HOMEWORK 7 **DUE: Mon., May 14** **NAME:**

DIRECTIONS:

- Turn in your homework as **SINGLE-SIDED** typed or handwritten pages.
- **STAPLE** your homework together. Do not use paper clips, folds, etc.
- **STAPLE** this page to the front of your homework.
- Be sure to write your name on your homework.
- Show all work, **clearly and in order**.

You will lose point 0.5 points for each instruction not followed.

Questions	Points	Score
1	2	
2	3	
3	1	
4	1	
5	1	
6	1	
7	1	
Total	10	

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Problem 1: (2 points) Find the accumulation points of the following sets in $\mathbb{R}$.

(a) (0.5 points) $S = (0, 1)$.

(b) (0.5 points) $S = \left\{(-1)^n + \frac{1}{n} | n \in \mathbb{N}\right\}$.

(c) (0.5 points) $S = \mathbb{Q}$.

(d) (0.5 points) $S = \mathbb{Z}$.

Problem 2: (3 points)

(a) (1 point) Find an infinite subset of $\mathbb{R}$ that does not have an accumulation point in $\mathbb{R}$.

(b) (1 point) Find a bounded subset of $\mathbb{R}$ that does not have an accumulation point in $\mathbb{R}$.

(c) (1 point) Find a bounded infinite subset of $\mathbb{Q}$ that does not have an accumulation point in $\mathbb{Q}$.

Problem 3: (1 point) Let $S \subset \mathbb{R}$. Suppose every neighborhood of $x \in S$ contains infinitely many points of S. Prove that x is an accumulation point of S. *(This is the second half of the proof from class.)*

Problem 4: (1 point) Show that the arbitrary union of open sets in $\mathbb{R}$ is open. That is suppose $\{U_i\}_{i \in \mathcal{I}}$ is a collection of open sets in $\mathbb{R}$. Prove that $\cup_{i \in \mathcal{I}} U_i$ is also open. *Note: $\mathcal{I}$ need not be a denumerable set of indices.*

Problem 5: (1 point) Show, by example, that an infinite intersection of open sets in $\mathbb{R}$ is not necessarily open (you will still need to prove that your example is not open).

Problem 6: (1 point) Show, by example, that an infinite union of closed sets in $\mathbb{R}$ is not necessarily closed.

Problem 7: (1 point) Show that a finite union of closed sets in $\mathbb{R}$ is a closed set in $\mathbb{R}$.