HOMEWORK 6 **DUE: Wed., Feb. 10** **NAME:**

DIRECTIONS:

- **STAPLE** this page to the front of your homework (don't forget your name!).
- Show all work, clearly and in order **You will lose points if you work is not in order**.
- When required, **do not forget the units**!
- Circle your final answers. **You will lose points if you do not circle your answers**.

Question	Points	Score
1	2	
2	4	
3	2	
4	2	
Total	10	

Problem 1: (2 points) Prove that for a vector field, $\mathbf{F} = (F_1(x,y,z), F_2(x,y,z), F_3(x,y,z))$, the divergence of the curl is zero. That is, prove

$$\nabla \cdot (\nabla \times \mathbf{F}) = 0.$$

Problem 2: (4 points) Determine which of the following vector fields is *not* a gradient vector field.

(a) (2 point) $\mathbf{F} = \left(x^2 + y^2\right)\mathbf{i} - 2xy\mathbf{j}$.

(b) (2 point) $\mathbf{F} = 3x^2y\mathbf{i} + (x^3 + y^3)\mathbf{j}$.

Problem 3: (2 points) Evaluate the following integral over the rectangle R given by $[0,2] \times [-1,0]$

$$\int\int_R \left[|y| \cos\left(\frac{\pi x}{4}\right)\right] dydx.$$

Problem 4: (2 points) Although Fubini's theorem holds for most functions we'll see in practice, you still need to be careful. For example, you can show that

$$\int_0^1 \int_0^1 \frac{x^2 - y^2}{\left(x^2 + y^2\right)^2} dydx = \frac{\pi}{4},$$

yet

$$\int_0^1 \int_0^1 \frac{x^2 - y^2}{\left(x^2 + y^2\right)^2} dxdy = -\frac{\pi}{4}.$$

Why does this not contradict Fubini's theorem?