## Trailer Proposal for Draftee Summer

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## 1 Abstract

Universal Studios ${ }^{\text {TM }}$ has agreed to produce Draftee Summer in association with TARNADO Productions, LLC. Following is a proposal for the design and implementation of the trailer for Draftee Summer.

In light of the budget allotted for the production of the trailer, the use of computer-generated imagery (CGI) will not be possible. The stunts will be performed by stunt actors instead. The "Specifications" section of this proposal includes detailed analysis of the fourteen stunts that will be performed in the trailer. Each has been evaluated for safety and plausibility. The set producers have evaluated the materials that will be needed for these stunts and confirmed their accessibility.

The "Design and Methods" section of this proposal includes a timeline of the trailer, which includes the duration of each stunt. The total time allotted for stunts is 5:00 minutes, and the total duration of the trailer is 7:00 minutes. The trailer is such that between stunts there will be contextual dialogue and panoramic views, which will give the audience an idea of what Draftee Summer is about and cause them to want to see it in the summer of 2012.

In "Evaluations" we summarize and compile the individual evaluation rubric after each stunt analysis. This will be further confirmation of the safety and plausibility of the stunts we plan to use in the trailer.

This trailer will appear in theaters showing action movies, as the target audience for Draftee Summer are fans of other action movies. Thus, it is integral for our stunts to appear dramatic and realistic. We plan to optimize the dramatic effect by using talented stunt actors, advanced machinery, and various film techniques, such as multiple camera angles and slow motion.

The stunts in the trailer will be highly technical and difficult yet safe for the trained stunt actors to perform.

The goal of this proposal is to demonstrate the feasibility of the stunts needed for the trailer of Draftee Summer to be performed by stunt actors.

## 2 Design

The budget allotted for Draftee Summer does not allow for CGI. In lieu of CGI, the production team for TARNADO Productions has choreographed and analyzed a sequence of fourteen stunts that will net a duration of five minutes.

All but one of these stunts involve stunt actors; therefore, the primary concern in evaluating these stunts is the actors' safety. We have designed these stunts with the actors in mind while maintaining a focus on the dramatic effect these stunts will provide, as this trailer is the first look audiences will have at Draftee Summer and we want to convince them of its dramatic and action-packed integrity.

### 2.1 Plot Overview

Nora Sagiv is your average American teenager. She washes the dishes after dinner every day, her favorite class in school is Biology, and she likes to spend time at the mall with her friends. As someone who has led a relatively sheltered life in middle American suburbia, Nora has cultivated a strong aversion to war.

Why, then, is the Israeli army so set on drafting her? Because she is eighteen, an Israeli citizen, and schooled in three languages. Nora is in danger of being drafted into the Israeli Defense Forces, as Israel is the only country in the world that conscripts women. However, due to her pacifist tendencies and in spite of the high rank with which the Israeli state attempt to lure her in, Nora has no desire to enter the military. The only way to escape a life of war-mongering is to run away.

The movie follows Nora's frantic, high-flying journey through the twisting streets of Tel Aviv. As she struggles to evade the supercilious men on her tail, she jumps out of buildings, dives into water, and covers miles of unpredictable urban terrain. Ultimately, Nora, with her athleticism and strong will, prevails over the clumsy-footed bureaucrats of the Israeli Defense Forces.

### 2.2 Timeline of Trailer

| Stunt | End time |
| :--- | :--- |
| Bus bombing | $0: 05$ |
| Ambulances, emotional camera angles | $0: 25$ |
| Frictional incline | $0: 50$ |
| Brachistochrone slide | $1: 10$ |
| Panoramic view of Tel Aviv, Israel | $1: 30$ |
| Spliced shots of IDF (Israeli army) | $1: 50$ |
| Parachuting soldiers | $2: 15$ |
| Officers meet to discuss Nora and her importance | $2: 40$ |
| Conference between officers and Nora | $2: 50$ |
| Car on a banked curve | $3: 10$ |
| Car flying off a bridge | $3: 35$ |
| Car lands in water | $3: 40$ |
| Floating car escape | $4: 15$ |
| Swimming against a current | $4: 55$ |
| Nora breaks down in urban Tel Aviv | $5: 00$ |
| Nora overcomes her breakdown, runs away from the camera | $5: 05$ |
| Helical run | $5: 25$ |
| Humorous banter between officers and Nora | $5: 30$ |
| Swinging on a rope | $5: 50$ |
| Jumping into an open window | $6: 05$ |
| Skateboarding down an incline | $6: 25$ |
| Bungee jump | $6: 50$ |
| Draftee Summer: Summer 2012 | $7: 00$ |

Table 2.2. The table above gives a timeline for the trailer of Draftee Summer. The "End time" denotes the point at which each event ends. The total time spent on action stunts is 5:00 and the time spent on contextual material is 2:00, totaling to 7:00.

## 3 Methods

The trailer consists of both action stunts and contextual dialogue. The dialogue serves to give the audience an idea of what Draftee Summer is about and to make them interested and invested in Nora and her conflict with the Israeli Defense Forces.

The stunts take up a majority of the time allotted to this trailer because the target audience for the movie is assumed to enjoy action movies. The challenge for the production team is incorporating as many stunts as possible given a seven-minute time limit, of which two minutes are reserved for dialogue and other contextual scenes, and the need to use live-action stunts.

Here we will explain our methodology for evaluating the stunts. We evaluate the stunt for feasibility and safety. Each stunt includes a rubric that evaluates it on the basis of its safety and feasibility. "Notes" details any constraints or special requirements for the performance and set-up of each stunt, e.g., the use of crystallized sugar instead of glass.

Mathematical justification as well as schematic diagrams for each of the stunts is included in "Specifications." The analyses also include data about the materials to validate the safety of the stunts.

### 3.1 Bus Bombing

The movie begins with a bus bombing, a common form of terrorist attack in Israel since the second intifada in $2000^{1}$. For the purpose of this movie, and our intent to start this trailer in medias res, we will assume the bus has been projected vertically upwards. The following calculations apply to the 5 m fall, which will be completely orchestrated by attaching a decrepit bus to a junk-yard magnet.

We use the idea that when the bus is at its peak in the air its total energy is in the form of potential energy and just before it hits the ground its total energy is in the form of kinetic energy ${ }^{2}$, which allows us to use the equation $U=K E$, where $U$ denotes potential energy and $K E$ denotes kinetic energy. This means that $m g h=\frac{1}{2} m v^{2}$. Further mathematical justification can be found in section 4.1 of "Specifications."

In terms of the trailer, this stunt is very important contextually, so for dramatic emphasis, we will film it in slow motion using multiple camera angles, so likely the entire stunt will be closer to 5 seconds.

[^0]
## Evaluation

Feasibility: Strong
Safety: Poses little risk; however, for safety people should keep their distance from the crane and hanging bus
Notes: We would need to gain access to a crane and a decrepit bus
Duration: 0:05

### 3.2 Frictional Incline

In this stunt, we want our heroine to run up a narrow incline, such as a ramp, a conveyor belt, or wooden plank. To make the scene especially dramatic, the incline's angle should be maximized. To ensure that our actress won't slide down the ramp, which could lead to serious injury, we need to choose a material for the incline that offers enough static friction. We can assume that the actress is wearing shoes with rubber soles.

We determine that if the angle between the incline and the ground is $\theta$ and the friction between the heroine's shoes and the incline is $\mu$, that the angle at which we can set the ramp is correlated with the friction between the two surfaces by $\theta=\arctan \mu$.

Using data about the friction coefficient for variety of materials, we can determine the maximum angle at which she can travel up the ramp, which is $\theta=49.27^{\circ}$ (see "Specifications" for further detail).
Evaluation
Feasibility: Strong
Safety: Poses little risk
Notes: The stunt double's shoes should be rubber-soled
Duration: 0:25

### 3.3 Brachistochrone Slide

In one part of our chase scene, the heroine slides from an elevated height to a lower height. We would like for her to accomplish this in the shortest time. It is assumed she starts at rest at the elevated height, stays in contact with the path, and is only acted on by a constant force of gravity (no friction). It turns out that the curve that we have constructed for this purpose is an inverted cycloid.

Evaluation
Feasibility: Strong
Safety: Poses little risk
Notes: The stunt actor's landing will be cushioned to prevent injury
Duration: 0:20

### 3.4 Parachuting Soldiers

We consider the problem of Israeli Defense Forces soldiers parachuting (from free fall). In the case of free fall, there will be two forces that need to be considered: gravity and the resistive drag force. A good model for the drag force is: $F_{\text {drag }}=\frac{1}{2} D \rho A v^{2},{ }^{3}$ where $D$ is the so-called "drag coefficient," $\rho$ is the density of air, $A$ is the cross-sectional area of the person, and $v$ is the speed.

We find that for a parachute of dimension $20 \mathrm{~m}^{2}$, the terminal velocity would be around $10 \mathrm{~m} / \mathrm{s}$, which would provide a safe landing.
Evaluation
Feasibility: Strong
Safety: Poses little risk to experienced parachuters
Notes: The stunt actor's landing will be cushioned to prevent injury
Duration: 0:25

### 3.5 Car on a Banked Curve

In this stunt, Nora's burly pursuers are driving their car on a banked curve, depicted in the movie as a highway off ramp that is at an angle $\theta$ with the horizontal. We will take the radius of the turn and speed of the car as given as we try to maximize $\theta$ so that the stunt looks as dangerous as possible.

We use the relationship between tangential and centripetal force to find the car's acceleration: $m a=m \frac{v^{2}}{r} .{ }^{4}$ and find that the angle can be as great as $68.2^{\circ}$; however, to guarantee safety we will use a $45^{\circ}$ angle.
Evaluation
Feasibility: Strong
Safety: Poses little risk

[^1]Notes: The car is traveling at a constant 80.0 mph , which we set as the velocity at which static friction plays no role ${ }^{5}$
Duration: 0:20

### 3.6 Car Flying Off a Bridge

Here, our stunt actress drives with a car from a bridge into a body of water. We will construct a ramp (of angle $\theta$ with the horizontal) to increase the distance $x$ which the car traveling at a velocity $v$ covers before hitting the water. The height of the bridge is $h$ and the height of the ramp is $h_{r}$. Our objective is to find out, given values for the aforementioned variables, how long the stunt will take.

We use projectile motion physics equations to evaluate this stunt:

$$
t=-v \sin \theta-\sqrt{\frac{(v \sin \theta)^{2}-4(4.9)\left(h+h_{r}\right)}{-9.8}} .6
$$

We find in "Specifications" that the car will be in the air for 3.38 seconds, and given dramatic film techniques, this stunt will take ten seconds. For the sake of safety, we will not have a stunt actor driving the car during the stunt and will execute it remotely to prevent fatal injuries.
Evaluation
Feasibility: Strong
Safety: Poses no risk to human, as it is unmanned
Notes: We would need a cheap car
Duration: 0:25

### 3.7 Car Lands in Water

In this stunt we want to know how much force the car needs to be able to withstand when it hits the water surface. We model this stunt on a Mazda3,

[^2]which is the most driven car in Israel ${ }^{7}$. We find that the tensile strength of steel is greater than the force the car hits the water, making this stunt safe.

From the projectile motion equations in 3.6 , we can model the path of the car by:

$$
\begin{aligned}
x(t) & =(v \cos \theta) t \\
y(t) & =\left(h+h_{r}\right)+(v \cos \theta) t-4.9 t^{2}
\end{aligned}
$$

We use these path equations to determine the car's acceleration, from which we can calculate its force. We then compare this force with the compressive force of steel and show that the car can withstand this crash. Please see "Specifications" for further detail.
Evaluation
Feasibility: Strong
Safety: Poses no risk to human, as it is unmanned
Notes: We would need a cheap car
Duration: 0:05

### 3.8 Floating Car Escape

In this part of the stunt sequence, we want our heroine to escape from the car unharmed. There are two ways for a person to escape out of a sinking car. Either, they can escape while the car is still above water and the water pressure against the sides is not big enough to make it impossible for them to open the door, or they can wait till the car is mostly flooded with water, so the pressure is equalized and they can open a door or window underwater. Since the second method can lead to panic and serious injury, we deem it safer for our action heroine to escape the car while it is yet afloat. To enable such an escape, we will make the car waterproof, to keep it from sinking. Also, when the actress escapes, she should open a window, not a door to leave the car.
Evaluation
Feasibility: Strong
Safety: Poses little risk

[^3]Notes: The car needs to be well-sealed, no water may enter, the stunt actress needs to escape through the window
Duration: 0:35

### 3.9 Swimming Against a Current

In this final part of the stunt sequence, we want the actress to swim ashore. The objective here is to create a suspenseful atmosphere by only providing a narrow stretch of shore accessible from the water. We want our stunt double to just barely reach the end of the accessible area. Since it is most intuitive for the actress to swim directly towards the shore, we want to know, given the velocity $v_{s}$ at which she can swim, the velocity $v_{r}$ at which the river is flowing, and her distance $d$ from the shore, how long the stretch $l$ of accessible area needs to be so she can get ashore.

We can then evaluate the time it will take her to swim ashore and then calculate her displacement along the shore using the general physics equation $d=\|v\| t,{ }^{8}$ where $\|v\|$ is the speed. We find in "Specifications" that she will be displaced 10 m .
Evaluation
Feasibility: Strong
Safety: Poses little risk
Notes: The shore should be longer than 10 m in case our swimmer cannot maintain her speed. Also, we will have to find a location with a relatively slow current. In case we cannot exactly match these current figures, the process for calculating the distance can be amended to accommodate the real conditions we will encounter upon production
Duration: 0:40

### 3.10 Helical Run

In this scene, Nora is being chased through a parking lot. Here we try to find the distance Nora can travel in the 20 -second time lot we have allotted for this stunt. We will approximate her path through the parking garage to be a helical path, which can be parameterized as:

$$
\overrightarrow{\mathbf{c}}(t)=(r \cos (\omega t), r \sin (\omega t), t)
$$

[^4]where $\omega$ is angular velocity and $r$ is the radius from the center axis to the path of motion. We find that she can travel 158.8 ft in the allotted 20 seconds.

## Evaluation

Feasibility: Strong
Safety: Poses little risk
Notes: The stunt actor's speed must be held constant at the rate of a 7:00 minute mile for the stunt to be completed as written above
Duration: 0:20

### 3.11 Swinging on a Rope

The primary safety concern for the rope swing stunt is the tensile strength of the rope used. The set-up is such that the friction between the rope and axel is negligible because the rope would be attached to a large metal washer that would have a steel axel running through the hole, and both the inner hole surface of the metal washer and the steel axel would be greased. This way, the friction between the surfaces is small and would have a negligible effect on the tension of the rope during the stunt.

We use the centripetal force equation to show that the rope can handle the tension of the stunt: $F_{\text {centripetal }}=T-m g \cos \theta=\frac{m v^{2}}{R} .{ }^{9}$ The rope data in "Specifications" shows that the rope can certainly tolerate the tension caused by the weight of the stunt actor.
Evaluation
Feasibility: Strong
Safety: Poses little risk
Notes: Friction between the rope and the axel must be minimized
Duration: 0:20

### 3.12 Jumping into an Open Window

Here, we will analyze a stunt in which our heroine jumps from a building through an open window. In order to optimize the impressiveness of the stunt, we wish to maximize the distance that she can travel. We will find that the horizontal displacement $d$ depends on the stunt double's initial takeoff angle $\theta$ with the horizontal.

[^5]We evaluate this stunt on the equation that has been derived for long jumpers,

$$
d=\frac{v^{2} \sin 2 \theta}{2 g}\left(1+\sqrt{1+\frac{2 g h}{v^{2} \sin ^{2} \theta}}\right),{ }^{10}
$$

and find that the optimal angle for take off is $22^{\circ}$.

## Evaluation

Feasibility: Strong
Safety: Poses little risk
Notes: As in the case of the long jump, which employs sand, the stunt double's landing should be cushioned
Duration: 0:15

### 3.13 Skateboarding Down an Incline

The primary concern for the downhill skateboarding stunt is air friction, which would reduce the speed of the stunt actor moving down the hill. We need to optimize the speed of the skateboarding actor such that it is safe to perform the stunt while maintaining dramatic effect. We find that the skateboarder's terminal velocity is

$$
v_{\text {terminal }}=\sqrt{\frac{2 m g \cos \theta}{C \rho A}},{ }^{11}
$$

which, given a maximum velocity that we determine for the purposes of safety, allows us to solve for the maximum angle at which this stunt can be performed safely.
Evaluation
Feasibility: Strong
Safety: Poses little risk
Notes: We would place cushioned material at the end of the incline for extra safety
Duration: 0:20

[^6]
### 3.14 Bungee Jump

The problem of bungee cord jumping is very complex, but we will attempt to use a simple model to describe the situation. We model the bungee cord as a spring and assume that there is no energy dissipation. Then the problem becomes one of simple harmonic motion. The oscillation resulting from our inital set-up is given as $x(t)=A \cos (\omega t+\pi)$, where $\omega=\sqrt{\frac{k}{m}}$. We find that a reasonable speed for the action hero to release at a lower level would be 10 $\mathrm{m} / \mathrm{s}$.
Evaluation
Feasibility: Strong
Safety: Poses little risk
Notes: The time for the stunt will vary slightly from the calculations, as real-world effects will apply. For this reason we utilize extra safety measures Duration: 0:25

## 4 Specifications

We present analyses of several stunts to be included in the trailer. Each stunt is intended to be used as a splice, ranging in duration from ten to thirty seconds. Each stunt is contextually tied to the rest because throughout the trailer the heroine will be running to escape from corrupt and dangerous army officers. It can therefore be assumed that each of the following stunts occurs while the heroine is running away from these belligerent men.

The following specifications are presented in order according to the timeline in Section 2.2. For dramatic effect, stunts may utilize slow motion, multiple camera angles, and splicing. These film-editing techniques elongate the time each stunt takes relative to real time, therefore, we must take into account these techniques when designating the time each stunt will take.

### 4.1 Bus Bombing

Figure 4.1. A schematic diagram of the bus falling from a height $h$. The $F$ denotes the force due to gravity, $M$ is the mass of the bus.


A few conditions we will create to simplify the stunt as well as to maintain safety, predictability, and reproducibility are that we will take a decrepit bus and strip it and remove the engine. This way, the center of mass in the horizontal component will be in the center, and the bus will land as close to flat on its wheels as possible rather than on an edge.

We will attach the bus to a junk-yard magnet. A typical bus has a mass of $4,726 \mathrm{~kg},{ }^{12}$ which would indicate that the force of gravity on such a bus would be $F_{g}=M g=46,314.8 N$.

There are electromagnets that can exceed gravity's force on the bus, which would be sufficient to hold it up before the drop ${ }^{13}$. The electromagnetic force of a junkyard magnet is $F=\frac{B^{2} A}{2 \mu_{0}}$, where $B$ is the magnetic field (for a typical iron core is 1.6 T ), $A$ is the cross-sectional area of the core, which we will take to be $0.05 \mathrm{~m}^{2},{ }^{14}$ and $\mu_{0}$ is the permeability of free space, equal to $4 \pi \times 10^{-7} \frac{T \cdot m}{A}$. Therefore, the force exerted by the junkyard crane is $F=50,955 \mathrm{~N}$.

This electromagnetic force is much greater than the force of gravity on the bus, so the junkyard crane is feasible for use in this stunt.

[^7]Now, we have to calculate the time the drop will take starting from a height of 5 m . Because the height is relatively small, air resistance is negligible, so we take the potential energy at the top to be equal to the kinetic energy at the bottom of the drop ${ }^{15}$ and solving for velocity we get $v_{f}=\sqrt{2 g h}$.

We use a well-known physics equation relating velocity, acceleration, and time ${ }^{16}$ : $v_{f}=v_{0}+a t$. Plugging in the $v_{f}$ from the energy equation, and noting that the initial velocity is $0 \mathrm{~m} / \mathrm{s}$, we can solve for $t$. Noting that the acceleration on the bus is only that due to gravity, we can substitute $g$ for $a$. Finally, substituting in our values in this equation, we can find the numerical time value, $t=1.01 \mathrm{~s}$. The amount of time the bus will spend in the air is therefore, 1.01 seconds.

### 4.2 Frictional Incline

We determine that if the angle between the incline and the ground is $\theta$ and the friction between the heroine's shoes and the incline is $\mu$, that the angle at which we can set the ramp is correlated with the friction between the two surfaces by $\theta=\arctan (\mu)$.

Table 4.2. Friction coefficients for common surfaces as well as their incline angles ${ }^{17}$

| Second Material | Friction coefficient | Angle (degrees) |
| :--- | :--- | :--- |
| Rubber | 1.16 (kinetic) | 49.27 |
| Concrete (dry) | 1.0 (static) | 45.00 |
| Asphalt (dry) | $0.5-0.8$ (kinetic) | $26.57-38.66$ |
| "Solids" | $1.0-4.0$ (static) | $45.00-75.96$ |

For those material combinations for which we could only determine the kinetic friction, we can say that the kinetic friction coefficient is a conservative estimate for the static friction coefficient. From the table, we see that the combination of rubber on rubber is rather promising, as it provides us with

[^8]the conservative estimate of an angle of $49.27^{\circ}$. Rubber on "solids" also seems very promising, though we would have to find out which solids give us the highest friction.

Figure 4.2. The normal force $(N)$ is perpendicular to the surface, friction $(F), \theta$ is the incline


For an object in equilibrium on an incline, we have

$$
m g \sin \theta=F_{\text {friction }} .
$$

Since $N=m g \cos \theta$ and $F_{\text {friction }} \leq \mu_{s} N$, to find the max angle allowed we set $m g \sin \theta=\mu m g \cos \theta$. Solving this, we get $\frac{\sin \theta}{\cos \theta}=\mu$, or $\tan \theta=\mu$ and $\theta=\tan ^{-1} \mu$. This angle is the angle at which we will incline the plane for this stunt.

### 4.3 Brachistochrone Slide

Figure 4.3. A schematic diagram of the parameterization of a cycloid


From Figure 4.3 we find that the parametric equations for a cycloid are:

$$
\begin{gathered}
x(t)=a(t-\sin t) \\
y(t)=a(1-\cos t)
\end{gathered}
$$

where $a$ is the radius of the circle that generates the cycloid. The parameter interval will be taken to be $0 \leq t \leq \pi$.

One of the first aspects to consider is the total distance along which the action hero slides (that is, the arc length of the curve). We compute this as follows:

$$
\begin{aligned}
L & =\int_{0}^{\pi} \sqrt{\left(\frac{d x}{d t}\right)^{2}+\left(\frac{d y}{d t}\right)^{2}} d t \\
& =\int_{0}^{\pi} \sqrt{a^{2}(1-\cos t)^{2}+a^{2} \sin ^{2} t} d t
\end{aligned}
$$

The length turns out to be $4 a$. This would be important to consider in constructing the curve, since we would like to know how much material is needed.

We should also take into account her final velocity as she exits the curve. This may be computed with conservation of energy: $\frac{1}{2} m v^{2}=m g h$, which gives $v=\sqrt{2 g h}$. Assuming a vertical displacement of 10 m , which is roughly equivalent to two stories of a building, the exiting velocity would be about $14 \mathrm{~m} / \mathrm{s}$, which is pretty safe.

### 4.4 Parachuting Soldiers

The natural thing to consider is the terminal velocity of the soldiers, which occurs when the drag force and gravitational force balance: $m g=\frac{1}{2} D \rho A v^{2}$. This gives us $v_{\text {terminal }}=\sqrt{\frac{2 m g}{D \rho A}}$. Most sources quote the terminal velocity of an average person as $60 \mathrm{~m} / \mathrm{s}$. Of course, this is probably not a safe speed for landing.

With a parachute, the terminal velocity is greatly diminished. Upon opening the parachute, the drag force increases dramatically to yield a net upward force on the person, and a new terminal velocity is eventually reached. With a parachute of area $20 \mathrm{~m}^{2}$, for example, the terminal velocity would be around $10 \mathrm{~m} / \mathrm{s}$, which is safe for landing.

Figure 4.4. The person of mass $m$ experiences the gravitational force of $m g$ downward and a resistive drag force of $F_{\text {drag }}$ upward.


### 4.5 Car on a Banked Curve

Let us set the velocity of the car at $80.0 \mathrm{mph}(35 \mathrm{~m} / \mathrm{s})$, which has been determined to be the maximum average velocity of the esteemed Jake and Elwood Blues' car during a Chicago-based chase scene in the 1980 movie The Blues Brothers ${ }^{18}$. Additionally, we will choose $50.0 \mathrm{~m}(164 \mathrm{ft})$ to be the radius of our turn-in fact; this is an arbitrary number, as we will need to construct the roadway, anyway, to place it at our desired angle $\theta$.

A car moving along a circular path with a constant speed has a changing velocity, because the car itself is constantly changing direction. So, the car's acceleration will point toward the center of the circle, such that $a=\frac{v^{2}}{r}$. Recall Newton's Second Law, which relates force to acceleration with the relationship $\sum F=m a$. That is, the mass of an object multiplied by its acceleration in a certain direction will give the force in that direction ${ }^{19}$. Combining the above equation, the equation for centripetal force can be written as $m a=m \frac{v^{2}}{r}$.

Under real-life conditions, the car will experience some static friction with the road surface that will oppose its forward motion. There is an ideal speed $\left(v_{\text {ideal }}\right)$ for which the force of static friction does not impact the car's motion, because the car has exactly the velocity that fulfills the aforementioned equation for centripetal force.

[^9]If the car's velocity varies from $v_{\text {ideal }}$, the force of static friction will act to pull the car either up or down the curve ${ }^{20}$. However, we will assume that our stunt car will move at $v_{\text {ideal }}$ with regards to the $\theta$ at which we construct the roadway. Because we are varying the angle, we can take an arbitrary velocity (in this case, 80.0 mph ) and set it as our ideal velocity.

Then, note that there is a normal force $N$ pointing perpendicular to the surface of the road, the downward force of gravity (equaled to the weight of the car, 3400 lbs ), and the net force is pointed inward. We can start to use our circular motion equations to relate $\theta$ to $v$. Note that we have not rotated our axes; $y$ and $x$ point vertically and horizontally, respectively.

Figure 4.5. The car moves along a road at angle $\theta$ with the horizontal. It experiences a normal force $(N)$, pointing up from the top of the car and orthogonal to the road, as well as a gravitational force of magnitude $m g$. Its net force $\left(F_{n e t}\right)$ points inward.


The car has no vertical velocity, so it also has no vertical acceleration. Thus, we can solve for $N$ in terms of our other constants and $\theta: \sum F_{y}=m a_{y} \Longrightarrow$ $N \cos \theta-m g=0 \Longrightarrow N=\frac{m g}{\cos \theta}$.

Now, let us deal with the $x$-component of the force: $\sum F_{x}=m a_{x} \Longrightarrow$ $-N \sin \theta=-m \frac{v^{2}}{r}$. We can plug in the equation for $N$ that we found above: $m g \tan \theta=\frac{m v^{2}}{r}$. Solve for $\theta$ to find that $\theta=\arctan \left(\frac{v^{2}}{g r}\right)$, and then plug in the values of our constants (note that, too, the acceleration due to gravity $\mathrm{g}=9.8 \mathrm{~m} / \mathrm{s}^{2}$ ) to find a maximum $\theta$ of $68.2^{\circ}$. This is a severe angle, but we can choose a more reasonable one of $45^{\circ}$ to achieve a comparably dramatic effect.

[^10]
### 4.6 Car Flying Off a Bridge

Using projectile motion, we can determine that the time the car spends in the air is modeled by

$$
t=-v \sin \theta-\sqrt{\frac{(v \sin \theta)^{2}-4(4.9)\left(h+h_{r}\right)}{-9.8}},{ }^{21}
$$

and the car's horizontal displacement is $x=(v \cos \theta) t{ }^{22}$
We know that the most widely sold car in Israel is the Mazda3 series, which has an estimated high-speed of about $170 \mathrm{~km} / \mathrm{h}$. For safety reasons we might want to limit the speed at which the car is driving to $145 \mathrm{~km} / \mathrm{h}$. Assuming that the bridge is about 8 m high, if there is no ramp, the car is in the air for 1.28 seconds and covers a distance of 51.47 meters. If we let the height of the ramp be 1.5 meters and its angle $20^{\circ}$ (the ramp would be 5.6 meters long), then the car would be in the air for 3.38 seconds and would hit the water 128.09 meters from the base of the bridge.

Figure 4.6. A diagram of the projectile motion of the car, where $v$ is the velocity of the car, $\theta$ and $h_{r}$ the angle and height of the ramp, $h$ the height of the bridge, and $x$ the horizontal distance the car covers before it hits the water.


We can describe the horizontal and vertical displacement as functions of time $(x(t)$ and $y(t)$, respectively) of the car launched at an angle $\theta$ at a height

[^11]$\left(h+h_{r}\right)$ traveling at an initial velocity of $v$ as the following:
\[

$$
\begin{aligned}
& x(t)=0+(v \cos \theta) t-\frac{1}{2} \cdot 0 \cdot t^{2} \\
& y(t)=\left(h+h_{r}\right)+(v \sin \theta) t-4.9 t^{2}
\end{aligned}
$$
\]

When the car lands in the water, its vertical height $y(t)$ will be equal to zero, so $0=\left(h+h_{r}\right)+(v \sin \theta) t-4.9 t^{2}$, which we can solve using the quadratic formula to get the time.

### 4.7 Car Lands in Water

From the above stunt, we know that the displacement of the car in the $x$ and $y$ direction can be described by

$$
\begin{aligned}
& x(t)=0+(v \cos \theta) t-\frac{1}{2} \cdot 0 \cdot t^{2} \\
& y(t)=\left(h+h_{r}\right)+(v \sin \theta) t-4.9 t^{2} .
\end{aligned}
$$

Figure 4.7. A diagram of the car as it hits the water. The letter $m$ denotes the mass and $F$ is the force with which the car hits the water.


The path of the car can be described by:

$$
\vec{c}(t)=(v \cos \theta) t \vec{i}+\left[\left(h+h_{r}\right)+\left((v \sin \theta) t-4.9 t^{2}\right)\right] \vec{j}
$$

We can therefore describe the velocity of the car as the derivative of its path vector:

$$
\vec{c}^{\prime}(t)=(v \cos \theta) \vec{i}+((v \sin \theta)-9.8 t) \vec{j} .
$$

And by the same logic, we can find the acceleration by differentiating the velocity vector:

$$
\vec{c}^{\prime \prime}(t)=-9.8 \vec{j}
$$

Since we know that the force $F$ on an object equals its mass $m$ times its acceleration, the downward force with which the car acts on the water, given that the Mazda3 weighs about an average of 1247.5 kg , is $F=12,225.5 \mathrm{~N}$.

The compressive stress ${ }^{23}$ that stainless steel can tolerate is approximately $1.70 \times 10^{8} \mathrm{~Pa},{ }^{24}$ and the steel used in cars is typically more reinforced, therefore this value would be higher. In any case, determining the force from the pressure by multiplying by surface area of the front of the car, which is 2.56 $\mathrm{m}^{2},{ }^{25}$ we get

$$
P=\frac{F}{A} \Longrightarrow F=4.35 \times 10^{8} \mathrm{~N}
$$

This value is far greater than the force the car exerts on the water, which means this stunt is safe and reasonable, though for practical purposes, we would not have a stunt actor perform this stunt from inside the car.

### 4.8 Floating Car Escape

Figure 4.8. Floating car, where $m$ is the mass of the car, $m g$ the downward gravitational force acting on the car, and $\rho V g$ the upward buoyancy acting on the car from the water.


The overall force acting on the car is equal to the force of gravity acting on the car minus the force of buoyancy ${ }^{26}$, or $F=m g-\rho V g$. The car will float if

[^12]the force of buoyancy is greater or equal to the force of gravity: $\rho V g \geq m g$. The density $\rho$ of water ${ }^{27}$ at $25^{\circ} \mathrm{C}$ (a good estimate for the temperature of river water in Tel Aviv in the summer) is $997.0479 \mathrm{~kg} / \mathrm{m}^{3}$. Furthremore, $V$, the water displaced by the car would probably be less than or equal to half the car's volume. Given the measurements of the Mazda3 Sedan ${ }^{28}$, that would be $V=\frac{1}{2}(4.511 \times 1.755 \times 1.465)=5.7991 \mathrm{~m}^{3}$.

This means that the weight of the car and the stunt actress combined cannot exceed 5781.94 kg , which, given that the Mazda3 has a curb weight of $1180-1315 \mathrm{~kg}$ and our actress weighs somewhere around 60 kg , is perfectly reasonable.

### 4.9 Swimming Against a Current

It seems reasonable to assume that our stunt double can swim at a constant velocity of 1.8 meters per second (current record for 50 meters freestyle is a little over two meters per second ${ }^{29}$ ) and that the river flows at a constant velocity parallel to the shoreline at 0.6 meters per second ${ }^{30}$. The distance to the shore can be 30 meters (so the stunt would take a maximum of 17 seconds; also, from the first part of the stunt we know that the car covers a distance of about 130 meters, so the river needs to be about 160 meters wide). In other words, $v_{s}=1.8 \mathrm{~m} / \mathrm{s}, v_{r}=0.6 \mathrm{~m} / \mathrm{s}, d=30 \mathrm{~m}$.

[^13]Figure 4.9. A diagram of the stunt actress swimming ashore, where $m$ is the mass of the swimmer, $v_{r}$ the velocity of the river, $v_{s}$ the velocity of the swimmer, $d$ the distance to the shore, and $l$ is the length of accessible shoreline.


The horizontal component of velocity can be represented as $\overrightarrow{v_{s}}=\left(v_{s}, 0\right)$. Similarly, the vertical component can be represented as $\overrightarrow{v_{r}}=\left(0, v_{r}\right)$.

Then, relating distance travelled to velocity and time with $d=\left\|v_{s}\right\| t$, where $\left\|v_{s}\right\|$ is the speed of the swimmer, we can solve for the time it will take the swimmer to reach the shore: $30=1.8 t \Longrightarrow t=1.6 \mathrm{~s}$.

The time it takes the swimmer to reach the shore is the same amount of time the current has to act on the swimmer and change her displacement along the shore. Using the same logic and equation as we did to find the time it takes her to swim to shore, we can find the distance the current displaces her by: $d=\left\|v_{r}\right\| t \Longrightarrow d=(0.6)(16.7)=10 \mathrm{~m}$. Therefore, she will hit the shore 10 m from her original position, so the shore must be 10 m long.

### 4.10 Helical Run

Figure 4.10 At time $t=0$, Nora starts off at the first X with some initial speed, which we will take to be $7.00 \mathrm{~min} / \mathrm{mile}$, or $3.83 \mathrm{~m} / \mathrm{s}$. Her speed stays constant as she runs up the helix for 20 seconds.


To find the distance she travels, we need to parameterize her path and then calculate the arc length from $0 \leq t \leq 20$. A helix can be thought of as a curve being traced out on the surface of a cylinder. A helical path can be parameterized as:

$$
\overrightarrow{\mathbf{c}}(t)=(r \cos (\omega t), r \sin (\omega t), t)
$$

where angular velocity $\omega$ is related to linear velocity $v$ by $\omega=\frac{v}{r}$.
In this case, let's take the radius of the cylinder (that is, the distance between the center of the parking garage and running Nora) to be 36 feet, which is a standard minimum radius for the outside wall of a circular ramp ${ }^{31}$. Let us also set her speed at a constant $7.00 \mathrm{~min} / \mathrm{mile}$ pace, which is the equivalent of $12.6 \mathrm{ft} / \mathrm{s}$. Thus, the parameterization becomes:

$$
\overrightarrow{\mathbf{c}}(t)=\left(36 \cos \left(\frac{12.6}{36} t\right), 36 \sin \left(\frac{12.6}{36} t\right), t\right)
$$

[^14]Then, taking the derivative of the path yields the velocity function, on the same interval:

$$
\overrightarrow{\mathbf{c}^{\prime}}(t)=\left(-12.6 \sin \left(\frac{12.6}{36} t\right), 12.6 \cos \left(\frac{12.6}{36} t\right), t\right)
$$

Now, plugging this into the formula for arc length and using the appropriate $t$ values (0 to 20), we find that

$$
L=\int_{t_{i}}^{t_{f}}\left\|\overrightarrow{\mathbf{c}}^{\prime}(t)\right\| d t=158.8 \mathrm{ft}
$$

This means that we can film Nora being chased up 158.8 ft of the parking garage in 20 s of footage. Since the surface she runs on will be even, it is reasonable to expect that the stunt actor maintain a constant pace. Moreover, because the stunt poses little risk to the actor, it is highly feasible.

### 4.11 Swinging on a Rope

Figure 4.11. The rope swing is a uniform circular motion problem, with an inward radial force $T$ and a constant downward gravitational force $m g$. The rectangle represents the position of the actor.


The rope swing is a circular motion problem, with an inward radial force of tension $T$ and a constant downward gravitational force of $m g$.

Breaking the gravitational force into components, the gravitational component in the opposite direction of tension is $m g \cos \theta$. Therefore, the centripetal force is the total inward radial force, which includes tension and the radial component of gravitational force is:

$$
F_{\text {centripetal }}=T-m g \cos \theta=\frac{m v^{2}}{R}
$$

To solve for tension, a second equation is needed to allow us to solve for velocity. Energy is conserved, and therefore:

$$
0+m g R=\frac{1}{2} m v^{2}+m g(R-R \cos \theta)
$$

where the potential energy is set to be 0 at the bottom of the circle, and so $v=\sqrt{2 g r \cos \theta}$.

Substituting this value of $v$, we can solve for tension to get $T=3 m g \cos \theta$. We are considering the range $0 \leq \theta \leq \frac{\pi}{2}$. Note that $T \leq 3 m g$, and we achieve the maximium tension when $\theta=0$, which is at the bottom of the circle.

In order for this stunt to be completed safely, this max tension must be less than the sum of the tensile strength of the rope and the circumferential strength of the steel bar.

The average tensile strength of a typical rock climbing rope is 6500 lb and the safe working load is $450 \mathrm{lb}^{32}$, which in itself is greater than three times the weight of the stunt actor. The average modulus of elasticity for steel is $29,000 \mathrm{ksi}^{33}$. Therefore, the ratio of stress to elasticity is the percent of the length of the steel rod that will stretch downward under too much stress; however, the large modulus of elasticity indicates that the steel rod requires a great deal of downward force to reach deformation. The sum of tensile strength and circumferential strength is much greater than three times the weight of the stunt actor, therefore, this stunt is safe and feasible.

### 4.12 Jumping into an Open Window

Past research has been conducted on the nature of the long jump, which very closely mirrors the stunt at hand. In performing the long jump, the

[^15]jumper gathers horizontal velocity by taking a running start, takes off from the ground at angle with respect to the ground, and minimizes her rotational velocity over the course of her flight.

Although $\theta=45^{\circ}$ optimizes the displacement of an object in projectile motion, a smaller angle is more ideal for a long jumper. This is because her primary concern is to maximize $d$, not to obtain vertical height; the jumper would actually have to slow her approach to gather the vertical thrust necessary for a $45^{\circ}$ departure, thereby decreasing her $v_{x}$ and, subsequently, the $d$ that she can achieve. The following is a schematic representation of the long jump:

Figure 4.12. The maximum $d$ the jumper can achieve relates to a variety of factors, including her take-off angle and her center of mass's vertical displacement. The rectangle represents the location of the actor and $v$ is her initial velocity.


The jumper's horizontal displacement has been found to relate to $\theta$ by

$$
d=\frac{v^{2} \sin 2 \theta}{2 g}\left(1+\sqrt{1+\frac{2 g h}{v^{2} \sin ^{2} \theta}}\right) \cdot{ }^{34}
$$

Combining this speed-angle formula with that for the displacement of an object in projectile flight, we find that the optimal $\theta$ is well under $45^{\circ}$, and in fact is about $22^{\circ}$ (recall that this is approximately the take-off angle that long jumpers strive to achieve).

[^16]
### 4.13 Skateboading Down an Incline

Figure 4.13. A force diagram showing the components of each force acting on the stunt actor. $N$ is the normal force pointing perpendicular to the plane of motion, $M g$ is the gravitational force pointing downward, and $F$ is the frictional force (both surface and air friction) pointing opposite the direction of motion.


For large objects moving at relatively high velocity, the frictional drag is proportional to the square of the velocity, $F_{d r a g}=-\frac{1}{2} C \rho A v^{2}$, where $C$ is the drag coefficient, $\rho$ is the air density, $A$ is the cross-sectional area of the object, and $v$ is the velocity.

The air density can be calculated based on the ideal gas law, $P V=n R T$, where $P$ is pressure, $V$ is volume, $n$ is the number of moles of gas, $R$ is the gas constant, and $T$ is temperature (in Kelvin). The air does not adhere perfectly to the ideal gas law, but it provides an approximation that is close enough to reality for this calculation.

Because it is implausible to know the number of moles of gas in this scenario, we understand the moles/volume quantity to be related to air density in such a way that $\rho=\frac{P}{R T}$. The environmental conditions can be determined such that the temperature outside can be 70 degrees Fahrenheit and 14.6916 psi (typical of sea level conditions, such as Chicago), which would give us an air density of $0.074887 \mathrm{lbs} / \mathrm{ft}^{3}$.

The stunt actor should be about 5 ' 6 " and 120 lbs , which would give her a cross sectional area of about $924 \mathrm{in}^{3}=6.42 \mathrm{ft}^{3}$. This calculation is based on the assumption that the average width of the stunt actor is 14 ".

A typical value for the drag coefficient of a human moving at a relatively high velocity (which would be between a stationary man and a ski-jumper, two situations for which the drag coefficients are known) is 1.0-1.3. The drag coefficient is dimensionless by definition.

The stunt actor will reach her terminal velocity when the drag force equals the gravitational force component in the direction of motion. In this case, this gives $v=\sqrt{\frac{2 m g \cos \theta}{C \rho A}}$. Let us assume that for safety reasons we do not want her velocity to exceed 30 mph . The maximum angle at which we could incline the plane of her motion would be given by $\frac{1}{2} C \rho A v^{2}=m g \cos \theta$, or $\theta=\arccos \left(\frac{C \rho A v^{2}}{2 m g}\right)$. Substituting in the values for each of these constants, and assuming her weight is $120 \mathrm{lbs}, \theta=89.94^{\circ}$. The fact that the angle can be so nearly perpendicular to the ground indicates that this stunt is perfectly safe if performed at a more reasonable, 40 degree angle.

### 4.14 Bungee Jump

The problem of bungee cord jumping is very complex, but we will attempt to use a simple model to describe the situation. We model the bungee cord as a spring, and assume that there is no energy dissipation. Then the problem becomes simple harmonic motion. (Note that if we wanted to consider damping, the differential equation would become $\frac{d^{2} x}{d t^{2}}+\frac{b}{m} \frac{d x}{d t}+\frac{k}{m} x=g$, where we have modeled the damping force to be proportional to $v$. The solution is oscillatory in nature, but enveloped by exponentially decaying functions. That is, the amplitude of the oscillation is time-varying and decays exponentially.)

Suppose that the person of mass $m$ is attached to a spring with spring constant $k$. In the equilibrium position, the person is stretched a distance $\frac{m g}{k}$ from the top. Let this position be the origin; then the oscillation will be about this point. Assume that initially the person is a distance $A$ above the equilibrium position, so the spring is compressed by $A$. That is, the position at $t=0$ is $x=-A$ (where the positive direction is downward). Then the oscillation is given by $x(t)=A \cos (\omega t+\pi)$, where $\omega=\sqrt{\frac{k}{m}}$. For example, it would take $\pi \sqrt{\frac{m}{k}}$ to go a distance $2 A$ downward (a displacement of $A$ from the equilibrium position), which is half a cycle.

This can provide a rough estimate of the time it would take for our hero to reach a lower level of a building, for example, by stepping off $\left(v_{0}=0\right)$ the ledge of a higher level. It should be noted that a relatively stiff spring (large $k$ ) would be chosen for this purpose, because in fact if a spring streches too far from its equilibrium position it behaves in a non-Hookian manner.

Since velocity is the derivative of position, $v(t)=-A \omega \sin (\omega t+\pi)$. To ensure safety, the velocity should not be too high at the desired position of
release. A reasonable speed might be $10 \mathrm{~m} / \mathrm{s}$, as in the case of the parachuting problem above.

Figure 4.14. The object of mass $m$ is attached to a spring of spring constant $k$. The two forces acting on the object are that of gravity $m g$ and the restoring spring force $k x$.


## 5 Evaluation of Proposal

The budget for the trailer of Draftee Summer does not allow for the use of computer-generated imagery (CGI). This limitation forced the production team of TARNADO Productions, LLC. to choreograph stunts that could be safely performed by stunt actors. This proposal has demonstrated that the choreographed stunts are both feasible and safe for use in the trailer.

There are, however, limitations to some of the mathematical justifications. We make many conditional assumptions, especially in regards to friction (air, static, and kinetic). When these stunts are performed real-world effects will occur, which will slightly alter the calculations. Overall, however, the calculations provide good approximations for the conditions necessary to perform these stunts.

In the future, we could extend these calculations to computer models, where we could include real-world effects, which would provide even better approximations for these stunts.

Another limitation to this proposal is the fact that these stunts are very expensive to execute. Many of them require access to disposable cars and heavy machinery. Though our budget could not accommodate the use of CGI, we assume it could accommodate the expensive supplies we require for
the performance of these stunts. We acknowledge that we made many assumptions; however, the purpose of this proposal was to justify the use of stunt actors in making a legitimate action movie trailer. If TARNADO Productions, LLC., accepts the expenditures for this trailer, then this proposal is sufficient to show that the stunts are feasible and safe.

## 6 Conclusion

After careful calculations, we have mapped out a 7:00 minute trailer for the dramaction movie Draftee Summer that incorporates fourteen live-action stunts. Normally, many of these stunts would be embellished by computer animation; however, we have done our mathematics to ensure that we produce stunning action sequences while eschewing the mammoth costs of CGI. Shooting for the movie has already begun, and its release date has been set for Summer 2012. The trailer will be released in Summer 2011.

In addition to serving as a roadmap for the trailer, our proposal demonstrates the feasibility of live-action stunts. We have shown that stunt actors can perform perfectly harrowing feats with the buoying force of mathematics ensuring their safety. As such, we hope that it will inspire future production teams to look at mathematics as a viable source of cinematical thrill and establish TARNADO Productions, LLC. as a formidable force in the filmmaking industry

## 7 Acknowledgements

Draftee Summer's production team would like to thank TARNADO Productions, LLC. for its unceasing and generous support for applied mathematics. In an age when computer generated graphics have rendered mathematics nearly obsolete in the filmmaking industry, TARNADO finances technical acumen just as much as creative genius. As a result, its films reflect a deep understanding of the physical laws that govern our world, rather than merely representing an ostentatious display of wealth. Thanks to TARNADO, we were able to produce an action movie with a message, something that moviegoing audiences rarely find; for this opportunity, we are eternally grateful.

We would also like to thank Dr. Eva M. Strawbridge, who served as a sounding board for many of our harebrained ideas and encouraged us to
take intellectual risks. Without her continual advice and insights, we might have found ourselves asking our stunt doubles to run at $36 \mathrm{ft} / \mathrm{s}(24.5 \mathrm{mph})$ or perform other superhuman tasks. We appreciate her criticisms and her confidence, and we trust that she will enjoy seeing Draftee Summer brought to theaters near her by dint of her own efforts.

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