Which Dice Win At Chutes & Ladders, or "Chuteless & Ladderless"

Stephen Lucas* [1] Darren Glass [2]

[1]Department of Mathematics and Statistics, James Madison University, Harrisonburg VA

[2]Department of Mathematics, Gettysburg College, Gettysburg PA 17325

August 5 2019

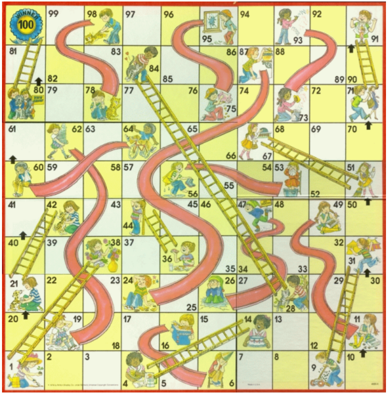
MOVES Conference

Outline

- The Games
- Past Work – Minimize Average Game Length
- Probability of Winning with Difference Dice
- Chutes & Ladders, One Die
- Chutes & Ladders, Multiple Dice
- Chuteless & Ladderless, One Die
- Chuteless & Ladderless, Multiple Dice
- Chuteless & Ladderless, Probability of Getting Stuck

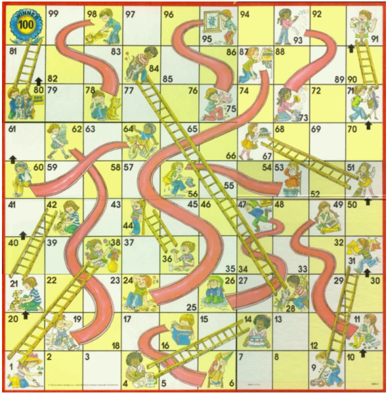
Chutes & Ladders

Chutes & Ladders is a classic game where you roll a die to move, and sometimes go down a chute (or snake) or up a ladder.



Chutes & Ladders

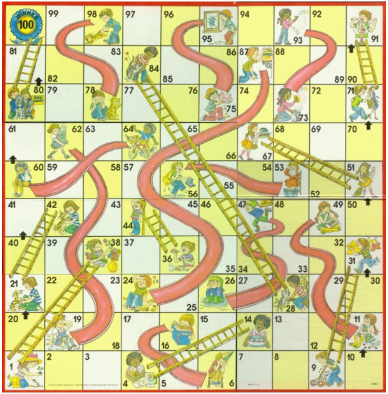
Chutes & Ladders is a classic game where you roll a die to move, and sometimes go down a chute (or snake) or up a ladder.



Snakes & Ladders originated in India (2nd century BC, AD, 13th century AD?).

Chutes & Ladders

Chutes & Ladders is a classic game where you roll a die to move, and sometimes go down a chute (or snake) or up a ladder.



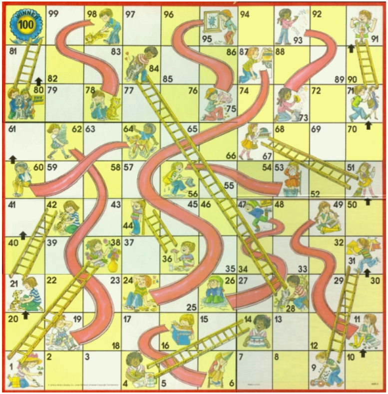
Snakes & Ladders originated in India (2nd century BC, AD, 13th century AD?).

Imported to Victorian Britain.

JAMES MADISON UNIVERSITY

Chutes & Ladders

Chutes & Ladders is a classic game where you roll a die to move, and sometimes go down a chute (or snake) or up a ladder.



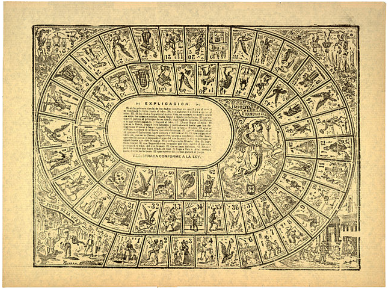
Snakes & Ladders originated in India (2nd century BC, AD, 13th century AD?).

Imported to Victorian Britain.

US version (children scared of snakes) by Milton Bradley, 1943.

Variants

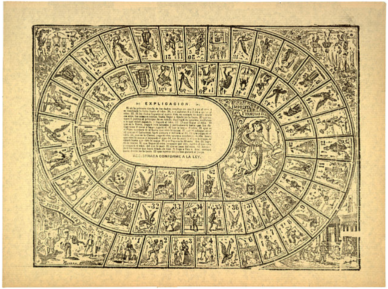
- Game of the Goose, originally 16th century Europe, played by Thomas Jefferson at Monticello.



Goose: move again, shortcuts, chutes, lose turns, back to beginning.

Variants

- Game of the Goose, originally 16th century Europe, played by Thomas Jefferson at Monticello.



Goose: move again, shortcuts, chutes, lose turns, back to beginning.

- "Chuteless & Ladderless" is Chutes & Ladders with no chutes and no ladders, allows for easier mathematical analysis.

JAMES MADISON UNIVERSITY

Past Work

- Chutes & Ladders was first modeled using Markov chains by Daykin, Jeacocke & Neal in 1967.

Past Work

- Chutes & Ladders was first modeled using Markov chains by Daykin, Jeacocke & Neal in 1967.
- Althoen, King & Schilling showed the average length of a game is 39.23 moves in 1993, and it has since become a standard linear algebra example.

Past Work

- Chutes & Ladders was first modeled using Markov chains by Daykin, Jeacocke & Neal in 1967.
- Althoen, King & Schilling showed the average length of a game is 39.23 moves in 1993, and it has since become a standard linear algebra example.
- Cheteyan, Hengeveld & Jones showed that the shortest average game length is 25.81 moves with a die of size 15 in 2001.

Past Work

- Chutes & Ladders was first modeled using Markov chains by Daykin, Jeacocke & Neal in 1967.
- Althoen, King & Schilling showed the average length of a game is 39.23 moves in 1993, and it has since become a standard linear algebra example.
- Cheteyan, Hengeveld & Jones showed that the shortest average game length is 25.81 moves with a die of size 15 in 2001.
- Glass, Lucas & Needleman showed that without chutes or ladders, the shortest average game length is 26 with a die of size 13. A six sided die requires 33.33 moves.

Is Minimizing Average Length of Game Best?

The probability distribution for number of moves required to finish turns out to be very long tailed.

Is Minimizing Average Length of Game Best?

The probability distribution for number of moves required to finish turns out to be very long tailed.

What if we actually want to win, not minimize number of moves?

Is Minimizing Average Length of Game Best?

The probability distribution for number of moves required to finish turns out to be very long tailed.

What if we actually want to win, not minimize number of moves?

What if we use two six sided dice instead of one?

Is Minimizing Average Length of Game Best?

The probability distribution for number of moves required to finish turns out to be very long tailed.

What if we actually want to win, not minimize number of moves?

What if we use two six sided dice instead of one? Does moving faster towards the end negate the lower chance of a move near the end reaching the last square, and the chance of not being able to finish at all?

Is Minimizing Average Length of Game Best?

The probability distribution for number of moves required to finish turns out to be very long tailed.

What if we actually want to win, not minimize number of moves?

What if we use two six sided dice instead of one? Does moving faster towards the end negate the lower chance of a move near the end reaching the last square, and the chance of not being able to finish at all? This question motivated this project.

JAMES MADISON UNIVERSITY

Markov Chains

A Markov chain is a sequence of random variables where the state of the random variable at some time t only depends on the value at the previous time $t-1$.

Markov Chains

A Markov chain is a sequence of random variables where the state of the random variable at some time t only depends on the value at the previous time $t-1$. Formally, $P(X_{t+1} = x | X_1 = x_1, X_2 = x_2, \ldots X_t = x_t) = P(X_{t+1} = x | X_t = x_t)$.

Markov Chains

A Markov chain is a sequence of random variables where the state of the random variable at some time t only depends on the value at the previous time $t-1$. Formally, $P(X_{t+1} = x | X_1 = x_1, X_2 = x_2, \ldots X_t = x_t) = P(X_{t+1} = x | X_t = x_t)$. Often, the probabilities are independent of time.

Markov Chains

A Markov chain is a sequence of random variables where the state of the random variable at some time t only depends on the value at the previous time $t-1$. Formally, $P(X_{t+1} = x | X_1 = x_1, X_2 = x_2, \ldots X_t = x_t) = P(X_{t+1} = x | X_t = x_t)$. Often, the probabilities are independent of time.

Given a finite number of possible states associated with $1, 2, \ldots, n$, the probability distribution satisfies

$$x^{(t+1)} = x^{(t)} P, \quad p_{ij} = P(X_{t+1} = j | X_t = i).$$

Markov Chains

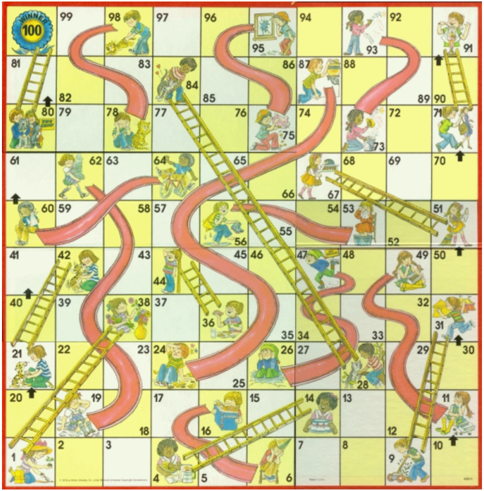
A Markov chain is a sequence of random variables where the state of the random variable at some time t only depends on the value at the previous time $t-1$. Formally, $P(X_{t+1} = x | X_1 = x_1, X_2 = x_2, \ldots X_t = x_t) = P(X_{t+1} = x | X_t = x_t)$. Often, the probabilities are independent of time.

Given a finite number of possible states associated with $1, 2, \ldots, n$, the probability distribution satisfies

$$x^{(t+1)} = x^{(t)} P, \quad p_{ij} = P(X_{t+1} = j | X_t = i).$$

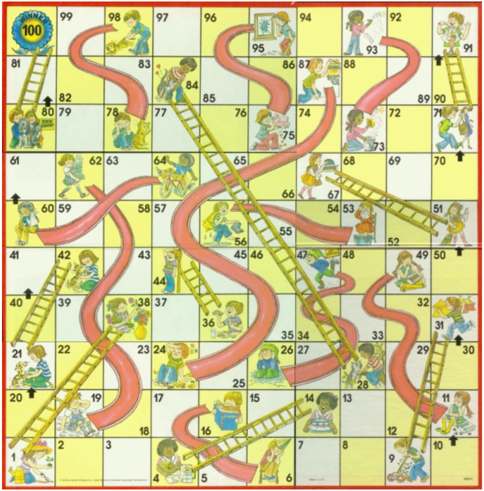
If $P = \begin{pmatrix} Q & R \\ \mathbf{0}^T & 1 \end{pmatrix}$, the first element of $(I_t - Q)^{-1}\mathbf{1}$ is the average number of steps.

Markov Chutes & Ladders



Initially, probability 1/6 at $(38, 2, 3, 14, 5, 6)$.

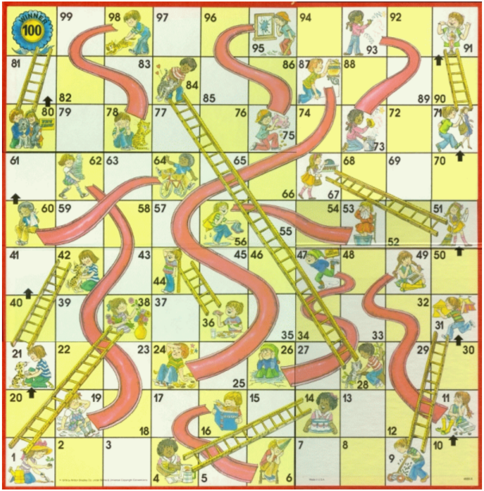
Markov Chutes & Ladders



Initially, probability $1/6$ at $(38, 2, 3, 14, 5, 6)$. Probability p at 48, next step probabilities $p/6$ added to $(11, 50, 66, 52, 53, 54)$.

JAMES MADISON UNIVERSITY

Markov Chutes & Ladders



Initially, probability $1/6$ at $(38, 2, 3, 14, 5, 6)$.
Probability p at 48, next step probabilities $p/6$ added to $(11, 50, 66, 52, 53, 54)$.
$\mathbf{x}^{(t+1)T} = \mathbf{x}^{(t)T} P$ with vectors of length 101.

JAMES MADISON UNIVERSITY

Chutes & Ladders Results

Calculate the probability distribution at every time step, look at proportion.

Chutes & Ladders Results

Calculate the probability distribution at every time step, look at proportion.

- Six sided die: fastest finish is 7 moves.

Chutes & Ladders Results

Calculate the probability distribution at every time step, look at proportion.

- Six sided die: fastest finish is 7 moves.
- 50%: 32 (mean 39),

Chutes & Ladders Results

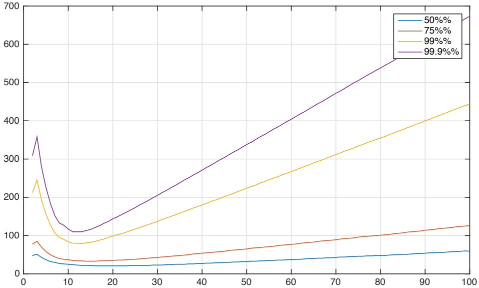
Calculate the probability distribution at every time step, look at proportion.

- Six sided die: fastest finish is 7 moves.
- 50%: 32 (mean 39), 75%: 50, 99%: 128, 99.9%: 184.

Chutes & Ladders Results

Calculate the probability distribution at every time step, look at proportion.

- Six sided die: fastest finish is 7 moves.
- 50%: 32 (mean 39), 75%: 50, 99%: 128, 99.9%: 184.
- Best die: Twelve sided (ish).



Cumulative Finished Distribution

Updating probabilities, $\mathbf{x}^{(t+1)\,T} = \mathbf{x}^{(t)\,T} P$ with $\mathbf{x}^{(0)\,T} = [1, 0, \ldots, 0]$.

Cumulative Finished Distribution

Updating probabilities, $\mathbf{x}^{(t+1)\,T} = \mathbf{x}^{(t)\,T} P$ with $\mathbf{x}^{(0)\,T} = [1, 0, \ldots, 0]$. Let $F(t) = x_{101}^{(t)}$, the cumulative probability of having reached the last square in at most t moves.

Cumulative Finished Distribution

Updating probabilities, $\mathbf{x}^{(t+1)\,T} = \mathbf{x}^{(t)\,T} P$ with $\mathbf{x}^{(0)\,T} = [1, 0, \ldots, 0]$. Let $F(t) = x_{101}^{(t)}$, the cumulative probability of having reached the last square in at most t moves. Continue until $F(k)$ is sufficiently close to one, say $1 - 10^{-8}$.

Cumulative Finished Distribution

Updating probabilities, $\mathbf{x}^{(t+1)\,T} = \mathbf{x}^{(t)\,T} P$ with $\mathbf{x}^{(0)\,T} = [1, 0, \ldots, 0]$. Let $F(t) = x_{101}^{(t)}$, the cumulative probability of having reached the last square in at most t moves. Continue until $F(k)$ is sufficiently close to one, say $1 - 10^{-8}$.

If two players use different sized dice, they will have different Markov matrices P_1 and P_2, and different cumulative finished distributions F_1 and F_2.

Cumulative Finished Distribution

Updating probabilities, $\mathbf{x}^{(t+1)\,T} = \mathbf{x}^{(t)\,T} P$ with $\mathbf{x}^{(0)\,T} = [1, 0, \ldots, 0]$. Let $F(t) = x_{101}^{(t)}$, the cumulative probability of having reached the last square in at most t moves. Continue until $F(k)$ is sufficiently close to one, say $1 - 10^{-8}$.

If two players use different sized dice, they will have different Markov matrices P_1 and P_2, and different cumulative finished distributions F_1 and F_2. Keep playing until both $F_1(t)$ and $F_2(t)$ are sufficiently close to one.

Cumulative Finished Distribution

Updating probabilities, $\mathbf{x}^{(t+1)\,T} = \mathbf{x}^{(t)\,T} P$ with $\mathbf{x}^{(0)\,T} = [1, 0, \ldots, 0]$. Let $F(t) = x_{101}^{(t)}$, the cumulative probability of having reached the last square in at most t moves. Continue until $F(k)$ is sufficiently close to one, say $1 - 10^{-8}$.

If two players use different sized dice, they will have different Markov matrices P_1 and P_2, and different cumulative finished distributions F_1 and F_2. Keep playing until both $F_1(t)$ and $F_2(t)$ are sufficiently close to one.

If a player uses multiple dice, the same approach with more complicated Markov matrices works.

Cumulative Finished Distribution

Updating probabilities, $\mathbf{x}^{(t+1)\,T} = \mathbf{x}^{(t)\,T} P$ with $\mathbf{x}^{(0)\,T} = [1, 0, \ldots, 0]$. Let $F(t) = x_{101}^{(t)}$, the cumulative probability of having reached the last square in at most t moves. Continue until $F(k)$ is sufficiently close to one, say $1 - 10^{-8}$.

If two players use different sized dice, they will have different Markov matrices P_1 and P_2, and different cumulative finished distributions F_1 and F_2. Keep playing until both $F_1(t)$ and $F_2(t)$ are sufficiently close to one.

If a player uses multiple dice, the same approach with more complicated Markov matrices works. But finished when at end or stuck.

Which Dice Win

Assuming moves are made simultaneously, player one wins on move k is they reach the last square on that move (probability $F_1(k) - F_1(k-1)$) and player two doesn't reach the final square up to move k (probability $1 - F_2(k)$).

Which Dice Win

Assuming moves are made simultaneously, player one wins on move k is they reach the last square on that move (probability $F_1(k) - F_1(k-1)$) and player two doesn't reach the final square up to move k (probability $1 - F_2(k)$). So

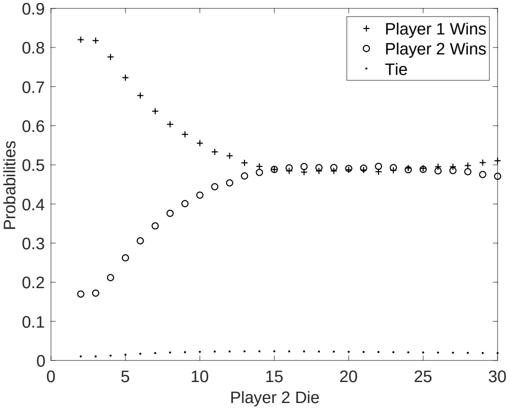
$$P(\text{Player 1 wins}) = \sum_{k=1}^{\infty} (F_1(k) - F_1(k-1))(1 - F_2(k)),$$

$$P(\text{Player 2 wins}) = \sum_{k=1}^{\infty} (F_2(k) - F_2(k-1))(1 - F_1(k)), \quad \text{and}$$

$$P(\text{Tie}) = \sum_{k=1}^{\infty} (F_1(k) - F_1(k-1))(F_2(k) - F_2(k-1)).$$

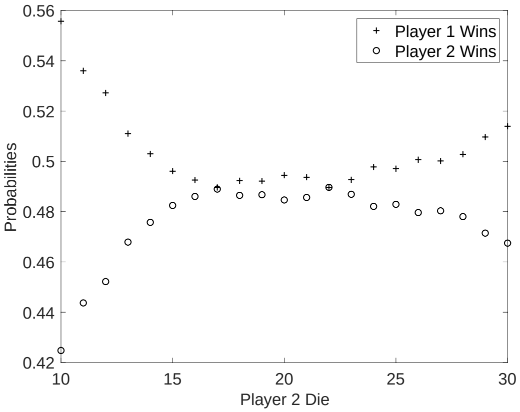
Chutes & Ladders, One Die

The minimum average number of moves uses a die of size 15. If player one uses a die of size 15 and player two uses a die of size two to thirty:



Chutes & Ladders, Best Single Die

The best die is size 22, second best is size 17. 22 vs 17: 0.48962 vs 0.48897, tie 0.02140.



Chutes & Ladders, One Die Versus Two Dice

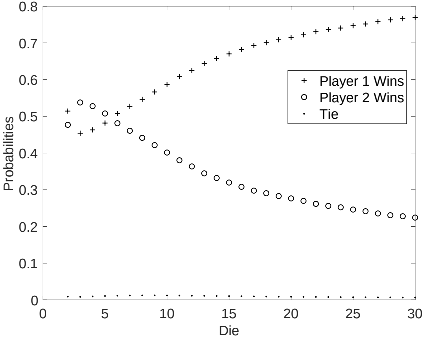
Two dice means we move more quickly, but could get stuck and finishing is less likely on any move near the end.

Chutes & Ladders, One Die Versus Two Dice

Two dice means we move more quickly, but could get stuck and finishing is less likely on any move near the end. One die versus two of the same kind:

Chutes & Ladders, One Die Versus Two Dice

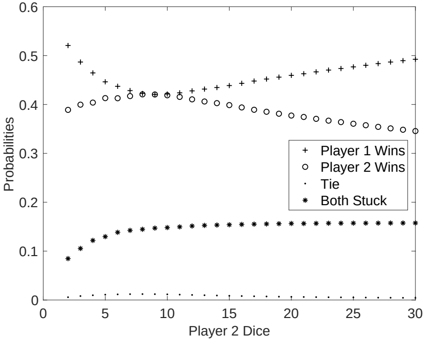
Two dice means we move more quickly, but could get stuck and finishing is less likely on any move near the end. One die versus two of the same kind:



Two dice are better with sizes $3, 4, 5$. Six sided, one die wins!

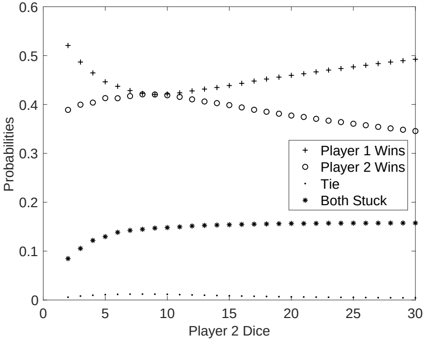
Chutes & Ladders, Best Two Dice

Both using two dice, nine sided is best, probability of both stuck is about 0.1471.



Chutes & Ladders, Best Two Dice

Both using two dice, nine sided is best, probability of both stuck is about 0.1471.



A single 22 sided die still beats every pair by substantial margins.

Chuteless & Ladderless, Board Length 100

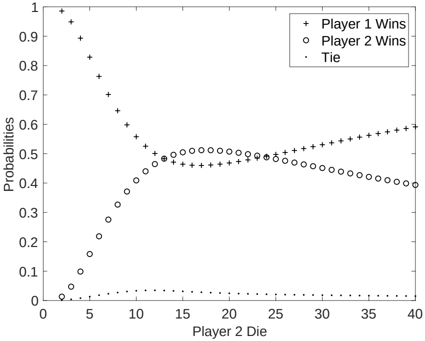
Without any chutes or ladders, we can easily vary the length of the board as well as the die size.

Chuteless & Ladderless, Board Length 100

Without any chutes or ladders, we can easily vary the length of the board as well as the die size. With length 100, minimum average game length uses die size 13.

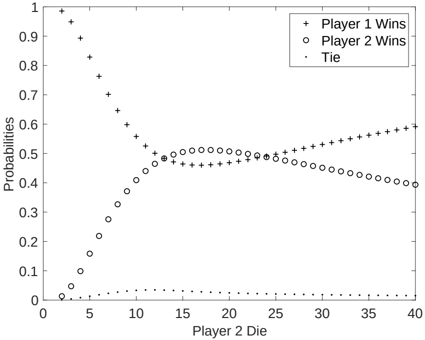
Chuteless & Ladderless, Board Length 100

Without any chutes or ladders, we can easily vary the length of the board as well as the die size. With length 100, minimum average game length uses die size 13.



Chuteless & Ladderless, Board Length 100

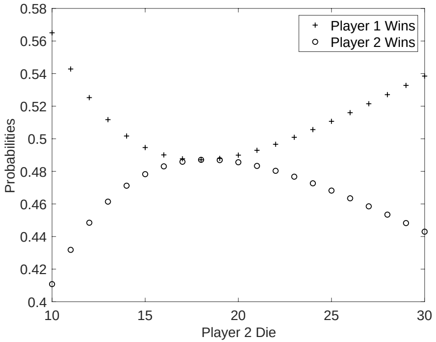
Without any chutes or ladders, we can easily vary the length of the board as well as the die size. With length 100, minimum average game length uses die size 13.



Fourteen through twenty three do better than thirteen.

Chuteless & Ladderless, Best Die, Board Length 100

Comparing all the possibilities, best die size is eighteen.

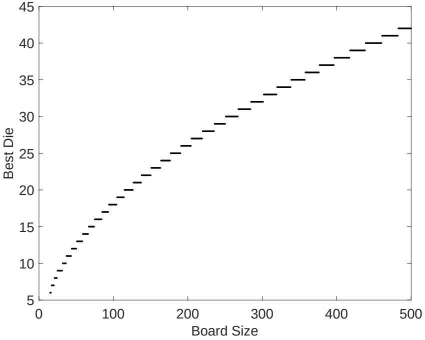


Chuteless & Ladderless, Best Die, Varying Board Length

For every board length (15 to 500), we can test each die against all others and find the one that wins most often.

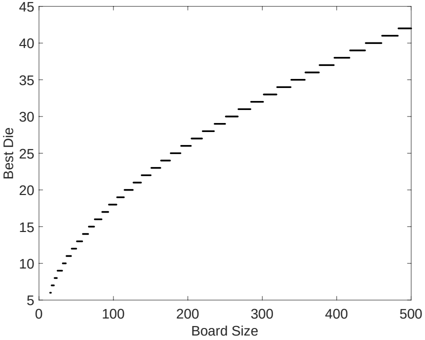
Chuteless & Ladderless, Best Die, Varying Board Length

For every board length (15 to 500), we can test each die against all others and find the one that wins most often.



Chuteless & Ladderless, Best Die, Varying Board Length

For every board length (15 to 500), we can test each die against all others and find the one that wins most often.



A simple fit suggests the best die is proportional to the square root of the board size.

Chuteless & Ladderless, Best Die, Small Board Length

For small boards,

p	d	p	d
5	5	11	6
6	3	12	6
7	4	13	$\star$
8	4	14	6
9	5	15	6
10	5	16	6

Chuteless & Ladderless, Best Die, Small Board Length

For small boards,

p	d	p	d
5	5	11	6
6	3	12	6
7	4	13	$\star$
8	4	14	6
9	5	15	6
10	5	16	6

For board length 13,
seven (0.4638) beats six (0.4631),

Chuteless & Ladderless, Best Die, Small Board Length

For small boards,

p	d	p	d
5	5	11	6
6	3	12	6
7	4	13	$\star$
8	4	14	6
9	5	15	6
10	5	16	6

For board length 13,
seven (0.4638) beats six (0.4631),
six (0.4580) beats five (0.4574),

Chuteless & Ladderless, Best Die, Small Board Length

For small boards,

p	d	p	d
5	5	11	6
6	3	12	6
7	4	13	$\star$
8	4	14	6
9	5	15	6
10	5	16	6

For board length 13,
seven (0.4638) beats six (0.4631),
six (0.4580) beats five (0.4574),
and five (0.4634) beats seven (0.4617).

Chuteless & Ladderless, Best Die, Small Board Length

For small boards,

p	d	p	d
5	5	11	6
6	3	12	6
7	4	13	$\star$
8	4	14	6
9	5	15	6
10	5	16	6

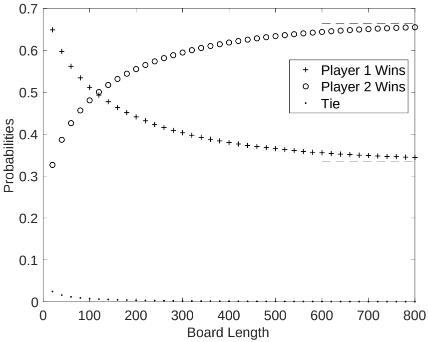
For board length 13,
seven (0.4638) beats six (0.4631),
six (0.4580) beats five (0.4574),
and five (0.4634) beats seven (0.4617).
Nontransitive!

Chuteless & Ladderless, one versus two six sided dice

Board length 100, one 0.511864, two 0.480613, tie 0.007523.

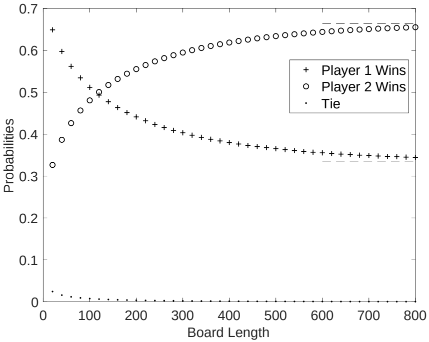
Chuteless & Ladderless, one versus two six sided dice

Board length 100, one 0.511864, two 0.480613, tie 0.007523.



Chuteless & Ladderless, one versus two six sided dice

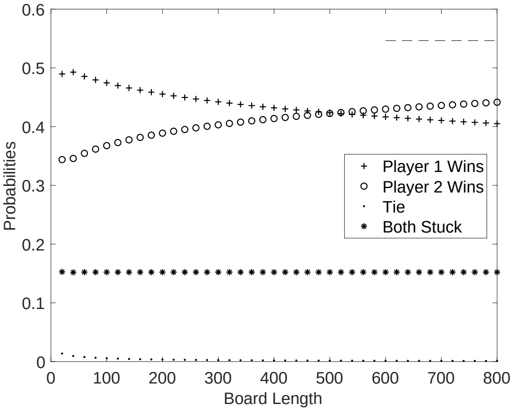
Board length 100, one 0.511864, two 0.480613, tie 0.007523.



One versus two, crossover at 116.
One versus three looks similar, crossover at 1279.

Chuteless & Ladderless, two versus three six sided dice

Crossover at 508



Combined Results

- Board length 10 to 115, one beats two beats three.

Combined Results

- Board length 10 to 115, one beats two beats three.
- Board length 116 to 507, two beats one beats three.

Combined Results

- Board length 10 to 115, one beats two beats three.
- Board length 116 to 507, two beats one beats three.
- Board length 1278 and up, three beats two beats one.

Combined Results

- Board length 10 to 115, one beats two beats three.
- Board length 116 to 507, two beats one beats three.
- Board length 1278 and up, three beats two beats one.
- Board length 508 to 1277, two beats one beats three beats two.

Combined Results

- Board length 10 to 115, one beats two beats three.
- Board length 116 to 507, two beats one beats three.
- Board length 1278 and up, three beats two beats one.
- Board length 508 to 1277, two beats one beats three beats two. Non-transitive!

Combined Results

- Board length 10 to 115, one beats two beats three.
- Board length 116 to 507, two beats one beats three.
- Board length 1278 and up, three beats two beats one.
- Board length 508 to 1277, two beats one beats three beats two. Non-transitive!
- Board length 890, two (0.658) beats one (0.341), one (0.521) beats three (0.478), and three (0.446) beats two (0.401).

Other Non-Transitive Examples

- A: 234499, B: 116688, C: 335577, A beats B beats C beats A all probabilities 5/9.

Other Non-Transitive Examples

- A: 234499, B: 116688, C: 335577, A beats B beats C beats A all probabilities 5/9.
- Two sided dice, board size 22 to 48, two beats one beats three beats two.

Other Non-Transitive Examples

- A: 234499, B: 116688, C: 335577, A beats B beats C beats A all probabilities 5/9.
- Two sided dice, board size 22 to 48, two beats one beats three beats two.
- Three sided dice, board size 19 to 51, non-transitive.

Other Non-Transitive Examples

- A: 234499, B: 116688, C: 335577, A beats B beats C beats A all probabilities 5/9.
- Two sided dice, board size 22 to 48, two beats one beats three beats two.
- Three sided dice, board size 19 to 51, non-transitive.
- Ten sided dice, non-transitive about four to seven thousand.

Other Non-Transitive Examples

- A: 234499, B: 116688, C: 335577, A beats B beats C beats A all probabilities 5/9.
- Two sided dice, board size 22 to 48, two beats one beats three beats two.
- Three sided dice, board size 19 to 51, non-transitive.
- Ten sided dice, non-transitive about four to seven thousand.
- Simulation with three players, length 890, one 0.178, two 0.398, three 0.423.

Chuteless & Ladderless, Two Dice Stuck

Adjusting the Markov chain approach to multiple absorbing states, we can find the probability of getting stuck.

Chuteless & Ladderless, Two Dice Stuck

Adjusting the Markov chain approach to multiple absorbing states, we can find the probability of getting stuck. Asymptotic as board size increases for given dice.

Chuteless & Ladderless, Two Dice Stuck

Adjusting the Markov chain approach to multiple absorbing states, we can find the probability of getting stuck. Asymptotic as board size increases for given dice.

Dice	Probability Stuck	Fraction
2d2	0.361111111111111	13/36
2d3	0.344907407407407	149/432
2d4	0.339423076923077	$\frac{29506}{87563}$
2d5	0.336968810916180	$\frac{34573}{102600}$
2d6	0.335688649974364	$\frac{317543}{945945}$
2d10	0.333990844573179	
2d20	0.333436370180405	
2d50	0.333341021092870	
2d100	0.333334348292267	

Chuteless & Ladderless, Two Dice Stuck

Adjusting the Markov chain approach to multiple absorbing states, we can find the probability of getting stuck. Asymptotic as board size increases for given dice.

Dice	Probability Stuck	Fraction
2d2	0.361111111111111	$13/36$
2d3	0.344907407407407	$149/432$
2d4	0.339423076923077	$\frac{29506}{87563}$
2d5	0.336968810916180	$\frac{34573}{102600}$
2d6	0.335688649974364	$\frac{317543}{945945}$
2d10	0.333990844573179	
2d20	0.333436370180405	
2d50	0.333341021092870	
2d100	0.333334348292267	

Darren has proven that as the die gets large, probability stuck $\to 1/3$.

Chuteless & Ladderless, Three Dice Stuck

With three large dice on a very long board (numerically), probability finishing approaches 6/11, second last square 3/11, third last square 2/11.

Chuteless & Ladderless, Three Dice Stuck

With three large dice on a very long board (numerically), probability finishing approaches 6/11, second last square 3/11, third last square 2/11.

Further numerical evidence suggests that if W_k is the probability of finishing with k big dice on a very long board, probabilities of getting stuck on squares $n-1, n-2, \ldots, n-k+1$ approach $W_k/2, W_k/3, \ldots, W_k/k$.

Chuteless & Ladderless, Three Dice Stuck

With three large dice on a very long board (numerically), probability finishing approaches 6/11, second last square 3/11, third last square 2/11.

Further numerical evidence suggests that if W_k is the probability of finishing with k big dice on a very long board, probabilities of getting stuck on squares $n-1, n-2, \ldots, n-k+1$ approach $W_k/2, W_k/3, \ldots, W_k/k$.

So $W_1 = 1, W_2 = 2/3, W_3 = 6/11, W_4 = 12/25, W_5 = 60/137$, ..., reciprocal of the Harmonic numbers.

Conclusion

- We have seen that the best dice for winning don't minimize the average length of a game.

Conclusion

- We have seen that the best dice for winning don't minimize the average length of a game.
- One six sided die is better than two in the classic game.

Conclusion

- We have seen that the best dice for winning don't minimize the average length of a game.
- One six sided die is better than two in the classic game.
- Choosing the right board length, we discover non-transitive behavior when playing Chuteless & Ladderless with one, two and three dice.

Conclusion

- We have seen that the best dice for winning don't minimize the average length of a game.
- One six sided die is better than two in the classic game.
- Choosing the right board length, we discover non-transitive behavior when playing Chuteless & Ladderless with one, two and three dice.
- We have proven probability of visiting a square only depends on die side averages, and asymptotic for two dice.

Conclusion

- We have seen that the best dice for winning don't minimize the average length of a game.
- One six sided die is better than two in the classic game.
- Choosing the right board length, we discover non-transitive behavior when playing Chuteless & Ladderless with one, two and three dice.
- We have proven probability of visiting a square only depends on die side averages, and asymptotic for two dice.
- Future work: Need to prove asymptotic result for k dice,

Conclusion

- We have seen that the best dice for winning don't minimize the average length of a game.
- One six sided die is better than two in the classic game.
- Choosing the right board length, we discover non-transitive behavior when playing Chuteless & Ladderless with one, two and three dice.
- We have proven probability of visiting a square only depends on die side averages, and asymptotic for two dice.
- Future work: Need to prove asymptotic result for k dice, what die sides are best on the standard board (at least one side one, no sides one hundred).

Conclusion

- We have seen that the best dice for winning don't minimize the average length of a game.
- One six sided die is better than two in the classic game.
- Choosing the right board length, we discover non-transitive behavior when playing Chuteless & Ladderless with one, two and three dice.
- We have proven probability of visiting a square only depends on die side averages, and asymptotic for two dice.
- Future work: Need to prove asymptotic result for k dice, what die sides are best on the standard board (at least one side one, no sides one hundred).

Thank You