The Speed and Power of Rumors

Fame, the great ill, from small beginnings grows:
Swift from the first; and ev'ry moment brings
New vigor to her flights, new pinions to her wings.
Soon grows the pigmy to gigantic size;
Her feet on earth, her forehead in the skies.
-Vergil, *Aeneid* IV.173 ff.

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Abstract

Rumors affect everyone, whether a student or business titan. From friendships to marriages—no matter the relationship—rumors have the ability to destroy in a particularly virulent fashion. There is no better reason to seek to understand their mechanism(s), speed, and effect on a person's reputation – all of which, we believe, can be explained mathematically. We consider several aspects (such as gossip transmission rate) of a community and quantify them, and demonstrate how such variables affect of spread as well as the interest and intrigue generated over time. Finally, we compare different types of rumors, and attempt to quantify the different characteristics of rumors, for example distinguishing between rumors like "Area 51 Contains Aliens"[1] and rumors like "Alex has cooties."[2]

Problem Statement

Our research began after speaking to Grandpa Brandt, who sagaciously quoted Benjamin Franklin: "Three may keep a secret, if two of them are dead." Converting this maxim into a mathematical model, we can see that for any *g* number of people who have heard gossip and serve as constant spreaders of a rumor with a transmission rate, *γ,* for some time, *t,* the differential equation that describes the spread of rumors over time is

$$\frac{dg}{dt} = g\gamma .$$

This can be easily solved as

$$g = e^{\gamma t + c} = c'e^{\gamma t} .$$

Hence, as *t* tends towards infinity, the spread of gossip tends towards infinity as well. At this point, reality begins to greatly contradict our model. The simple truth of the matter is that rumors occur every day and do not infect the whole of humanity. Furthermore, the population of humanity does not tend towards infinity as time tends toward infinity. Clearly, to accurately model the spread of rumors among people, limiting factors must be applied, such as the size and susceptibility of a community to gossip. In addition, a separate model must be proposed to explain conditions variable transmission rates.

While modeling the spread of rumors among a population illuminates their infectious quality, the spread is mainly significant due to the social repercussions they may have for the subject of a rumor. An entirely new model must be proposed to deal with the question of how rumors affect a person's reputation. This needs to account for the unpredictable nature of rumors to both help and hinder the reputations of those the rumors concern.

Ultimately, any model, no matter how theoretically sound or beautiful, is only as good as its ability to predict or model a given physical phenomena. Rather than going to various

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[1] http://www.unexplained-mysteries.com/forum/index.php?showtopic=41132
[2] A particularly scarring personal 1st grade experience.

schools, offices, and houses of worship and "infecting" a person in these communities, waiting several months to watch the ensuing chaos, and incurring hundreds of thousands of dollars in slander and libel lawsuits, we will use a few lines of computer code to model the random behavior of humans, by using a computerized version of Euler's method to solve our given differential equations.

Model Design

Our first consideration is the ability of the human brain to forget. Whatever the reason, lack of interest will most likely set into people who gossip (or possibly other, better rumors will replace the original). Hence, we should introduce a boredom coefficient α into our equation, affecting g, the population of gossips. Furthermore, we should know that the only way gossip can spread is if a gossiping person comes into contact with a non-gossiping person. The modified equation would then look something like this:

$$\frac{dg}{dt} = gp\gamma - g\alpha.$$

We should also distinguish between different types of non-gossipers. First, there are those in the community of a weaker moral fiber who are susceptible to become gossipers by their disposition and proximity in social networks, geography, or other attributes. On the other hand, there are those who are not gossipers but are also not susceptible to becoming gossipers. We will denote the susceptible population as p and the unsusceptible population as u. While u may seem insignificant as of thus far, we will assume that there is an exchange between u and p- possibly individuals change in their characteristics or in their proximity to gossipers. We will call this rate of exchange (more specifically, the rate of migration into p from u) δ. This constant's effect however, will be changed by the value of p, since the greater the susceptible population, the more likely unsusceptible people will be to become susceptible- this results in the term $p\delta$. Furthermore, the people who become bored with the rumor after being gossipers will transfer into the unsusceptible population- this results in the term αg in the unsusceptible population rate equation. So, the equations for the rates of these populations would be:

$$\frac{dp}{dt} = -gp\gamma + p\delta,$$

and

$$\frac{du}{dt} = \alpha g - p\delta.$$

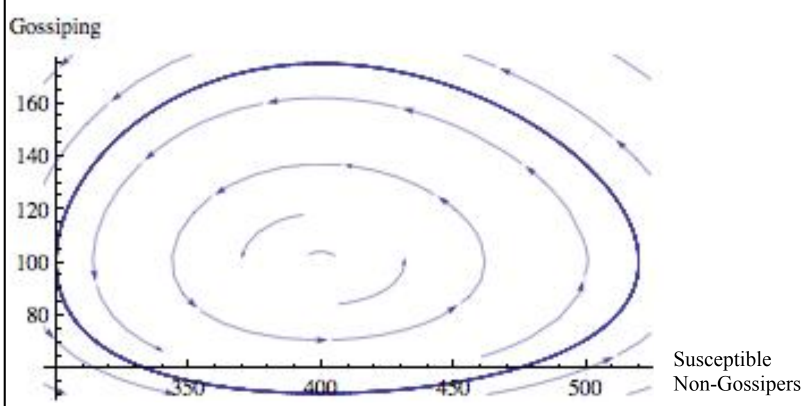


Figure 1: Cyclical equilibrium

Now our differential equations can be seeded with initial conditions, or graphed into flow lines to see the overall trend of the system. Figure 1 has taken both options and superimposed one on the other[3]. Here we can see that a steady state equilibrium has been established (given the arbitrary standard positive constants for γ, δ, and α), and the solution is orbiting around the center corresponding to the seed value of 100 gossipers and 300 susceptible non-gossipers. This model well supports the "Area 51" style rumor – a rumor that has been around for a long time and refuses to die down. Every so often there is a spike in the amount of interest in extraterrestrial life, but this often subsides when experts from the scientific community and government assure us that such facts are untrue.[4] This statistic can be verified when we remember that about 95% of Americans have heard or read rumors about UFOs flying around the night skies, but only 57% believe they are real.[5] There is a fairly constant 38% who have heard but do not believe, either because they used to believe and have either been converted to a naysayer, or they heard and never believed in the first place. As new Americans enter the community (by immigration or birth), they will in turn become susceptible to the rumors and will either gossip or disregard.

Now that we've seen a model for a steady state rumor, it's productive to look at a model for a rumor that's decaying- a rumor that is less and less interesting or gossip-worthy over time. For this model, rather than just the straight boredom coefficient α, we add a mechanism for changing this coefficient over time as β, the rate of change in the boredom coefficient, times *t*, the time:

$$\frac{dg}{dt} = g(p\gamma - (\alpha + \beta t)).$$

The other equations become

$$\frac{dp}{dt} = -gp\gamma + p\delta,$$

and

[3] Generated with Mathematica using the NDsolve and StreamPlot functions.
[4] That's not to say the authors of this paper believe such lies. The truth is out there.
[5] https://www.cia.gov/library/center-for-the-study-of-intelligence/csi-publications/csi-studies/studies/97unclass/ufo.html

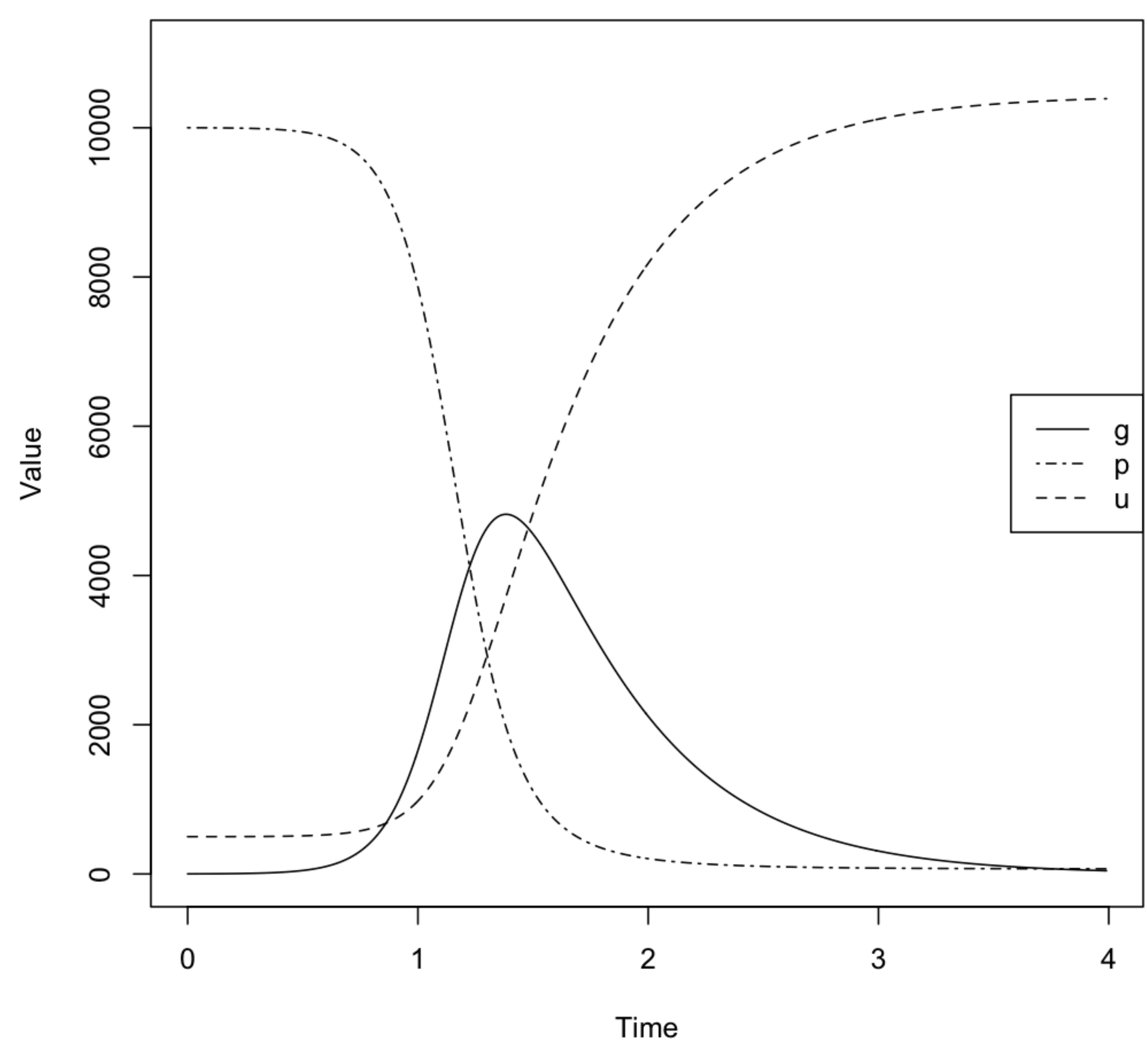
$$\frac{du}{dt} = (\alpha + \beta t)g - p\delta \, .$$

Thus, illustrating that the total population of individuals is constant,

$$\frac{dg}{dt} + \frac{dp}{dt} + \frac{du}{dt} = 0.$$

Here we see that our coefficient of boredom grows larger and larger with time with a positive β. This will send the amount of gossipers to 0 as time tended towards a very large value. Note that all our constants have been without units thus far, as this is dependent on what units we choose for our constants. Let's take a look at a modeling of the equation with initial values g(0) = 1, u(0) = 500, p(0) = 10000, and constants of $\alpha = .02$, $\beta = .0001$, $\gamma = .00001$ and $\delta = .000001$. Here the time series are shown in Figure 2. For more information on the computer modeling of these equations, see Appendix A. For information on the plot development, see Appendix B.

Figure 2: **G, P, and U Plotted vs. Time**



For this model, we can see that the gossip is initially overpowering to the reduction terms. The spike in gossipers rises from 1 to almost a maximum of 4820, but then it gradually levels off towards zero (slower than it rises, mind you). While this curve may appear vaguely Gaussian, by careful analysis with standard statistical tools such as the Normal Q-Q Plot, we observed that the distribution of gossipers is not Normal. This result is fitting with the common knowledge that gossip is very easy to spread, and it often takes a much longer time for it to fall. Meanwhile, the number of unsusceptible non-gossipers consistently increases, and the number of susceptible non-gossipers consistently decreases. Hence, at this point we've modeled different types and its growth and decline in various communities or populations.

Discussion and Conclusion

Our model succeeded in producing generally expectable results as to a rumor's growth and typical later decline. In its introduction of a boredom coefficient linearly dependent on time, the model was able to broadly distinguish between rumors and take each rumor's individual characteristics into account. This, of course, does not completely accurately quantify a rumor's attributes, as only going up to a linear dependence on only the variable of time will only at best generally mirror reality. Nonetheless, such an inclusion made it possible for our model to explain, for example, both rumors which skyrocketed and waned only slightly and ones which increased initially but quickly declined in popularity.

Furthermore, our inclusion of exchange between unsusceptible and susceptible segments of the total population provided a rough mechanism for mirroring difference in people's tolerance of and general ability to receive gossip and become gossipers. Again, the model only did so assuming migration between these two groups happened at a constant rate independent of time, so while it was better than nothing, it could not fully depict the changes in susceptibility of the total population over time.

As our model shows, for a model whose characteristic boredom term increases over time (the sum $\alpha + \beta t$ increases due to a positive β), the number of gossipers will initially rise, then reach some maximum, and then fall towards zero. Meanwhile, assuming a positive flow of individuals from the unsusceptible to susceptible population, the number of unsusceptible individuals will eventually decrease to zero as well. Finally, the number of susceptible individuals will probably decrease at first (depending on the relative magnitudes of the rumor's transmission rate and the migration rate between unsusceptible and susceptible populations) and then increase to consist of the total population being considered.

At a basic level, our model takes into account the essential components of the problem of rumor modeling and manages to fit the general shape of a rumor's popularity. Using a variant of the Lotka-Volterra predator-prey models (more specifically the parasite model), the modeling of the interaction between gossipers and the susceptible population in being a constant times their product, since the interaction between the two groups should be linearly dependent on each of their populations. One limitation of even this fairly unobjectionable statement, however, is that the whole system of categorizing and quantifying people to such a degree belies the actual more amorphous nature of rumors and the chances of individuals encountering and adopting them. Fundamentally, however, the model is sound.

Rumors are not simple phenomena and are naturally hard to model. Through the use of differential equations with multiple constants and variables, however, such a process can be roughly shown, with the same characteristic behavior that we would expect in real life.

Software Used

For Phase Diagram:
Wolfram's Mathematica 7.0.2

For Time Series Modeling:
Groovy 1.6.7, JVM 1.6.0_17
R 2.10.1 64-bit

Works Cited

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4. Coppex, F. Droz, M. Lipowski, A. *Lotka-Volterra Model of Macro-Evolution on Dynamical Networks*.

5. Hethcote, H. 1976. Qualitative analyses of communicable disease models. Mathematical Biosciences 28 (3-4): 335-356.

Author Contributions

Alexander Brandt: Mathematica coding and modeling, model design, abstract, problem statement

Mark Fornace: Model design, discussion and conclusion, general editing

Jens Jensen: Groovy simulation code and R plot, abstract, citations, Vergil quotation ☺, proofreading and editing

Yiannis Moses: Problem Statement, Citations, general proofreading and editing

Appendix A
Simulation Code

```
/* simulation.groovy
 *
 * Simulation for Final Project in Math 201. Alex Brandt, Mark Fornace, Jens Jensen, Yiannis
Moses.
 */


// for simplicity, def map with input settings rather than implementing cli, constants must make
sense as no error checking
def set = [
        timerange : 10,
        timestep : 0.01,
        population : 10000, // initial population
        alpha: 0.02, // alpha = boredom coeff
        beta: 0.0001, // beta = boredom decay coeff
        gamma : 0.00001,  // gamma = transmission rate
        delta : 0.000001 // delta = induction rate for population
]


// def map to hold state
def state = [
        p : set.population,
        g : 1,
        u: 500
]

println "t\tg\tp\tu\ttot"   // print header
// step from 0 to timerange by timestep
for (t = 0; t <= set.timerange; t += set.timestep) {
  // dump current state to output
  output = [t, state.g, state.p, state.u, (state.p + state.g + state.u)]
  println output.join("\t")

  // compute temp variables to hold changes so we are not modifying state as we calculate changes
  // cast them as float or Groovy will use Java's arbitrary-precision BigDecimal class
  tempg = (float)(state.g * (state.p * set.gamma - (set.alpha + set.beta * t)))
  tempp = (float)(state.p * set.delta - state.g * state.p * set.gamma)
  tempu = (float)((set.alpha + set.beta * t) * state.g - state.p * set.delta)

  state.g += tempg
  state.p += tempp
  state.u += tempu
}
```

Appendix B
R Code for Plot

```
sim = read.table("sim2.txt", header=TRUE)
attach(sim)
plot(t[1:400], g[1:400], type="l", ylim=range(0,11000), ylab="Value", xlab="Time", main="G, P,
and U Plotted vs. Time")
points(t[1:400],p[1:400],type="l", lty=4)
points(t[1:400],u[1:400],type="l", lty=2)
legend("right",c('g','p','u'), lty=c(1,4,2))
```