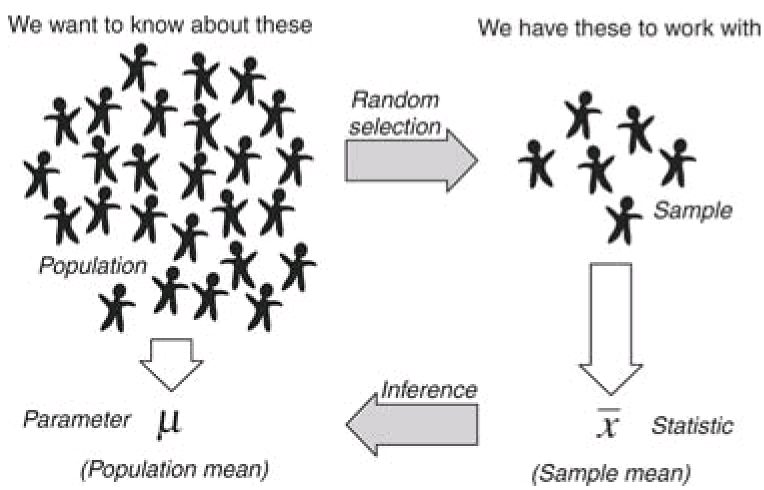
Inference

Statistical inference uses sample statistics to make decisions and predictions about population parameters.



In this course we are primarily interested to make inference about two population parameters:
population mean (**μ**) using the statistic $\bar{x}$ and
population proportion (***p***) using the statistic $\hat{p}$ **.**

Two types of statistical inference: (i) **estimation** and (ii) **test of hypothesis**.

Estimation

A **point estimate** is a single number that is the "best guess" obtained using sample data for the **parameter**.
An **interval estimate** is an interval of numbers computed using sample data, within which the **parameter** value is believed to fall.

To construct a confidence interval for a population parameter, we need to know the sampling distribution of the corresponding sample statistic.

Test of hypothesis

Test of hypothesis is a procedure for using sample data to decide between two competing claims (hypotheses) about a population parameter.

Test statistic = $\dfrac{\text{(sample statistic – hypothesized value of the parameter)}}{\text{standard error of the sample statistic}}$

In a statistical test, rejecting or failing to reject a claim depends on “how strong is the sample evidence”.
To determine the strength of the sample evidence, we need to know the sampling distribution of the corresponding sample statistic.

Central limit theorem from chapter 6 (section 6.3 & 6.4) gives us sampling distributions for two statistics: $\bar{x}$ and $\hat{p}$ **.**

Sampling Distribution of $\bar{x}$

Consider a population with mean **μ** and standard deviation **σ**, take ***k*** different random samples of size ***n*** from that population. The distribution of these ***k*** sample means is known as the sampling distribution of sample mean ($\bar{x}$ **).**
Sampling distribution of the sample mean ($\bar{x}$) has:

- mean **μ** (the same as the mean of the population).
- spread is described by the standard error, which equals the population standard deviation divided by the square root of the sample size: **σ/√n**.

If the population distribution is normal (or approximately normal), then the sampling distribution is **norma**l (or approximately normal) for **any** sample sizes. If the population distribution is non-normal or unknown, then the sampling distribution is approximately normal if sample size, $\boldsymbol{n \geq 30}$.

Sampling Distribution of $\hat{p}$

Consider a population with success proportion as ***p***, take ***k*** different random samples of size ***n*** from that population. The distribution of these ***k*** sample proportions is known as the sampling distribution of sample proportion ($\hat{p}$ **).**
Sampling distribution of sample proportion ($\hat{p}$ **)** has:

- mean = ***p*** and standard error $\sigma_{\hat{p}} = \sqrt{\dfrac{p(1-p)}{n}}$.
- If ***n*** is sufficiently large such that the expected numbers of outcomes of the two categories (success and failure), $\boldsymbol{np \geq 10}$ and $\boldsymbol{n(1-p) \geq 10}$, then this sampling distribution has a bell-shape. So sampling distribution of $\hat{p}$ is approximately normal.

Estimation

A **point estimate** is a single number that is the "best guess" obtained using sample data for the **parameter**.

Example: Each person in a random sample of 1100 "likely voters" (as defined by a professional polling organization) was questioned about his or her political views. Of those surveyed, 708 felt that "the economy's state" was the most urgent national concern.
Question: What is the point estimate of the population proportion of "likely voters" who thinks that "the economy's state" was the most urgent national concern?

The population proportion of likely voters, who felt that "the economy's state" was the most urgent national concern, is: p (the parameter).
The sample proportion of likely voters, who felt that "the economy's state" was the most urgent national concern, is: $\hat{p}$ (the statistic).

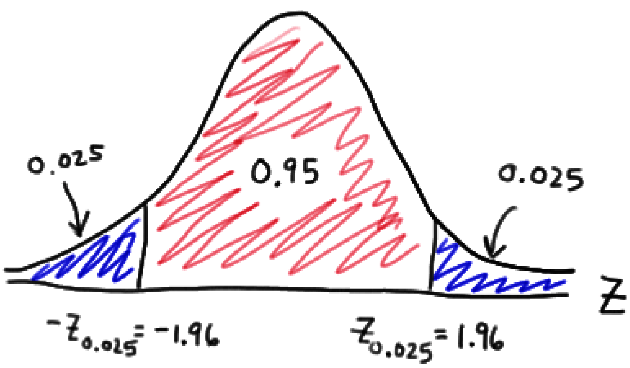
The point estimate of p is $\hat{p} = \frac{708}{1100} = 0.6436$.

A researcher is interested in a range of plausible values for the population proportion of "likely voters" instead of a single value (the point estimate) from the above example. So he decided to find an interval and chose the associated confidence level to be 95%.

The distribution of $\hat{p}$ is approximately normal because: $n\hat{p} = 707.96 > 10$ and $n(1 - \hat{p}) = 392.04 > 10$.
The 95% confidence interval of the population proportion is: $\hat{p} \pm$ *margin of error*.

Margin of error is: $m = (\sigma_{\hat{p}})Z_{\frac{\alpha}{2}} = 1.96\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$

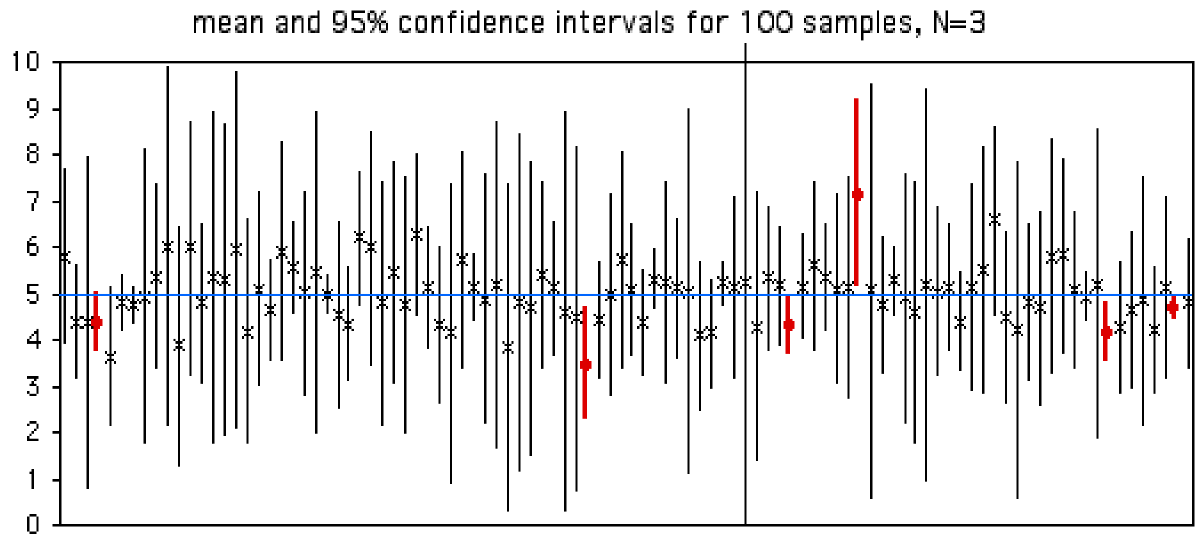
So the 95% C.I. is: $\hat{p} \pm 1.96\sqrt{\frac{\hat{p}(1-\hat{p})}{n}} = 0.6436 \pm (1.96) \times (.0144) = .6436 \pm .0282$



The 95% confidence interval is: $(0.6154 < p < 0.6718)$. We are 95% confident that the population proportion of likely voters, who felt that "the economy's state" was the most urgent national concern, is between 0.62 and 0.67.

Interpretation of Confidence Interval

The 95% confidence interval implies that, if 100 different random samples were selected from the same population and the interval estimation process is applied to each of these samples, out of the resulting 100 confidence intervals only 5 will miss the true parameter value.



In the above figure, the true parameter value is 5, represented by the blue line, the asterisks represent the sample statistic value for each interval, and red intervals are the ones that missed the true parameter value.

Test of hypothesis

A test of hypothesis or test of significance is a procedure for using sample data to decide between two competing claims (hypotheses) about a population parameter.

Null hypothesis is a statement that the parameter takes a specific value and it is denoted by $\mathbf{H_o}$.
Alternative hypothesis is a statement that the parameter takes some alternative value and it is denoted by $\mathbf{H_a}$.

Null hypothesis H_o: Parameter = Hypothesized value

Alternative hypothesis can have three different forms:
H_a: Parameter > Hypothesized value or
H_a: Parameter < Hypothesized value or
H_a: Parameter $\neq$ Hypothesized value

Test statistic is a quantity that measures the difference between the sample statistic and hypothesized value:

$$\text{Test statistic} = \frac{\text{(sample statistic – hypothesized value)}}{\text{standard error of the sample statistic}}$$

If there is a large difference between the sample evidence (sample statistic or point estimate) and the hypothesized value, the *probability that the H_o is true becomes very small*, so H_o should be rejected.
P(H_o is true) = small, how small a probability is required to reject the null hypothesis?
A probability of 5% or less is the commonly used criterion for rejecting the H_o. So if P(H_o is true) $\leq$ 0.05, we reject the H_o.
The probability that is used as the criterion for rejection is known as the "*level of significance*" for the test and it is denoted by α (lowercase Greek letter alpha).
The level of significance $\alpha = 0.05$ implies that H_0 is rejected when the test statistic value is only 5% likely to be as extraordinary as just seen, under the assumption that the null hypothesis is true.

***P*-value**

It is a measure of inconsistency between the hypothesized value of a parameter and observed sample statistic.

P-value is the probability, assuming that H_o is true, of obtaining a test statistic value at least as inconsistent with H_o as what actually observed.

H_o should be rejected if ***p*-value < α.**
H_o should not be rejected if ***p*-value ≥ α**.

Determination of P-value depends on the H_a.
(Let test statistic be denoted by z and its value by calculated z)

Steps in test of hypothesis:

1. Hypotheses: Specify the H_o and H_a, and also the *level of significance* (α).

2. Assumptions:
   a. Random sample
   b. Distribution shape (normal or approximately normal)

3. Compute the "Test statistic" value.

4. Determine the *p*-value (or critical value).

5. Conclusion: Based on the *p*-value (or critical value) reject or fail to reject the H_o.