HOMEWORK 1 **NAME:**

DIRECTIONS:

- Turn in your homework as **SINGLE-SIDED** typed or handwritten pages.
- **STAPLE** your homework together. Do not use paper clips, folds, etc.
- **STAPLE** this page to the front of your homework.
- Be sure to write your name on your homework.
- Show all work, **clearly and in order**.

You will lose point 0.5 points for each instruction not followed.

Questions	Points	Score
1	0.5	
2	0.5	
3	1	
4	1	
5	6	
6	1	
Total	10	

Problem 1: (0.5 points) Read "A Guide to Writing Mathematics" by K. Lee. List and describe five items pertaining to writing math of which you were previously unaware. *(Use full sentences.)*

Problem 2: (0.5 points) Sections I and II of "Writing Proofs" by T. Hsu. List and describe five items pertaining to writing mathematical proofs of which you were previously unaware. *(Use full sentences.)*

Problem 3: (1 point) Let $A = \{b, f, p\}$.

(a) (0.5 points) List all the subsets of A.

$\emptyset$, $\{b\}$, $\{f\}$, $\{p\}$, $\{b, f\}$, $\{b, p\}$, $\{f, p\}$, A.

(b) (0.5 points) List all the proper subsets of A.

$\emptyset$, $\{b\}$, $\{f\}$, $\{p\}$, $\{b, f\}$, $\{b, p\}$, $\{f, p\}$

Problem 4: (1 point) In each of the following, form a set whose elements are the symbols of the given word or phrase (remember, a 'space' is a symbol too).

(a) (0.25 points) BUBBLE

$\{B, U, L, E\}$

(b) (0.25 points) MATHEMATICS IS FUN!

$\{M, A, T, H, E, I, C, S, `space', F, U, N, !\}$

(c) (0.25 points) MADMAN

$\{M, A, D, N\}$

(d) (0.25 points) HUH?

$\{H, U, ?\}$

Problem 5: (6 points) Let A, B, C be subsets of some universal set X. Label the following true or false. If true, prove. If false, prove false or provide a counter example.

(a) (1 point) $A \cap (B \cap C) = (A \cap B) \cap C$. **True**

Proof: Since we want to show equality, we need to show *(I)* $A \cap (B \cap C) \subseteq (A \cap B) \cap C$ and *(II)* $(A \cap B) \cap C \subseteq A \cap (B \cap C)$.

Part I: We want to show that for all $x \in A \cap (B \cap C)$ then $x \in (A \cap B) \cap C$. So let $x \in A \cap (B \cap C)$. By definition of $\cap$ this means that $x \in A$ and $x \in B \cap C$, the latter of which implies that $x \in B$ and $x \in C$. Hence $x \in A \cap B$ since it is in both A and B. This in tern implies $x \in (A \cap B) \cap C$ since it is in both $A \cap B$ and C. Again, the choice of x was arbitrary and this approach holds for all $x \in A \cap (B \cap C)$.

Part II: The procedure is the same as Part I above.

Q.E.D.

(b) (1 point) $A \setminus B = B \setminus A \iff B = \emptyset$. **False**

Counter example: Suppose $A = B \neq \emptyset$.

(c) (1 point) $\emptyset \subset \emptyset$. **False**

Proof: If this were true, then there must exist some $x \in \emptyset$ on the left that is not in $\emptyset$ on the right. But this is a contradiction. Q.E.D.

(d) (1 point) $A \cap B = X \iff A = X$ and $B = X$. **True**

Proof: Since we wish to prove an if and only if statement, we will need to show *(I)* $A \cap B = X \Longrightarrow A = X$ and $B = X$ and *(II)* $A = X$ and $B = X \Longrightarrow A \cap B = X$ where X is the universal set.

Part I: Assume $A \cap B = X$. We want to show that $X = A$ and $X = B$. Let $x \in X$, then $x \in X = A \cap B \Longrightarrow x \in A$ and $x \in B$. Hence $X \subseteq A$ and $X \subseteq B$, but X is the universal set so $A \subseteq X$ and $B \subseteq X$. Therefore $X = A$ and $X = B$.

Part II: Assume $X = A$ and $X = B$. We want to show that $X = A \cap B$. Let $x \in X \Longrightarrow x \in A$ and $x \in B$ by our assumption. Then by definition of $\cap$, $x \in A \cap B$. Hence $X \subseteq A \cap B$. But again, X is the universal set with $A \subseteq X$ and $B \subseteq X$ so $A \cap B \subseteq X$. Therefore $X = A \cap B$.

Q.E.D.

(e) (1 point) $(A \cap B)^c = A^c \cup B^c$. **True**

Proof: We want to show equality so we must again show *(I)* $(A \cap B)^c \subseteq A^c \cup B^c$ and *(II)* $A^c \cup B^c \subseteq (A \cap B)^c$

Part I: Let $x \in (A \cap B)^c$. Then by definition of the complement, $x \notin A \cap B$ which means that $x \notin A$ and/or x.

case 1: Suppose $x \notin A$. Then $x \in A^c \Longrightarrow x \in A^c \cup B^c$ by definition of $\cup$.

case 2: Suppose $x \notin B$. Then $x \in B^c \Longrightarrow x \in A^c \cup B^c$ as before.

Part II: Similarly $A^c \cup B^c \subseteq (A \cap B)^c$.

Q.E.D.

(f) (1 point) $A \cap (B \triangle C) = (A \cap B) \triangle (A \cap C)$. **True**

Proof: Here we will use $\iff$ statements instead of doing the proof in two parts. Let $x \in A \cap (B \triangle C) \iff x \in A$ and $x \in B$. This is true if and only if $x \in B \setminus C$ or $x \in C \setminus B$.

If $x \in B \setminus C \iff x \in B$ and $x \notin C \iff x \in A \cap B$ and $x \notin A \cap C \iff x \in (A \cap B) \setminus (A \cap C) \iff x \in (A \cap B) \triangle (A \cap C)$.

or

If $x \in C \setminus B$ the procedure is the same.

Hence $A \cap (B \triangle C) = (A \cap B) \triangle (A \cap C)$.

Problem 6: (1 point) In 1895, George Cantor gave the following definition: “By a set we shall understand any collection into a whole of definite distinguishable objects of our intuition or thought. The objects will be called members of the collection.” In 1902, the philosopher and mathematician Bertrand Russel constructed and object of his intuition satisfying Cantor’s definition of a set, but that lead to a logical contradiction (Russell’s Paradox):

> *Consider a village in which there lives one barber. The barber does not shave people who shave themselves. Moreover, he shaves all those people who do not shave themselves.*

Analyze this situation. Why does it result in a paradox?

Answer: If the barber shaves himself, then he is one of those that he does not shave. On the other hand, if he does not shave himself, then he is one of those that he does shave. So there cannot be such a barber.