HOMEWORK 5 **DUE: Friday, Feb. 5** **NAME:**

DIRECTIONS:

- **STAPLE** this page to the front of your homework (don't forget your name!).
- Show all work, clearly and in order **You will loose points if you work is not in order**.
- When required, **do not forget the units**!
- Circle your final answers. **You will loose points if you do not circle your answers**.

Question	Points	Score
1	1	
2	2	
3	2	
4	2	
5	3	
Total	10	

Problem 1: (1 point) Recall the "Amazing Steve" from your second Homework Assignment. What is the force which which he hits the ground after his 'flight'?

Problem 2: (2 points) Suppose Steve is fired form the cannon with an angle of inclination of $\theta = 45$ degrees and that he hits the ground 500 meters from the cannon. What, then, was Steve's initial speed?

Problem 3: (2 points) Let $\mathbf{c}(t)$ be a path, $\mathbf{v}(t)$ its velocity, and $\mathbf{a}(t)$ the acceleration. Suppose $\mathbf{F}$ is a C^1 mapping of $\mathbb{R}^3$ to $\mathbb{R}^3$, $m > 0$, and $\mathbf{F}(\mathbf{c}(t)) = m\mathbf{a}(t)$ (Newton's second law). Prove that

$$\frac{d}{dt}\left[m\mathbf{c}(t) \times \mathbf{v}(t)\right] = \mathbf{c}(t) \times \mathbf{F}(\mathbf{c}(t)).$$

What can you conclude if $\mathbf{F}(\mathbf{c}(t))$ is parallel to $\mathbf{c}(t)$?

Problem 4: (2 points) Define the unit tangent vector T of the path x as the normalization of the velocity vector; that is,

$$\mathbf{T} = \frac{\mathbf{x}'(t)}{||\mathbf{x}'(t)||}.$$

Prove that $\frac{d\mathbf{T}}{dt}$ is perpendicular to $\mathbf{T}$ for all time, t.

Problem 5: (3 points) Consider the helix $\mathbf{x}(t) = (a\cos t, a\sin t, bt)$, $0 \leq t \leq 2\pi$.

(a) (1 point) What is the total length, $L(\mathbf{x})$, of the path?

(b) (2 points) Define a new parameter $s = t\sqrt{a^2 + b^2}$. Suppose we define the curvature κ of a path $\mathbf{x}$ as the angular rate of change in the direction of $\mathbf{T}$ per unit change in distance along the path. That is, define

$$\kappa = \frac{||d\mathbf{T}/dt||}{ds/dt}.$$

What is the curvature of the helix?