P_2 = \frac{\$2.25}{(0.10 - 0.06)} = \$56.18$$

³ The model was first presented by Myron J. Gordon in "Dividends, Earnings and Stock Prices," *Review of Economics and Statistics*, May 1959, pp. 99-105.

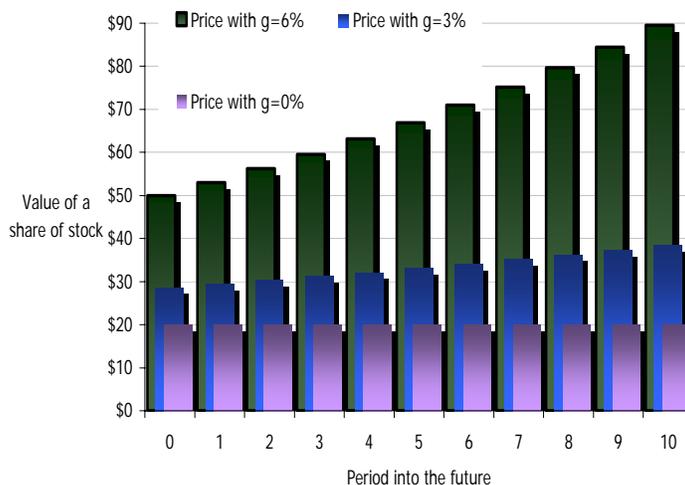
What is the growth in the price of this stock? From today to the end of one period, the price grew $\$53/\$50 - 1 = 6$ percent.⁴ From the end of the first period to the end of the second period, the price grew $\$56.18/\$53 - 1 = 6$ percent.

Because we expect dividends to grow each period, we also are expecting the price of the stock to grow through time as well. In fact, the price is expected to grow at the same rate as the dividends: 6 percent per period. For a given required rate of return and dividend -- in this case $r_e = 10$ percent and $D_1 = \$2$ -- we see that the price of a share of stock is expected to grow each period at the rate g .

We can see this in Exhibit 1, where the price of a stock with a dividend next period of \$2 is plotted over time for three different growth rates: 0 percent, 3 percent, and 6 percent.

Exhibit 1: Price per share over time using three different growth rates

$D_1 = \$2$
 $r_e = 10\%$
 $g = 0\%, 3\%, \text{ and } 6\%$



Example: Dividend valuation model

Problem

The Pear Company has a current dividend of \$3.00 per share. The dividends are expected to grow at a rate of 3% per year for the foreseeable future. If the current required rate of return on Pear Company stock is 10 percent, what is the price of a share of Pear common stock?

Solution

Given: $D_0 = \$3.00$; $g = 3\%$; $r_e = 10\%$

$D_1 = \$3.00 (1 + 0.03) = \3.09

$P_0 = \$3.09 / 0.07 = \mathbf{\$44.143}$

Example: Valuing a preferred share

Problem

If a preferred share has a \$25 par value, a dividend rate of 10.25 percent, and a required rate of return of 8 percent, what is its value?

Solution

Dividend = $D = \$25 (0.1025) = \2.5625

$P_0 = \$2.5625 / 0.08 = \mathbf{\$32.03}$

What if the dividends are expected to decline each year? That is, what if g is negative? We can still use the dividend valuation model, but each dividend in the future is expected to be less than the one before it. For example, suppose a stock has a current dividend of \$5 per share and the required rate of return is

⁴ Where did this formula come from? Consider $FV = PV (1 + i)^n$. If $n = 1$, then $FV = PV (1+i)$ and therefore $i = (FV/PV) - 1$.

10 percent. If dividends are expected to decline 3 percent each year, what is the value of a share of stock today? We know that $D_0 = \$5$, $r_e = 10\%$, and $g = -3\%$. Therefore,

$$P_0 = \frac{\$5(1 - 0.03)}{(0.10 + 0.03)} = \frac{\$4.85}{0.13} = \$37.31$$

Next period's dividend, D_1 , is expected to be \$4.85. We capitalize this at 13 percent: $10\% - (-3\%)$ or $10\% + 3\%$. What do we expect the price of the stock to be next period?

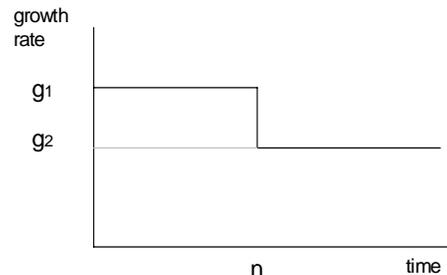
$$P_0 = \frac{\$5(1 - 0.03)^2}{(0.10 + 0.03)} = \frac{\$4.70}{0.13} = \$36.19$$

The expected price goes the same way as the dividend: down 3 percent each year.

B. Valuation of common stock with non-constant dividends

The dividend valuation model captures the valuation of stock whose dividends grow at a constant rate, as well as the valuation of stock whose dividends do not grow at all (that is, the perpetuity model, with $g = 0\%$). However, dividends may not be either a constant amount or have a constant growth. Dividends are declared by the company's board of directors. There is no obligation on the part of the board to pay dividends of any amount or periodicity. Dividends for a stock may simply not follow any discernable pattern or may not be paid at all.

We do observe that companies go through a life cycle, with fairly well-defined stages: high growth at the outset, then maturity, and, perhaps, decline. Accompanying these growth stages, the dividends of some companies grow not at a constant rate forever, but rather in stages. Any number of different future growth rates may be appropriate, depending on the company's circumstances.



To see how to value a stock whose dividends are neither constant or of constant growth, consider a stock whose dividends grow at two distinct rates: high growth initially and then maturing. We can use a modification of the dividend valuation model to capture two stages. In this case, we need to value the dividends that are expected in the first stage that grow at one rate, g_1 , and then the value of the dividends that are expected in the second stage, that grow at rate g_2 . If we assume that the second stage is the steady state forever, then the second stage is analogous to the dividend valuation model; that is, constant growth at g_2 .

Consider a share of common stock whose dividend is currently \$3 per share (that is, $D_0 = \$3$) and is expected to grow at a rate of 4 percent per year for three years (that is, $g_1 = 4\%$) and afterward at a rate of 2 percent per year after five years (that is, $g_2 = 2\%$). If the required rate of return is 10 percent, this becomes:

$$P_0 = \frac{\$3(1+0.04)}{(1+0.10)^1} + \frac{\$3(1+0.04)^2}{(1+0.10)^2} + \frac{\$3(1+0.04)^3}{(1+0.10)^3} + \frac{\$3(1+0.02)^4}{(1+0.10)^4} + \frac{\$3(1+0.02)^5}{(1+0.10)^5} + \dots$$

which is

$$P_0 = \frac{\$3.12}{(1+0.10)^1} + \frac{\$3.2448}{(1+0.10)^2} + \frac{\$3.37459}{(1+0.10)^3} + \frac{\$3.44208}{(1+0.10)^4} + \frac{\$3.51093}{(1+0.10)^5} + \dots$$

The present value of dividends received after the third year -- evaluated three years from today -- is the expected price of the stock in three years, P_3 , discounted to the present. The expected dividend in the fourth period, D_4 , is \$3.44208, so the price at the end of the third year is \$43.026:

$$P_3 = \frac{\$3.44208}{(0.10 - 0.02)} = \$43.026$$

$$P_0 = \frac{\$3.12}{(1+0.10)^1} + \frac{\$3.2448}{(1+0.10)^2} + \frac{\$3.37459}{(1+0.10)^3} + \frac{\$3.44208 / (0.10 - 0.02)}{(1+0.10)^3}$$

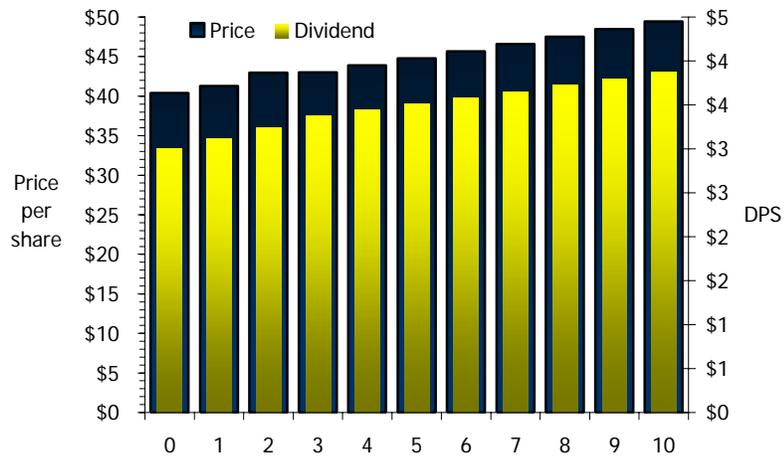
$$P_0 = \frac{\$3.12}{(1+0.10)^1} + \frac{\$3.2448}{(1+0.10)^2} + \frac{\$3.37459}{(1+0.10)^3} + \frac{\$43.026}{(1+0.10)^3}$$

$$P_0 = \$2.83636 + 2.68165 + \$2.53538 + \$32.32607 = \$40.37946$$

The value of a share of this stock today is \$40.37946, which is comprised of the present value of the dividends in the first three years (\$2.83636 + 2.68165 + 2.53538 = \$8.05339) and the present value of the dividends beyond three years, worth \$32.32607 today.

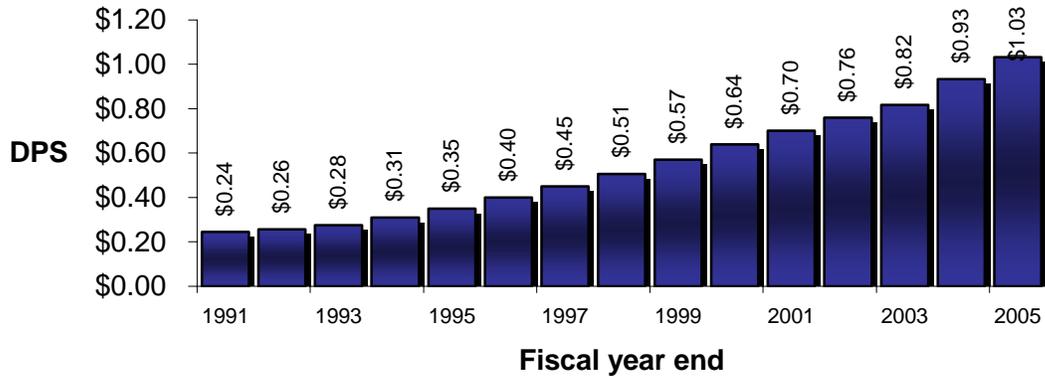
The price per share and dividends per share for the first ten years is as follows:

Period	Dividend per share	Price per share
0	\$3.00	\$40.38
1	\$3.12	\$41.30
2	\$3.24	\$42.96
3	\$3.37	\$43.03
4	\$3.44	\$43.89
5	\$3.51	\$44.76
6	\$3.58	\$45.66
7	\$3.65	\$46.57
8	\$3.73	\$47.50
9	\$3.80	\$48.45
10	\$3.88	\$49.42



Two-stage dividend example: Procter & Gamble, 1991-2005

Dividends per share (DPS) grew at an annual rate around 12-13% in the years 1993 through 2000, and slowed to 8-9% in the years 2001-04. The growth in dividends increased in 2005, which coincides with substantial restructuring of the company and the acquisition of Gillette.



Source of data: Mergent Online

As you can see, the valuation problem for a two-stage growth model is simply an extension of the financial mathematics that we used to value a stock for with a single growth rate. We can extend this approach further to cases in which there are three, four, or more different growth stages expected in a company's future.⁵

⁵ If there is no discernable pattern in terms of future growth rates, we are left with simply discounted an uneven series of expected dividends.

Example: Two-stage growth in dividends

Problem

A stock currently pays a dividend of \$2 for the year. Expected dividend growth is 20 percent for the next three years and then growth is expected to revert to 7 percent thereafter for an indefinite amount of time. The appropriate required rate of return is 15 percent. What is the value of this stock?

Solution

Cash flows:

Period	Cash flow
1	$CF_1 = D_1 = \$2 (1.20) = \2.40
2	$CF_2 = D_2 = \$2 (1.20)^2 = \2.88
3	$CF_3 = D_3 + P_3$ $D_3 = \$2 (1.20)^3 = \3.456 $P_3 = \$3.69792 / (0.15 - 0.07) = \46.224
	$CF_3 = \$3.456 + 46.224 = \49.68

Or, using equations:

$$P_0 = \frac{\$2.40}{(1+0.15)^1} + \frac{\$2.88}{(1+0.15)^2} + \frac{\$3.456}{(1+0.15)^3} + \frac{\$3.69792 / (0.15 - 0.07)}{(1+0.15)^3}$$

$$P_0 = \$2.086957 + 2.177694 + 2.272376 + 30.39303$$

$$P_0 = \$36.930057$$

C. Stock valuation and financial management decisions

We can relate the company's dividend policy to its stock value using the price-earning ratio in conjunction with the dividend valuation model.

Let's start with the dividend valuation model with constant growth in dividends:

$$P_0 = \frac{D_1}{r_e - g}$$

If we divide both sides of this equation by earnings per share, we can represent the dividend valuation model in terms of the price-earnings (P/E) ratio:

$$\frac{P_0}{EPS_1} = \frac{D_1 / EPS_1}{r_e - g} = \frac{\text{dividend payout ratio}}{r_e - g}$$

This tells us the P/E ratio is influenced by the dividend payout ratio, the required rate of return on equity, and the expected growth rate of dividends. For example, an increase in the growth rate of dividends is expected to increase the price-earnings ratio. As another example, an increase in the required rate of return is expected to decrease the price-earning ratio.

Another way of using this information is to estimate the required rate of return that is implied in a stock's current price. If we rearrange the last equation to solve for r_e ,

$$r_e = \frac{D_1 / \cancel{EPS_1}}{P_0 / \cancel{EPS_1}} + g = \frac{D_1}{P_0} + g,$$

we see that the required rate of return is the sum of the dividend yield and the expected growth rate.

Proctor & Gamble's Required Rate of Return

Using the equation:

$$r_e = \frac{D_1}{P_0} + g$$

and using current information on the dividend payout of 36 percent, a current dividend of \$1.03, a current stock price of \$59.56, and the expected growth rate of dividends of 11 percent, the required rate of return as of February 2006 is estimated as:

$$r_e = [\$1.03 (1.11) / \$59.56] + 0.11 = (\$1.1433 / \$59.56) + 0.11 = 0.0192 + 0.11 = 12.92\%$$

Source of data: Yahoo! Finance (stock price) and Mergent Online (dividend per share and earnings per share). The growth rate of dividends was estimated using the calculated 2004 to 2005 dividend growth rate.

2. Return on stocks

The return on stock is comprised of two components: (1) the appreciation (or depreciation) in the market price of the stock -- the **capital yield** -- and (2) the return in the form of dividends, -- the **dividend yield**:

$$\text{Return on stock} = \text{Capital yield} + \text{Dividend yield}.$$

If the stock does not pay dividends, then the entire return is the capital yield. If, on the other hand, the return is derived from both the change in the value of the stock and the cash flows from dividends, the return is more complicated.

A. Return with no dividends

Let's first ignore dividends. The return on common stock over a single period of time (e.g., one year) where there are no dividends is the change in the stock's price divided by the beginning share price:

$$\text{Return on a share of stock} = \frac{\text{End of period price} - \text{Beginning of period price}}{\text{Beginning of period price}} = \frac{P_1 - P_0}{P_0} = \frac{FV - PV}{PV}$$

If the period of time in which this spans is more than one year, we can determine the annual return using time value of money math,

where:

FV = Ending price

PV = Beginning price

n = number of years in the period

The annual return is calculated as:

$$\text{Annual return on a share of stock} = i = \sqrt[n]{\frac{FV}{PV}}$$

Solving for i gives us the average annual return, which is the geometric average return.

Let's see how this works. At the end of 1990, Multiclops stock was \$20 per share, and at the end of 2005 Multiclops stock was \$60 a share. The average annual return on Cyclops was:

$$\text{Return on Multiclops stock, 1990-2005} = \sqrt[15]{\frac{\$60}{\$20}} - 1 = \sqrt[15]{3} - 1 = 3^{1/15} - 1 = 7.599\%$$

TI-83/84 Using TVM Solver	HP10B
N = 15	20 +/- PV
PV = -20	40 FV
PMT = 0	15 n
FV = 40	i/YR
<i>Solve for i</i>	

Multiclops stock has an average (that is, geometric average) return of 7.599 percent per year.

B. Return with dividends at the end of the period

If there are no dividends, we simply compare the change in the price of the shares with the original investment to arrive at the return. However, if there are dividends, we need to consider them as cash inflows, as well as the change in the share's price, in determining the return. The simplest way to calculate the return is to assume that dividends are received at the end of the period:

Example: To simplify our analysis, let's ignore our stock broker's commission. Suppose we bought 100 shares of Purple Computer common stock at the end of 2003 at \$35.25 per share. We have invested $100 \times \$35.25 = \$3,525$ in Purple stock. During 2003, Purple Computer paid \$0.45 per share in dividends, so we earned \$45.00 in dividends. If we sold the Purple shares at the end of 2003 for 43 (\$43.00 per share, or \$4,300.00 for all 100 shares), what was the return on our investment? It depends on when the dividends were received. If we assume that the dividends were all received at the end of 2003, our return is calculated comparing the stock's appreciation and dividends to the amount of the investment:

$$\text{Return on Purple Computer stock} = (\$4,300.00 - 3,525.00 + 43.00) / \$3,525.00$$

$$\text{Return on Purple Computer stock} = 0.2321 \text{ or } 23.21 \text{ percent in 1990.}$$

We can break this return into its capital yield (that is, appreciation in the value of the stock) and dividend yield components:

$$\text{Return on Purple Computer stock} = \frac{\$4,300 - 3,525}{\$3,525} + \frac{\$43}{\$3,525} = 21.99\% + 1.22\% = 23.21\%$$

Most of the return on Purple stock was from the capital yield, 21.99 percent -- the appreciation in the stock's price.

But like most dividend-paying companies, Purple does not pay dividends in a lump-sum at the end of the year, but rather pays dividends at the end of each quarter. If we want to be painfully accurate, we could calculate the return on a quarterly basis and then annualize.

Consider the case of the Green Computer company stock. Suppose you buy Green Computer stock at the end of 2002 for \$40 per share. And suppose that Green paid dividends of \$2 at the end of 2003, \$3 at the end of 2004, no dividend in 2005, and \$2.5 at the end of 2006. And suppose you sell the stock at the end of 2006 for \$50. What is your return on Green Computer stock? To determine this, we first translate this information into cash flows and then determine the return, the internal rate of return. Be sure to combine the two cash flows that occur at the end of 2006: $\$2.50 + \$50 = \$52.50$.

End of period	Cash flow	The return is the internal rate of return on this set of cash flows, which is 10.11433 percent.
2002	-\$40.00	
2003	+\$2.00	
2004	+\$3.00	
2005	+\$0.00	
2006	+\$52.50	

TI-83/84	HP10B
{2,3,0,52.50} STO <i>listname</i> IRR(-40, <i>listname</i>)	40 +/- CF _i 2 CF _i 3 CF _i 0 CF _i 52.5 CF _i IRR

You will notice in many cases that the dividend yield is calculated by simply taking the ratio of the annual dividend to the beginning period price. The dividend yield quoted in the *Wall Street Journal*, for example, is the ratio of next year's expected dividend to today's share price. Whereas these short-cuts are convenient, remember that the true return should take into account the time value of money. This is especially important when you are considering large dividends relative to the stock price or when reinvestment rates are high.

Whereas these short-cuts are convenient, remember that the true return should take into account the time value of money. This is especially important when you are considering large dividends relative to the stock price or when reinvestment rates are high.

In the preceding example, we assumed that you sold the investment at a specific point in time, realizing the capital appreciation or depreciation in the investment -- that is, actually getting cash. But we can also think about a return without actually selling the investment. What if you didn't sell the Purple stock at the end of 2003? You would receive the dividends for 2003 whether or not you sold the stock at the end of the year, so you would have the dividend yield of 1.2 percent. Your investment would still have increased in value during the year, even if you didn't sell it. If you don't sell the Purple stock, you still have a capital yield for the year, it's just not *realized*. A capital gain on a stock you haven't sold is what many refer to as a "paper gain".

You can see that we can compute returns on investments whether or not we have sold them. In the cases where we do not sell the asset represented in the investment, we compute the capital yield (gain or loss) based on the market value of the asset at the point of time we are evaluating the investment.

It becomes important to consider whether or not we actually realize the capital yield only when we are dealing with taxes. We must pay taxes on the capital gain only when we realize it. As long as you don't sell the asset, we are not taxed on its capital appreciation.

3. Summary

The valuation of common stocks is difficult because you must value a future cash flow stream that is uncertain with respect to both the amount and the timing. However, understanding that stocks' dividends exhibit patterns helps us manage the valuation of these securities.

Investors are constantly valuing and revaluing common stocks as expectations about future cash flows change, whether this is the timing and amount or the uncertainty associated with these expected future cash flows. Though they may not each have the dividend valuation model, or some variation, in their head, we assume that they are rational and will value a stock according to the best estimates regarding the risks and rewards from investing in the stock.