Maximizing Output From Oil Reservoirs Without Water Breakthrough

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- Using variants on $\int_0^1 \frac{x^4(1-x)^4}{(1+x^2)}\,dx = \frac{22}{7} - \pi$, developed new series expansions for π where each term can add any number of digits of accuracy. Showed how $\int_0^1 \frac{x^m(1-x)^n(a+bx+cx^2)}{1+x^2}\,dx = (\pm)(z-\pi)$ to any accuracy with nonnegative integrand. In particular, $\displaystyle\int_0^1 \frac{x^8(1-x)^8(25+816x^2)}{3164(1+x^2)}\,dx = \frac{355}{113} - \pi$. *Accepted by the American Mathematical Monthly*

Current Work

- Placing a circle pack, *with C. Collins, K. Stephenson, UTenn*, **Numerical**
- Generalizing Love's problem, deformations of an elastic half-space, *with M. Bevis, Ohio*, **Applied**
- Game theoretic considerations for truels and gruels, *with D. Lanphier, W. Kentucky & J. Rosenhouse, JMU*, **Game theory**
- Generalized exponential distribution functions, *with H. Hamdan, JMU*, **Probability/Numerical**
- Convergence of numerical methods for an autocatalytic reaction, *with P. Warne, JMU*, **Applied/Numerical**
- Fractal structures in adjacency matrices for graphs, *with J. Filar, UniSA*, **Pure/Numerical**
- Bounded continued fraction and Pierce representations of reals and rationals, **Analysis**

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Other Potential Undergraduate Research Topics

- Improved bounds for the kissing problem: how many spheres can touch a central sphere, all of the same radius, in n-dimensions? Implement a new optimization algorithm. **Numerical**
- An integer-valued logistic equation: investigate the effect of assuming integer solutions only for this standard population model. Do we still see period doubling to chaos in the same way? **Applied/chaos theory**
- Analyze "Dreidel:" using a Markov chain approach find out what are your chances of winning, and how long a game lasts. **Probability theory**
- Sudoku uniqueness: build an efficient Sudoku solver, then use it to investigate the distribution of how many solutions there are for initial set-ups with various conditions. **Computational/Recreational**
- Nontransitive dice: find sets of dice where on average A beats B beats C beats A, and find what conditions are required. **Game theory**
- Minimal Goldbach Sets: what small sets of numbers appear to satisfy the Goldbach conjecture, like twin primes. **Numerical/number theory**

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- March 14 – π-day, **Formulas for π**: a wander through many of the various formulas for calculating π, most with easy to follow derivations.

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Outline

- Description of the problem
- Mathematical model
- Approximate position of surface
- Optimization methods
- Results
- Further directions

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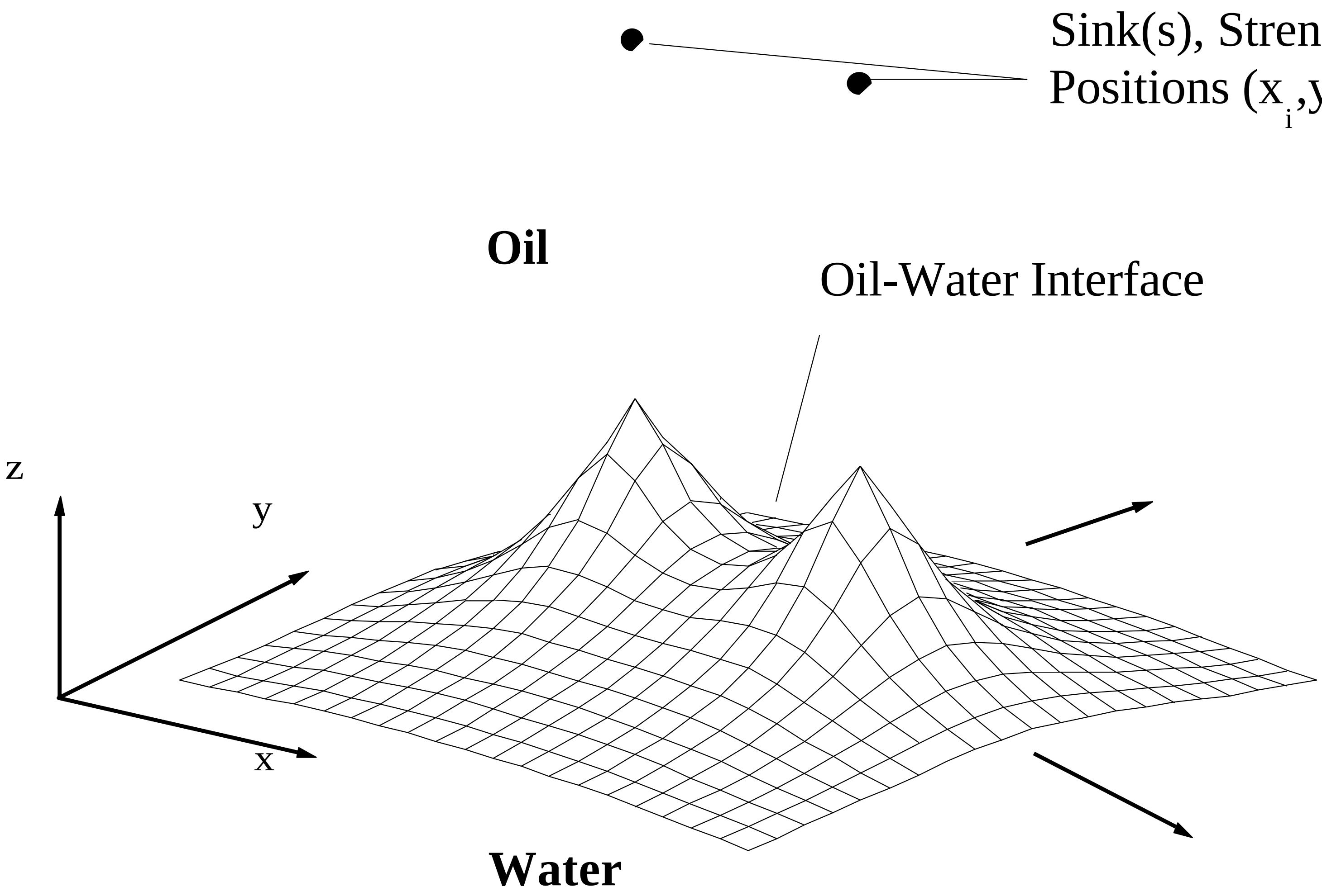
Geometry of the Model



Darcy's Law and Assumptions

$$\nabla \hat{p} = -\frac{\mu}{k}\mathbf{q}, \quad \text{with}$$

- μ - fluid viscosity
- k - permeability
- $\mathbf{q}$ - volume flux rate / unit area
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- Steady state problem

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- **u** and hence $\hat{p} \rightarrow 0$ at infinity, both fluids
- Suction pressure due to sink, flow rate m at (x', y', z')

$$p_s = \frac{-m\mu_{oil}}{4\pi k\sqrt{(x - x')^2 + (y - y')^2 + (z - z')^2}}$$

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Scaling

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- The dimensionless suction pressure becomes

$$\tilde{p}_s = \frac{-F}{4\pi\sqrt{(x-x')^2 + (y-y')^2 + (z-1)^2}}$$

where

$$F = \frac{m\mu_{oil}}{kz'^2(\rho_w - \rho_{oil})g} \sim \frac{\text{suction force}}{\text{density difference force}}.$$

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The Problem

We want to solve

$$\nabla^2 \tilde{p} = -\sum F_i \delta(\mathbf{x} - \mathbf{x_i'}),$$

with

$$\zeta + p = 0 \quad \text{on the lower boundary} \quad z = \zeta(x, y).$$

Given positions of sinks, what strengths maximize flow without the oil-water interface reaching the sinks? Call these critical values.

Past Work – Boundary Integral Method

For N sinks, each of strength F_i at positions (x_i', y_i', z_i'), we get

$$\frac{1}{2}\zeta(x_0,y_0) = \frac{1}{4\pi}\sum_{i=1}^{N}\frac{F_i}{[(x_0-x_i')^2+(y_0-y_i')^2+(\zeta(x_0,y_0)-z_i')^2]^{1/2}}+$$

$$\frac{1}{4\pi}\iint_{-\infty}^{\infty}\frac{\zeta\left(\frac{\partial\zeta}{\partial x}(x-x_0)+\frac{\partial\zeta}{\partial y}(y-y_0)-(\zeta(x,y)-\zeta(x_0,y_0))\right)}{[(x-x_0)^2+(y-y_0)^2+(\zeta(x,y)-\zeta(x_0,y_0))^2]^{3/2}}dx\,dy.$$

Solution Method

Solved by a form of pointwise iterative procedure:

- Initially approximate ζ by the small parameter expansion
- Solve at $(N+1) \times (N+1)$ points on a grid in the region

$$-x_{max} \leq x \leq x_{max}, \qquad -y_{max} \leq y \leq y_{max}$$

- In the far field, approximate the solution by the small parameter expansion
- Use some integration procedure to find the required integrals at each point, taking into account infinite extent and the singularity
- Use a bicubic spline interpolation for ζ values required between solution points

The Muskat Model

Balance suction pressure field (assuming half-plane solution) with gravitational restoring force.

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For single sink at height one,

$$\zeta(r) = \frac{F}{4\pi}\left[\frac{1}{\sqrt{(\zeta(r)-1)^2+r^2}} + \frac{1}{\sqrt{(\zeta(r)+1)^2+r^2}}\right].$$

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For N sinks at positions (x_i, y_i, z_i), $i = 1, \ldots, N$, surface is solution of

$$\zeta(x,y) = \sum_{i=1}^{N} \frac{F_i}{4\pi}\left[\frac{1}{r_i} + \frac{1}{r_i'}\right], \text{ with } \begin{aligned} r_i &= \sqrt{(z_i-\zeta(x,y))^2+(x-x_i)^2+(y-y_i)^2} \\ r_i' &= \sqrt{(z_i+\zeta(x,y))^2+(x-x_i)^2+(y-y_i)^2} \end{aligned}$$

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- Use secant method to solve for each (x, y), with previous solution as initial guess
- Care required in deciding on stability
- Solves extremely quickly, but is approximate

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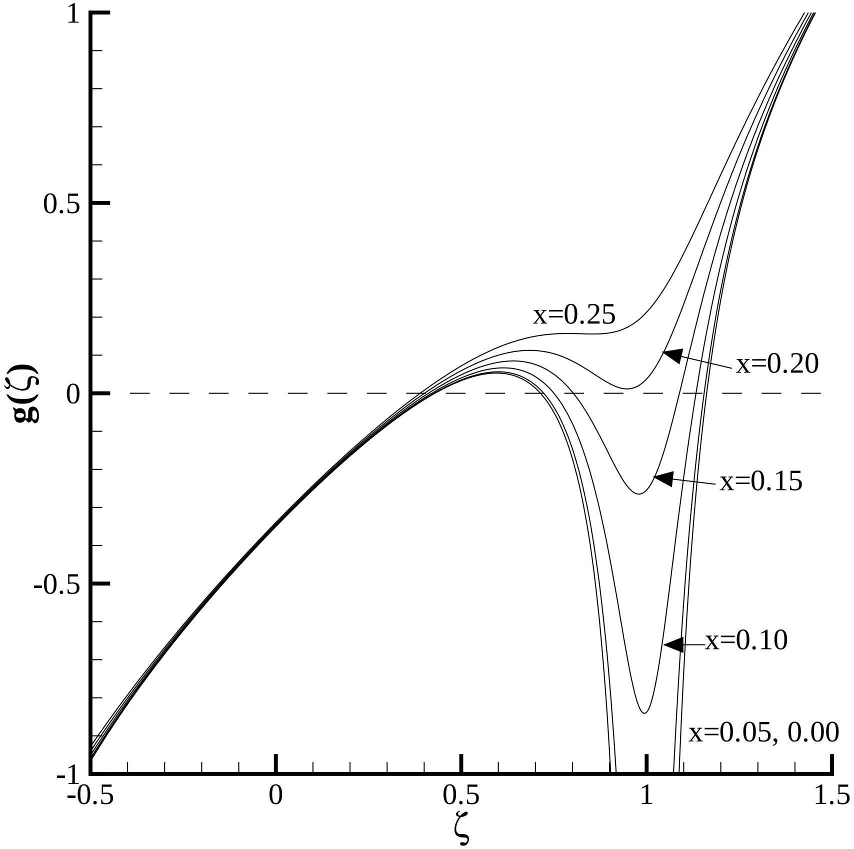
Stability – Multiple Solutions

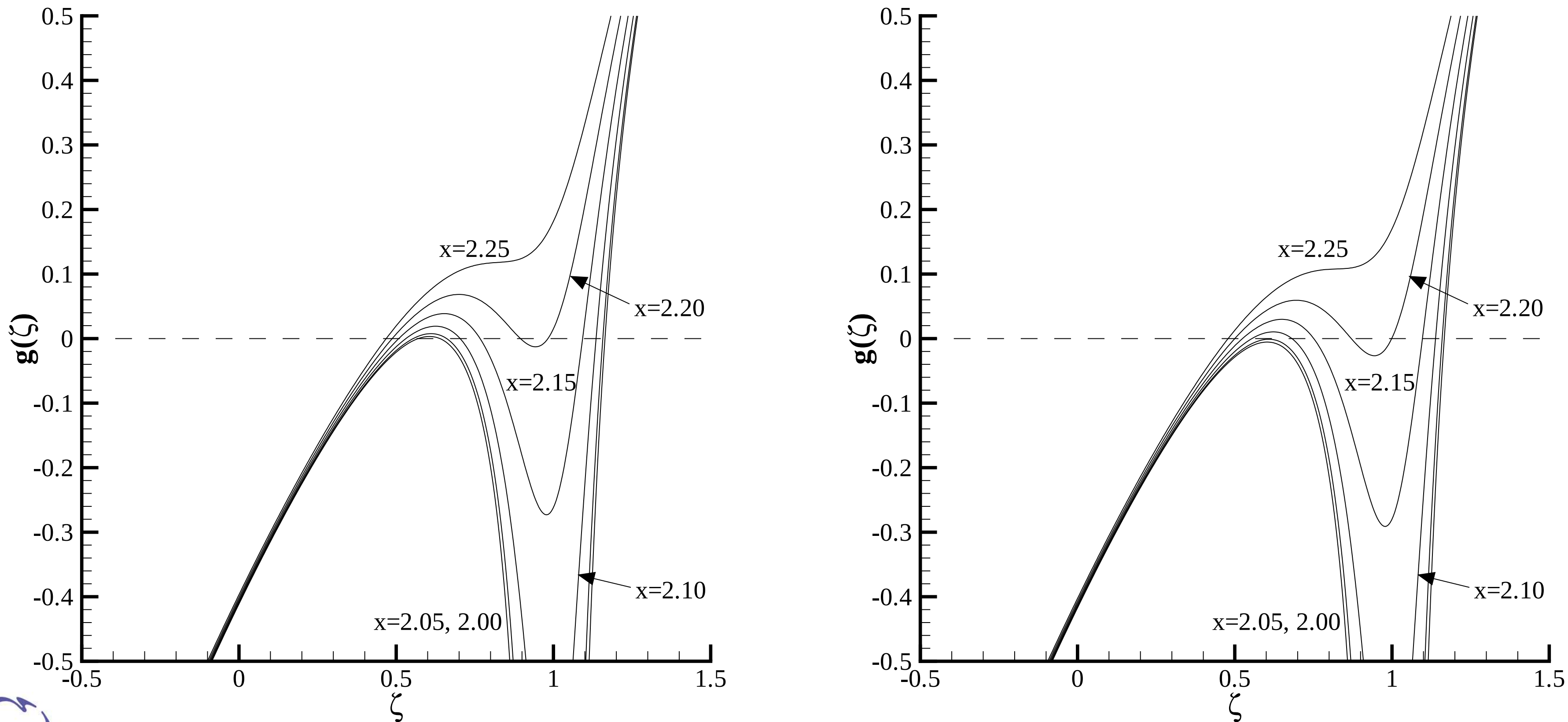
For a single sink strength 2.2 at (0,0,1), surface at x is the solution of

$$g(\zeta) = \frac{2.2}{4\pi}\left[\frac{1}{\sqrt{(\zeta-1)^2+x^2}}+\frac{1}{\sqrt{(\zeta+1)^2+x^2}}\right]-\zeta = 0.$$



Stability – Multiple Solutions

Two sinks at $(-2, 0, 1)$ and $(2, 0, 1)$, both $F = 2.08$ (left) and $F = 2.11$ (right):



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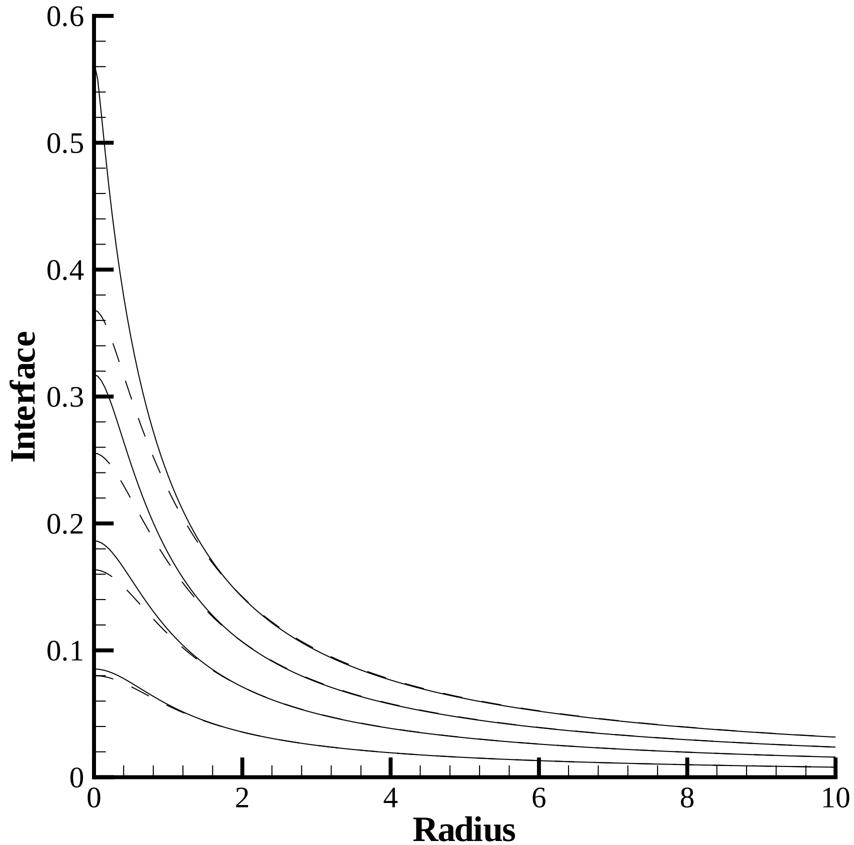
- When solving, start in far field and work in (mesh 0.005)
- Don't just evaluate under sink, maximum height or breakthrough can be elsewhere
- Use solution at previous point as next initial point
- Identify if sudden jumps or solution greater than one

Solution Error

The Muskat solution approximates the pressure field. Comparing Muskat and BIM solutions for a single sink at $(0, 0, 1)$, $F = 0.5, 1.0, 1.5, 2.0$:

Critical value of F (maximum with stable cone) is 2.05 for BIM, 2.418 for Muskat. Maximum heights are 0.661 and 0.5774 respectively.



Critical Sink Strengths – One Parameter

- n sinks equally spaced on a circle of a given radius.
- By symmetry, all sinks of same strength for maximum flow.
- Solve using bisection.

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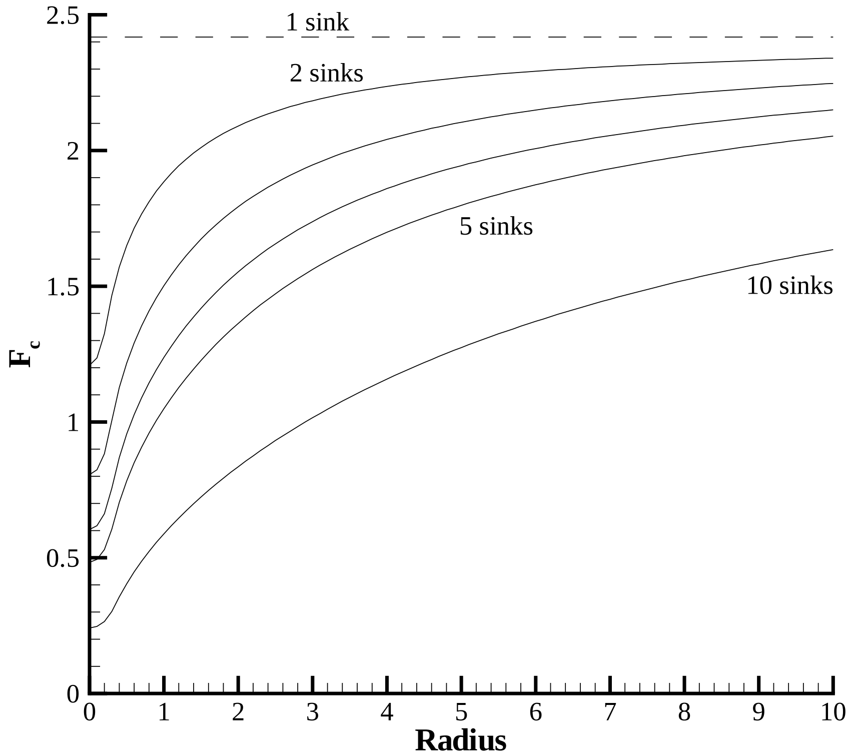
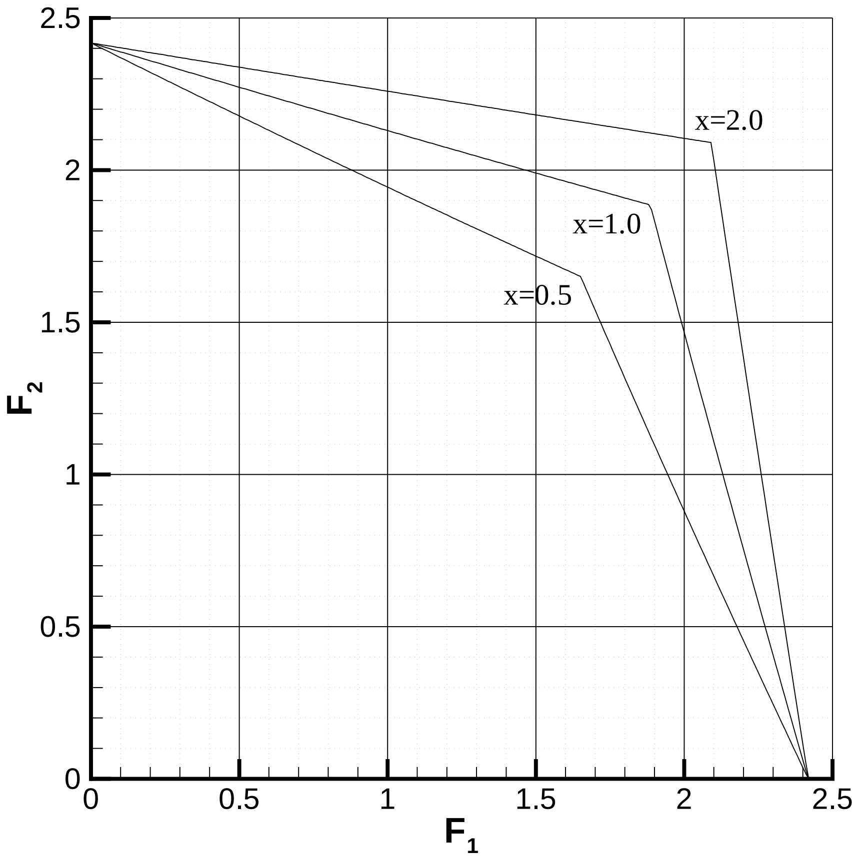


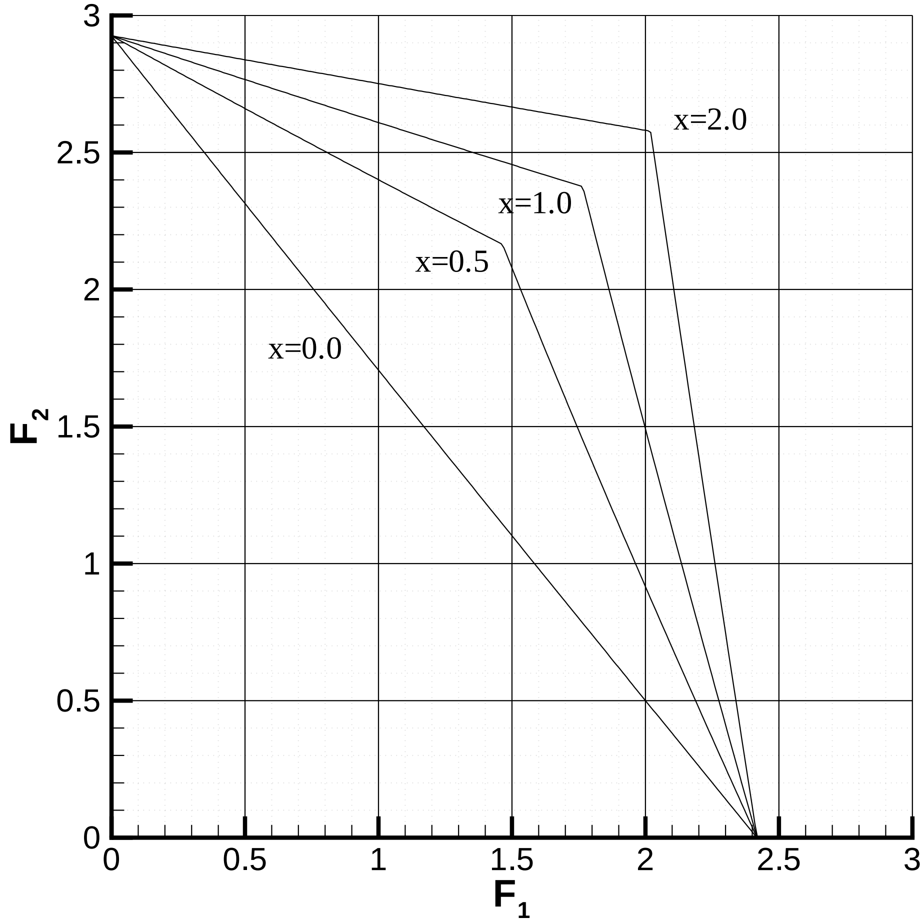
Feasible Regions – Two Sinks

- 2 sinks, F_1 at $(-x, 0, 1)$, F_2 at $(x, 0, 1)$, $x = 2, 1, 0.5$.
- For each F_1, find critical F_2 by bisection.



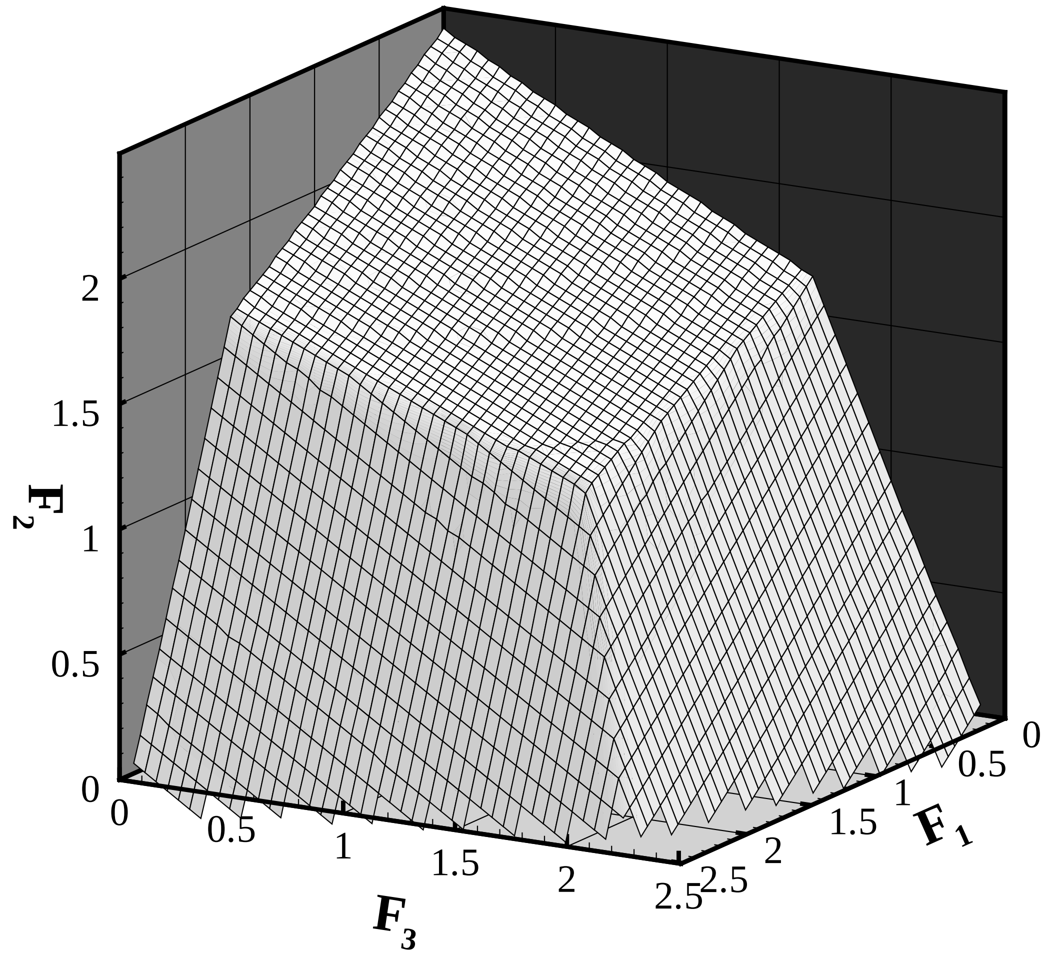
Feasible Regions – Two Skew Sinks

2 sinks, F_1 at $(-x, 0, 1)$, F_2 at $(x, 0, 1.1)$, $x = 2, 1, 0.5, 0.0$.



Feasible Region – Three Sinks Example

3 sinks, F_1 at $(-1, 0, 1)$, F_2 at $(0, 0, 1)$, F_3 at $(1, 0, 1)$.



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- Boundary is not smooth

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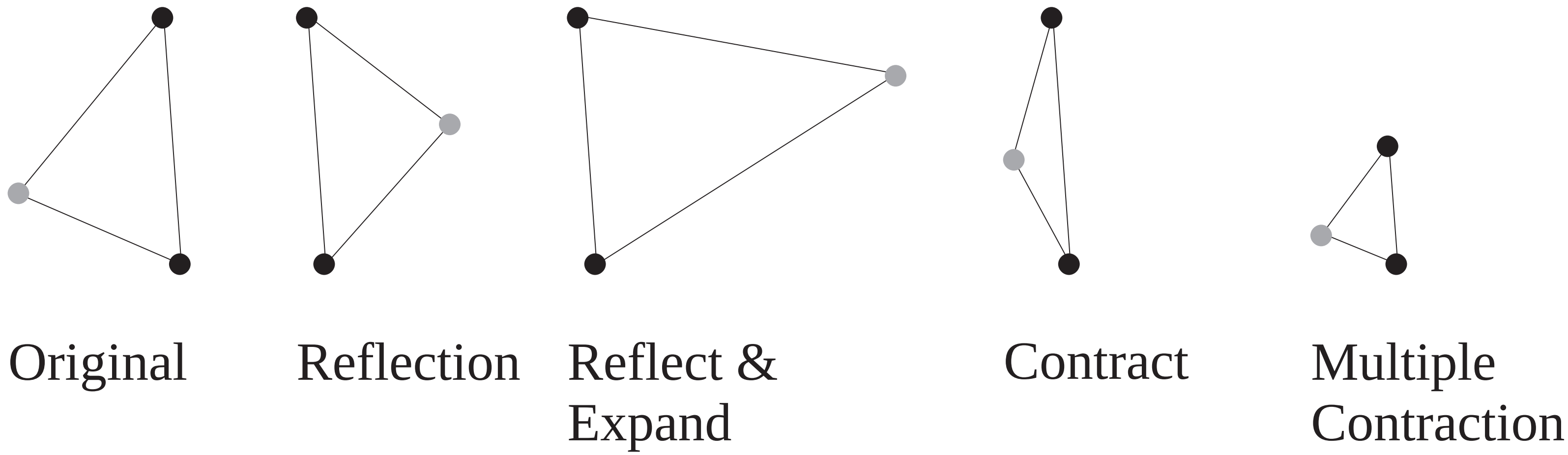

Constrained Nelder Mead Using Penalty Method

Minimize $G - \sum_{i=1}^{N} F_i$, where $G = \begin{cases} 0 & \text{in feasible region} \\ 1000 & \text{outside feasible} \end{cases}$

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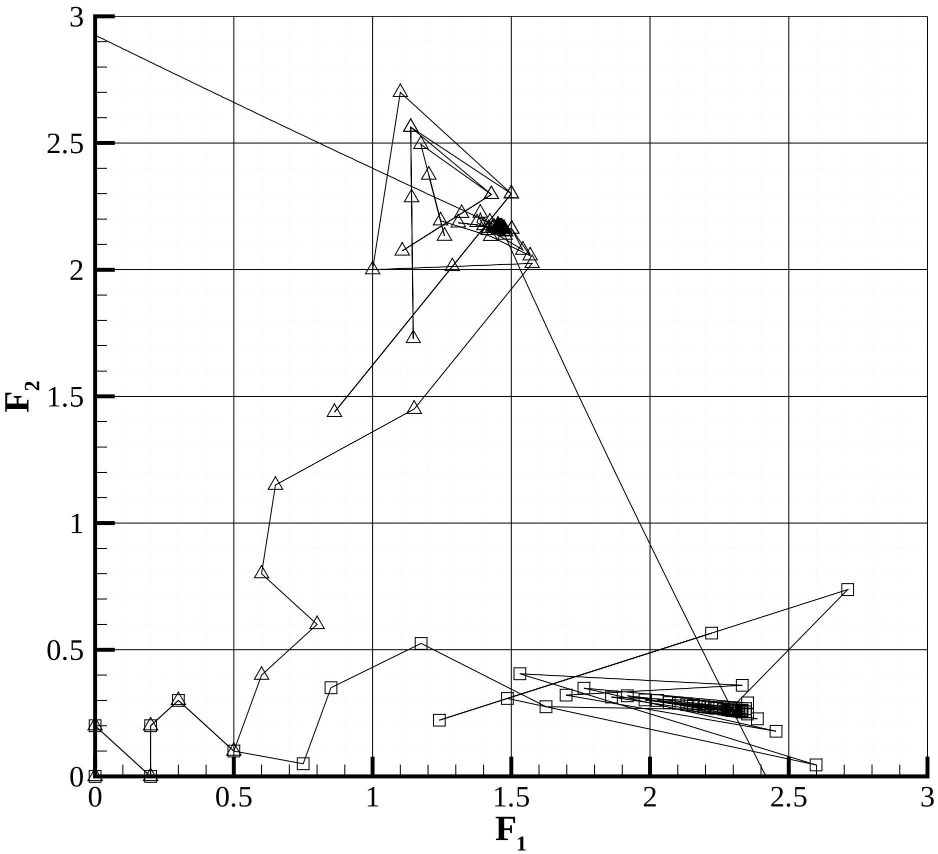
Doesn't deal with the discontinuity too well:



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Nelder Mead On Feasible Region Boundary

Given $F_1, F_2, \ldots, F_{n-1}$, calculate critical value for F_n, which is on the boundary. Then minimize $-\sum_{i=1}^{n} F_i$.

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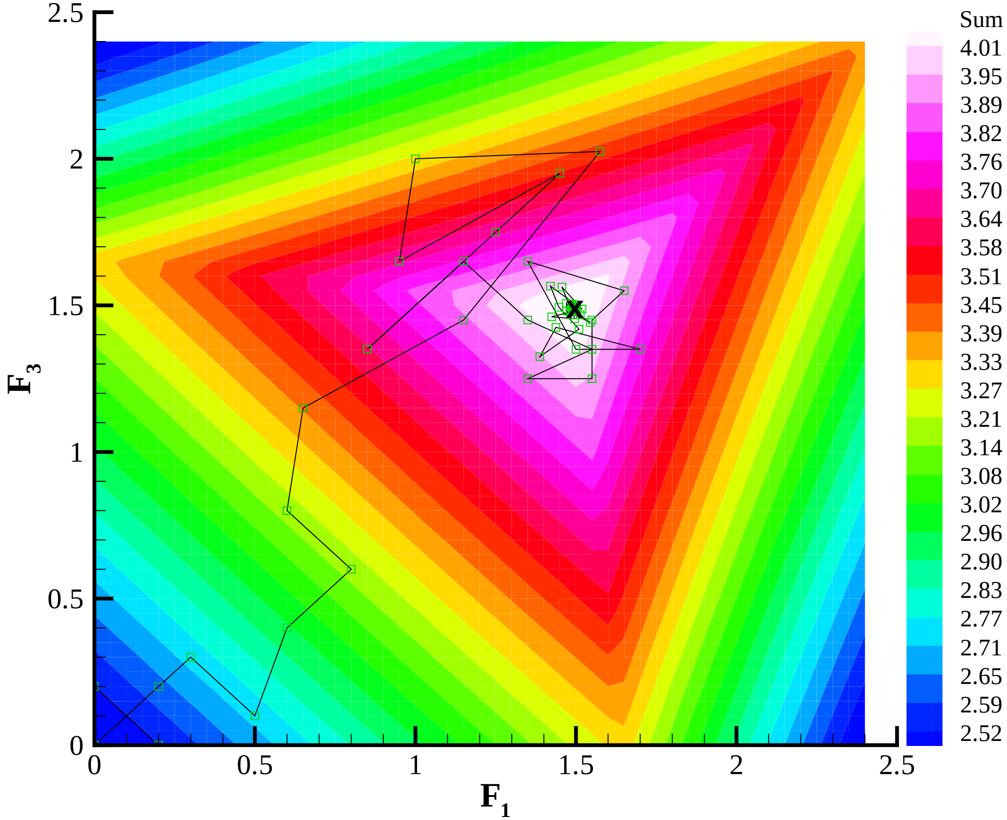
Works beautifully, 71 function evaluations with relative error 10^{-5} from (0,0):



Feasible Region – Three Sinks Again

3 sinks, F_1 at $(-x, 0, 1)$, F_2 at $(0, 0, 1)$, F_3 at $(x, 0, 1)$.

Using symmetry, assume $F_1 = F_3$, reduces to one parameter problem.

Find feasible regions with $x = 0.4, 0.5, \ldots, 1.0$.

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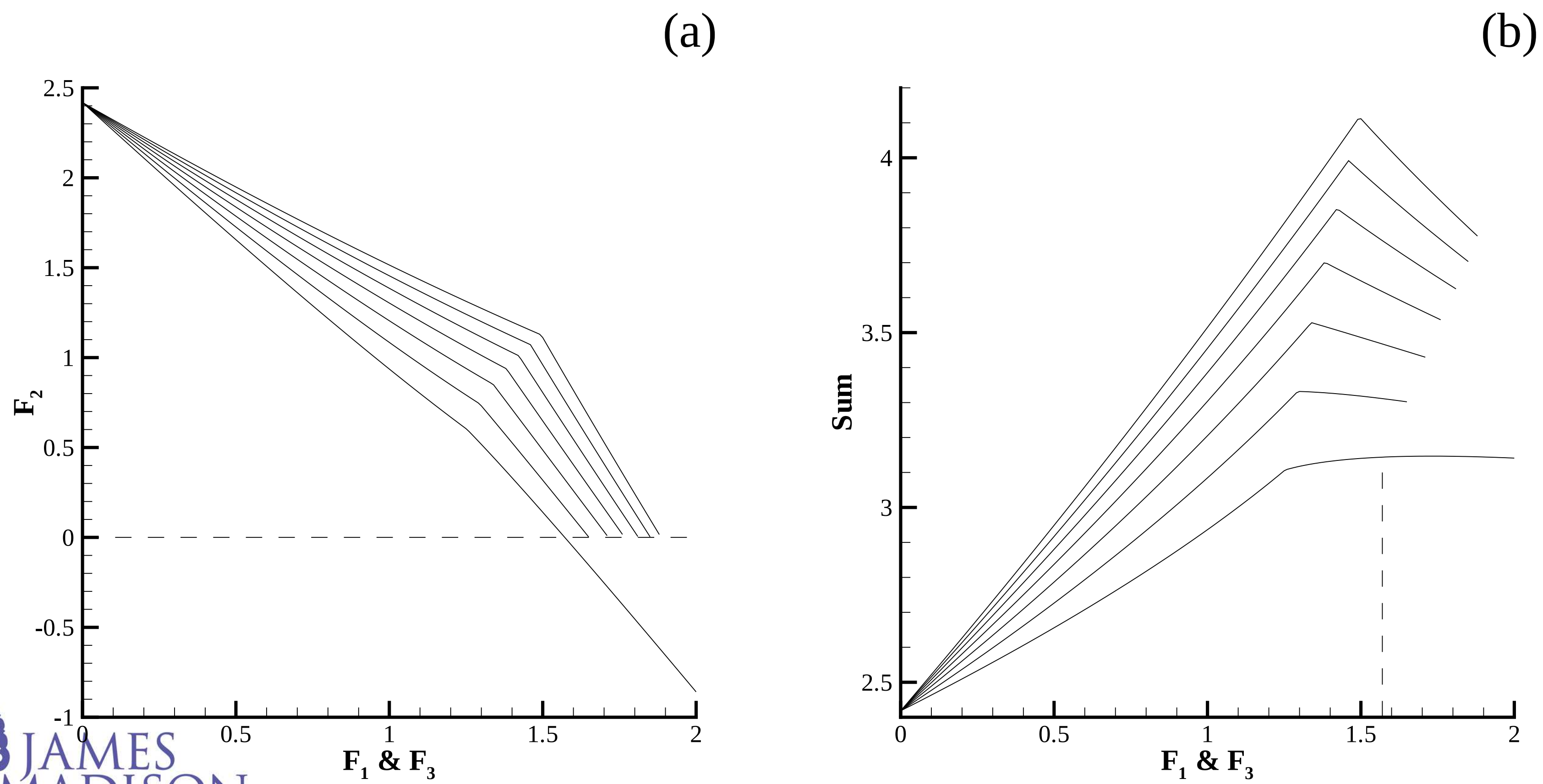

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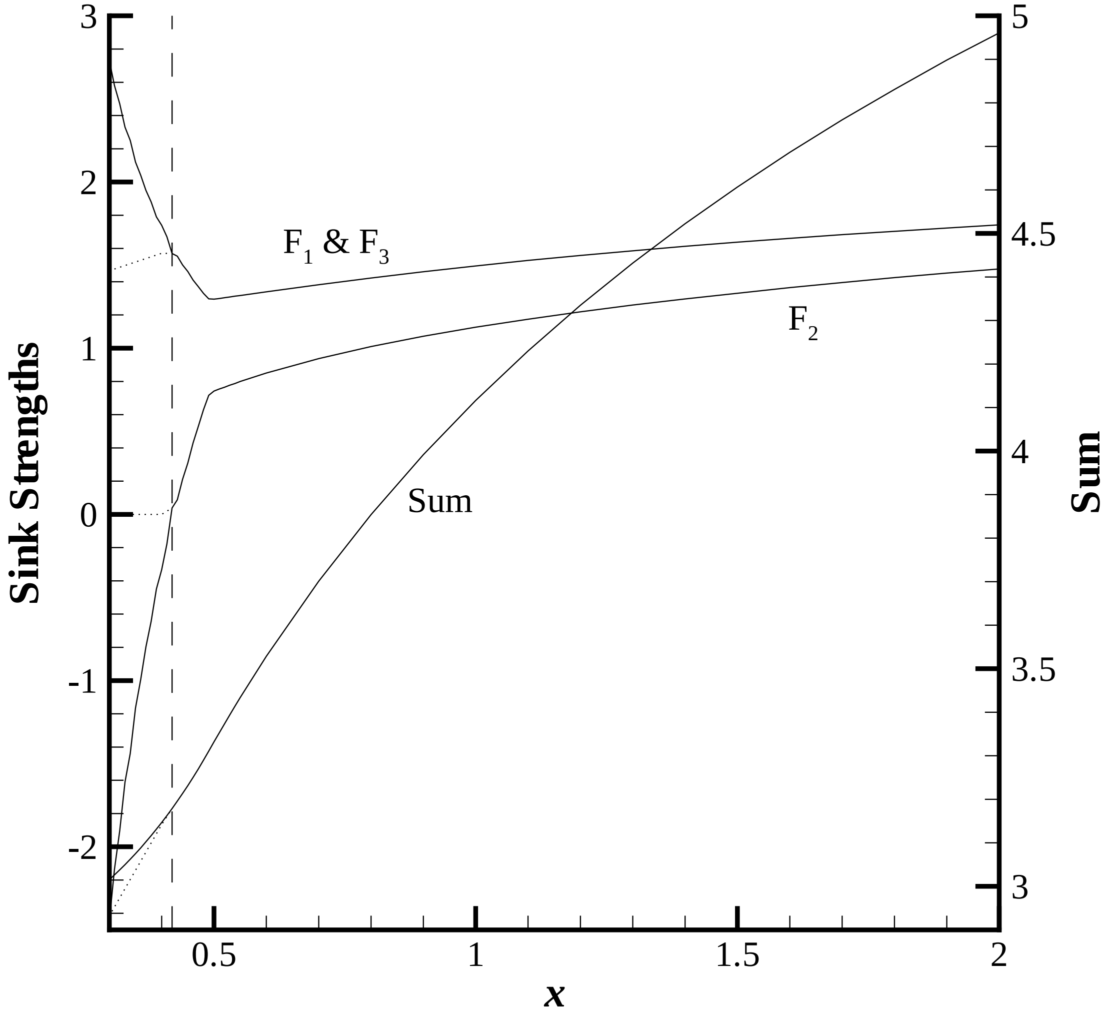
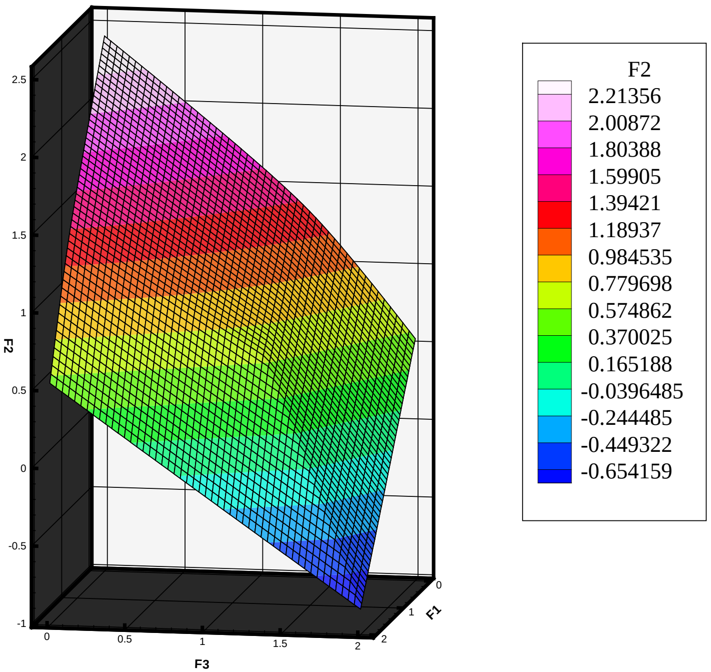
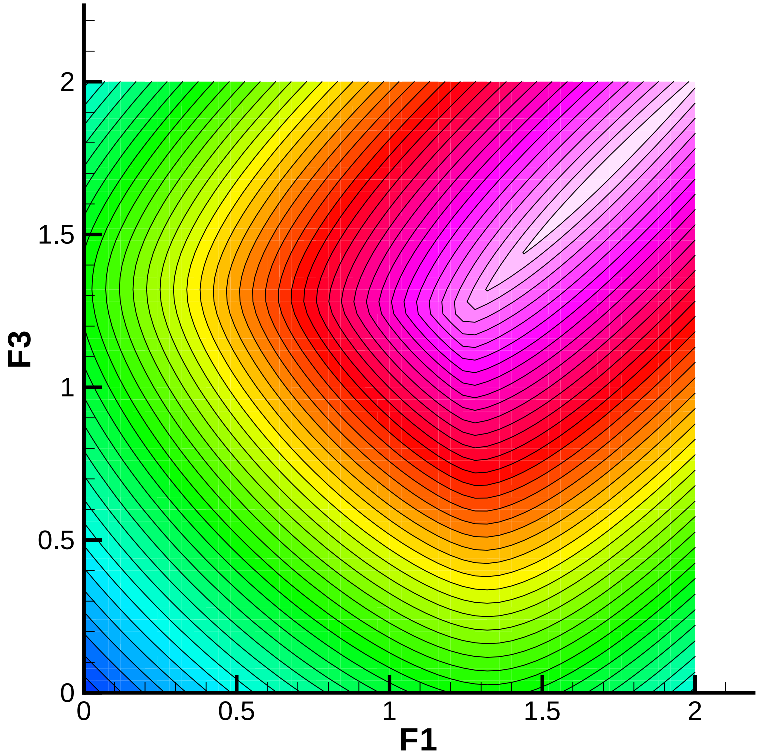


Feasible Region and Sum With $x = 0.4$





Nonnegative Constrained Nelder Mead

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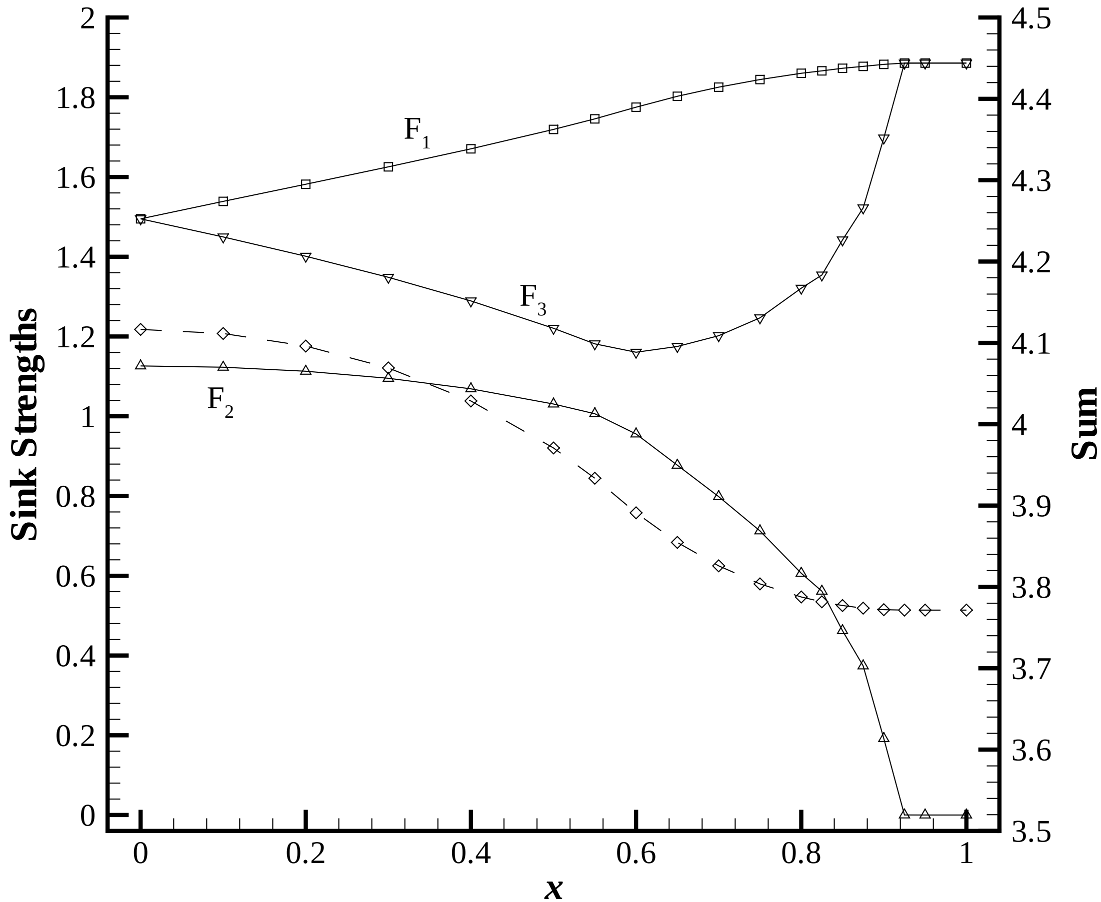
Accuracy 10^{-5} gives solutions (80 iterations)
1.555331, 0.032775, 1.555086, sum 3.143193.
Restarting here gives
1.564512, 0.014617, 1.564505, sum 3.143634.
Accuracy 10^{-7} gives (427 iterations)
1.571573, 0.000726, 1.571638, sum 3.143937.
Explicitly only two sinks:
1.571944, 0.000000, 1.572008, sum 3.143952.

Moving One Sink Between Two Fixed

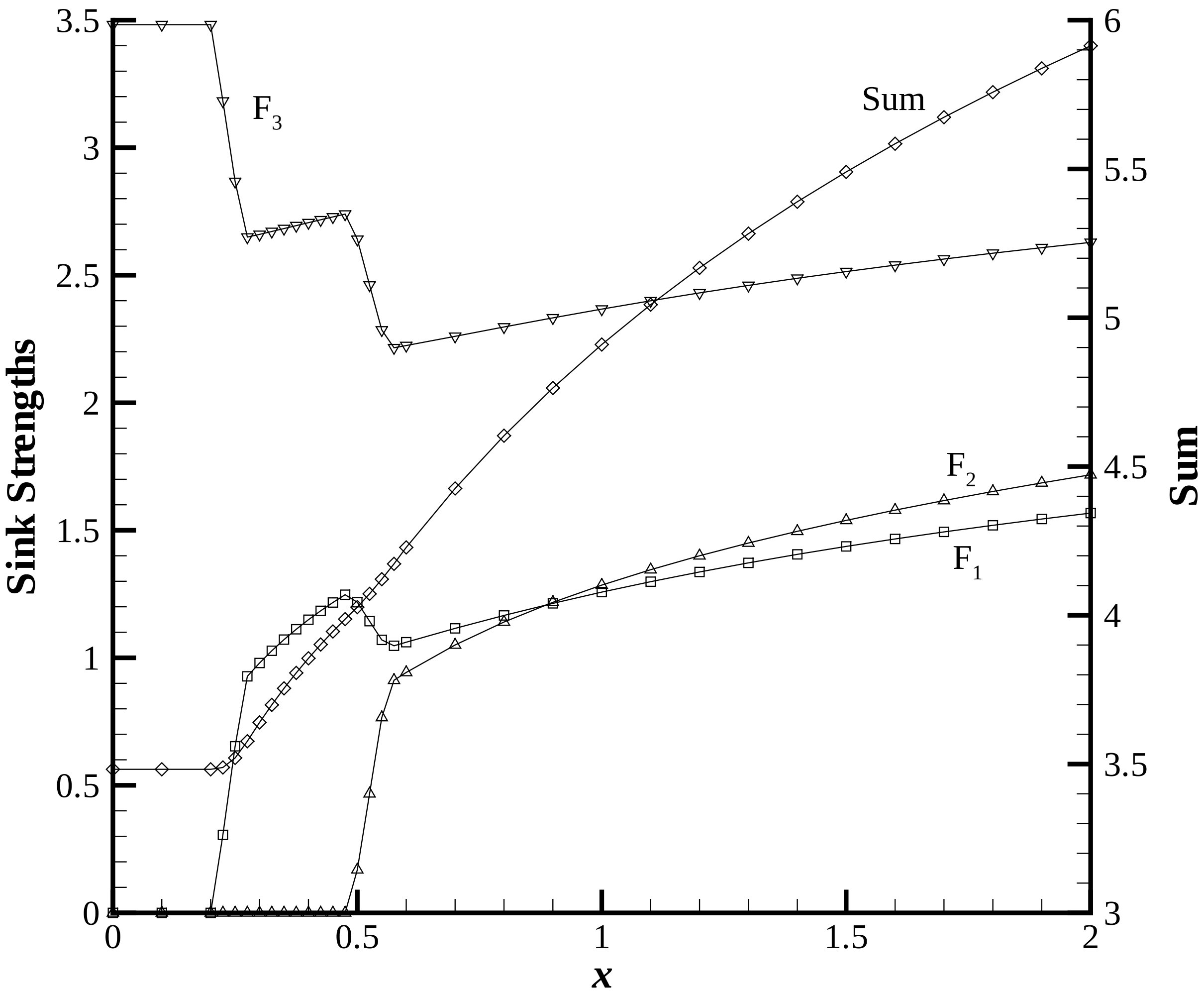
Three sinks, F_1 at $(-1, 0, 1)$, F_2 at $(x, 0, 1)$, F_3 at $(1, 0, 1)$.

Several times needed to restart simplex method – problem sufficiently ill conditioned that four digits accuracy lost in the individual sink strengths



Three Sinks at an Angle

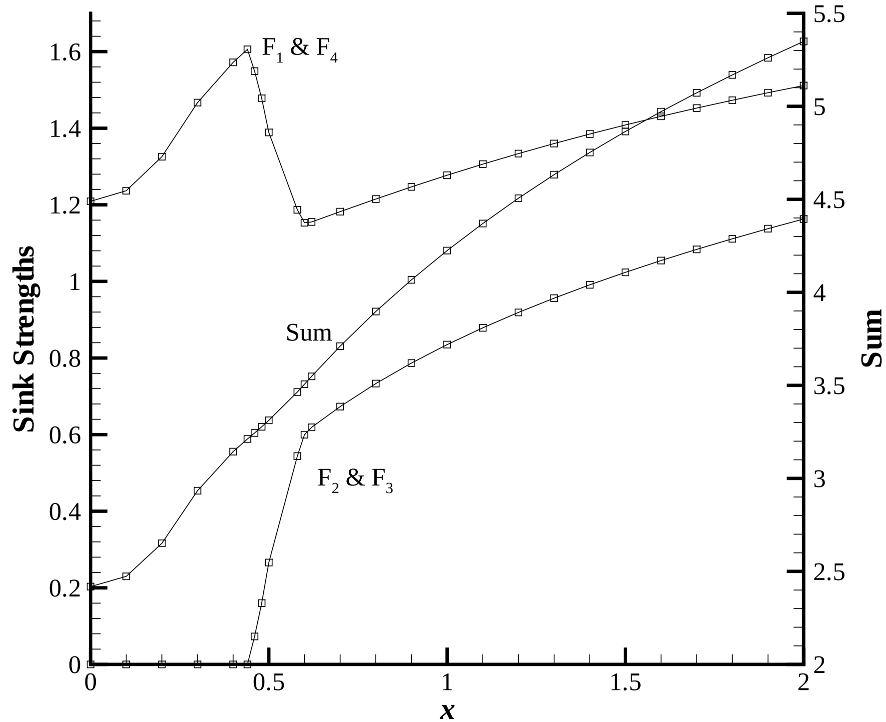
Three sinks, F_1 at $(-x, 0, 1)$, F_2 at $(0, 0, 1.1)$, F_3 at $(x, 0, 1.2)$.



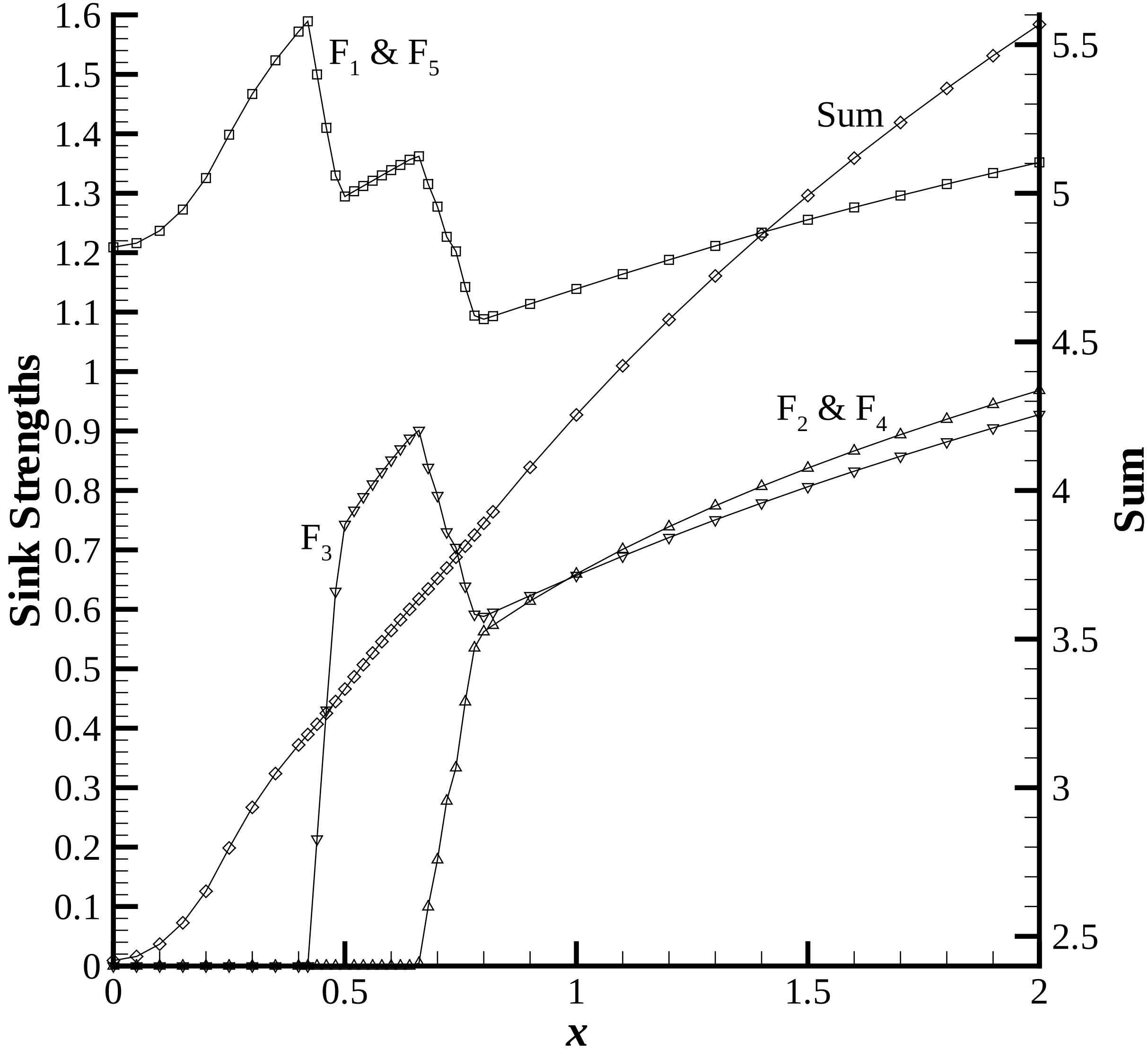
Four Sinks In A Row

Four sinks: F_1 at $(-x, 0, 1)$, F_2 at $(-x/3, 0, 1)$,
F_3 at $(x/3, 0, 1)$, F_4 at $(x, 0, 1)$.

Can assume $F_1 = F_4$ and $F_2 = F_3$ (two parameters) or run as is (four parameters). Gave identical answers until x is sufficiently small, where convergence difficulties arise.



Five Sinks In A Row



Sinks Evenly Distributed Between $(-2, 0, 1)$ and $(2, 0, 1)$

No. Sinks	1	3	5	7	9	11	13	15
Sum	2.418	4.961	5.568	5.785	5.873	5.907	5.921	5.938
Ind. Sinks								1.026
							0.896	0.054
						0.925	0.356	0.457
					1.014	0.483	0.422	0.328
				1.144	0.585	0.465	0.378	0.319
			1.351	0.731	0.550	0.439	0.367	0.318
		1.741	0.968	0.683	0.526	0.428	0.360	0.309
	2.418	1.478	0.927	0.668	0.519	0.424	0.358	0.310
		1.741	0.968	0.683	0.526	0.428	0.360	0.309
			1.351	0.731	0.550	0.439	0.367	0.318
				1.144	0.585	0.465	0.378	0.319
					1.014	0.483	0.422	0.328
						0.925	0.356	0.457
							0.896	0.054
								1.026

Sinks Evenly Distributed Between $(-2, 0, 1)$ and $(2, 0, 1)$

No. Sinks	2	4	6	8	10	12	14
Sum	4.182	5.348	5.701	5.838	5.894	5.915	5.930
Ind. Sinks							0.995
						0.895	0.147
					0.965	0.434	0.456
				1.072	0.530	0.436	0.350
			1.234	0.650	0.503	0.406	0.345
		1.511	0.833	0.608	0.478	0.394	0.336
	2.091	1.162	0.783	0.587	0.468	0.389	0.330
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		1.511	0.833	0.608	0.478	0.394	0.336
			1.234	0.650	0.503	0.406	0.345
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					0.965	0.434	0.456
						0.895	0.147
							0.995

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Sinks Evenly Distributed Between $(-0.5, 0, 1)$ and $(0.5, 0, 1)$

No. Sinks	1	3	5	7	9	11
Sum	2.418	3.331	3.331	3.331	3.331	3.331
Ind. Sinks						1.294
					1.294	0.000
				1.294	0.000	0.000
			1.294	0.000	0.000	0.000
		1.294	0.000	0.000	0.000	0.000
	2.418	0.742	0.742	0.742	0.742	0.742
		1.294	0.000	0.683	0.000	0.000
			1.294	0.000	0.000	0.000
				1.294	0.000	0.000
					1.294	0.000
						1.294

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Sinks Evenly Distributed Between $(-0.5, 0, 1)$ and $(0.5, 0, 1)$

No. Sinks	2	4	6	8	10	12
Sum	3.302	3.312	3.323	3.327	3.329	3.330
Ind. Sinks						1.282
					1.277	0.000
				1.279	0.000	0.000
			1.300	0.000	0.000	0.000
		1.389	0.000	0.000	0.000	0.000
	1.651	0.266	0.360	0.384	0.386	0.382
	1.651	0.266	0.360	0.384	0.386	0.382
		1.389	0.000	0.000	0.000	0.000
			1.300	0.000	0.000	0.000
				1.279	0.000	0.000
					1.277	0.000
						1.282

Dual/Triple Completion

Dual Completion: Place sink in water layer

Dual/Triple Completion

Dual Completion: Place <u>sink</u> in water layer

Triple Completion: Also place equal strength <u>source</u> further down in water layer

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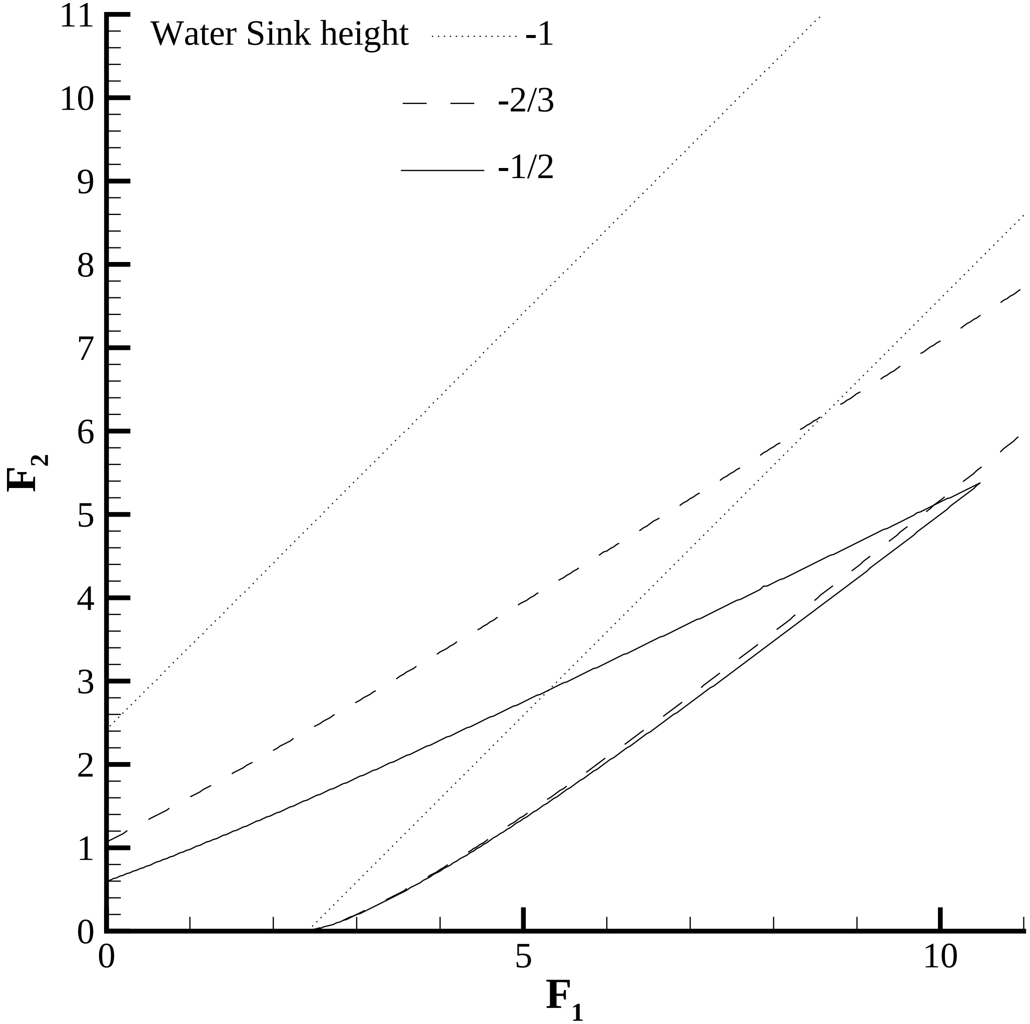
Dual/Triple Completion

Dual Completion: Place sink in water layer

Triple Completion: Also place equal strength source further down in water layer

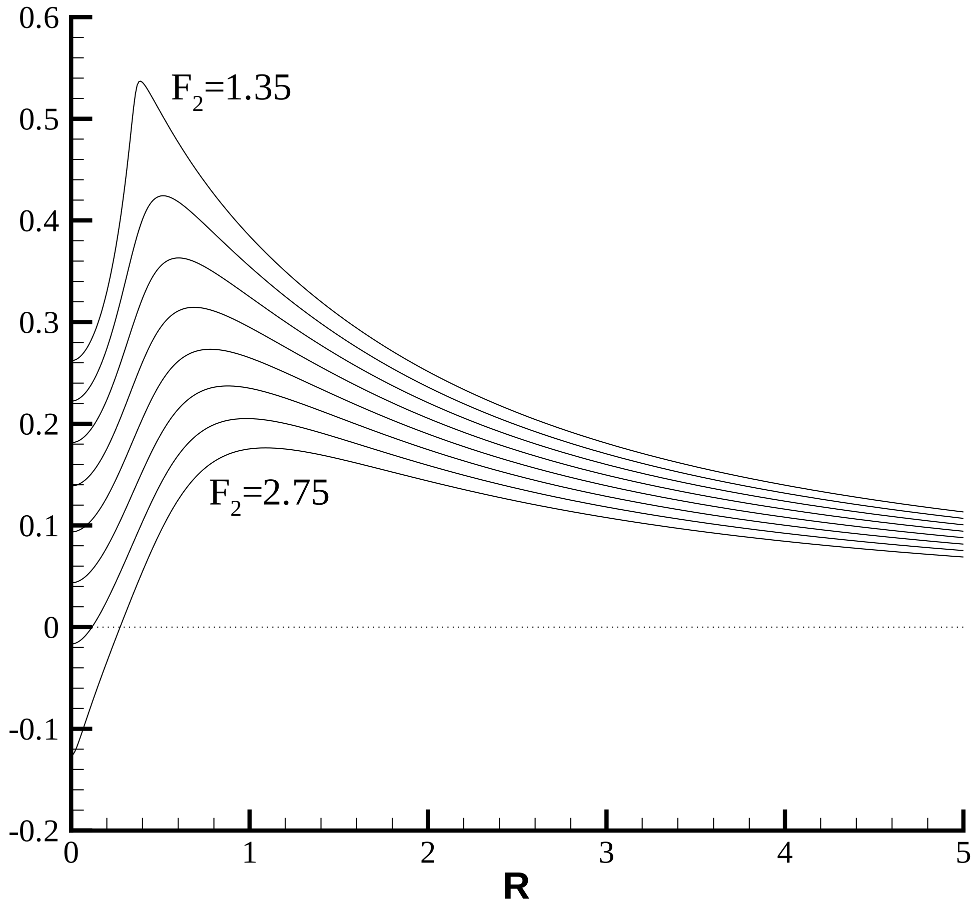
Muskat model formulation identical, just replace F_i values for water sinks by $-F_i\mu_w/\mu_{oil}$.

Muskat – Stable Dual Completion



Surface Shapes

$F_1 = 5$ at 1, $F_2 = 1.35, 1.75, \ldots, 2.75$ at -0.5



Conclusion

- Muskat model approximates interface height very quickly
- Developed a constrained nonnegative Nelder Mead method for finding optimal flow rates
- There is a minimal spacing beyond which additional sinks are superfluous
- Dual and triple completion alternative methods for delaying breakthrough

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Further work:

- BIM formulation for dual and triple completion
- Other interesting geometries?