Connected co-spectral graphs are not necessarily both Hamiltonian

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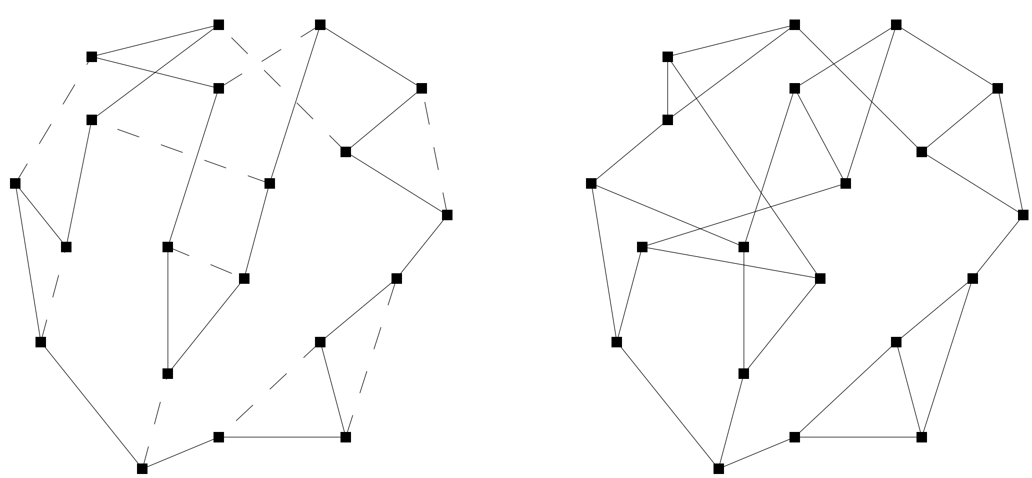
The spectrum of a graph is the set of eigenvalues of its associated adjacency matrix. Many important properties of a graph are related to its spectrum, as in for example Chung [1]. Connected graphs are ones where a path can be found between any pair of vertices. Co-spectral graphs are different graphs that have the same spectrum, and some of them can be constructed as in Godsil & McKay [2]. There are many examples of co-spectral graphs where one is connected while the other is not. Finally, a Hamiltonian cycle is a closed path that visits every vertex exactly once. Naturally, a graph needs to be connected to be Hamiltonian. We are not aware of any statements in the literature relating co-spectral graphs and Hamiltonicity.

The two graphs shown here are co-spectral with characteristic polynomial $432x + 3816x^2 + 7008x^3 - 7660x^4 - 28444x^5 - 3368x^6 + 40440x^7 + 19016x^8 - 28234x^9 - 19027x^{10} + 10744x^{11} + 9198x^{12} - 2226x^{13} - 2463x^{14} + 236x^{15} + 373x^{16} - 10x^{17} - 30x^{18} + x^{20}$. The left graph has a Hamiltonian cycle, while the right graph does not. One of the possible Hamiltonian cycles is shown as the solid lines, while dashed lines are used for arcs that are not part of the cycle. Thus the spectrum of a graph unfortunately does not contain enough information to decide whether a graph is Hamiltonian or not.



Note that these graphs are both cubic; each vertex is of degree 3. There are no co-spectral connected cubic graphs with less than twenty vertices, and about five thousand co-spectral pairs with twenty vertices. The vast majority of these are either both Hamiltonian or both not Hamiltonian, but there are five exceptions, including the set shown here.

The observation reported here was a biproduct of a more extensive study partially supported by the ARC grant DP0343028.

References

[1] F.R.K. Chung, *Spectral graph theory*, American Mathematical Society, Providence R.I., 1997.
[2] C.D. Godsil & B.D. McKay, Constructing cospectral graphs, *Aequationes Math.*, **25** (1982) 257–268.

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March 2005.