HOMEWORK 8 **DUE: Mon., May 21** **NAME:**

DIRECTIONS:

- Turn in your homework as **SINGLE-SIDED** typed or handwritten pages.
- **STAPLE** your homework together. Do not use paper clips, folds, etc.
- **STAPLE** this page to the front of your homework.
- Be sure to write your name on your homework.
- Show all work, **clearly and in order**.

You will lose point 0.5 points for each instruction not followed.

Questions	Points	Score
1	1	
2	1	
3	1	
4	1	
5	2	
6	1	
7	1	
8	1	
9	1	
Total	10	

Problem 1: (1 point) Let F be a field and F^n be a vector space. Prove that the set of canonical basis vectors, $S = \{\mathbf{e}_1, \mathbf{e}_2, ..., \mathbf{e}_n\}$, is a linearly independent set.

Problem 2: (1 point) Consider the vector space $V = F^n$. Let $\mathbf{v} \in V \setminus \{\mathbf{e}_1, \mathbf{e}_2, ..., \mathbf{e}_n\}$. Using only the definition of the canonical vectors, $\mathbf{e}_j$, and the definitions of linearly dependent and independent, prove that $\{\mathbf{e}_1, \mathbf{e}_2, ..., \mathbf{e}_n, \mathbf{v}\}$ is a linearly dependent set.

Problem 3: (1 point) Consider the set of vectors $\{\mathbf{v}_1, \mathbf{v}_2, ..., \mathbf{v}_\mathbf{m}\}$. Suppose $\mathbf{v}_j = \mathbf{0}$ for some j such that $1 \leq j \leq m$. Prove that this is a linearly dependent set.

Problem 4: (1 point) Let $\mathbf{v}_1$ and $\mathbf{v}_2$ be vectors in a vector space V. Show that the set $\{\mathbf{v}_1, \mathbf{v}_2\}$ is a linearly dependent set if and only if one of these vectors is a scalar multiple of the other.

Problem 5: (2 points) Determine by inspection if the given set is linearly dependent. You do not need to prove your answer.

(a) (0.5 points) $(1, 0)$, $(0, 1)$, $(\sqrt{2}, \pi)$.

(b) (0.5 points) $(1, 7, 6)$, $(2, 0, 9)$, $(3, 1, 5)$, $(4, 1, 8)$.

(c) (0.5 points) $(2, 3, 5)$, $(0, 0, 0)$, $(1, 1, 8)$.

(d) (0.5 points) $(-2, 4, 6, 10)$, $(3, -6, -9, -15)$.

Problem 6: (1 point) Let V be a vector space over a field F. Show $\{\mathbf{0}\}$ and V are subspaces of V.

Problem 7: (1 point) Let $V = \mathbb{Q}[x]$, and let W be the collection of all polynomials in $\mathbb{Q}[x]$ whose degree is less than or equal to a fixed non-negative integer n.

(a) (0.5 points) Prove that W is a subspace of V.

(b) (0.5 points) Find the dimension of W and justify your answer.

Problem 8: (1 point) Let F be a field and consider the vector space $V = F^n$ and for a fixed $m \leq n$, let $W = \{\mathbf{v} \in V |$ v is a linear combination of the basis vectors $\mathbf{e}_1, \mathbf{e}_1, ..., \mathbf{e}_m\}$.

(a) (0.5 points) Prove that W is a subspace of V.

(b) (0.5 points) Find the dimension of W and justify your answer.

Problem 9: (1 point) Let $V = \mathbb{R}$ be a vector field over $F = \mathbb{R}$. Let $a \in \mathbb{R}$. Consider $T_a : V \to V$ where $T_a(x) = ax$ for all $x \in \mathbb{R}$. Prove that T_a is a linear transformation.