Original Article

Optimizing acquisition and retention spending to maximize market share

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ABSTRACT To determine spending on acquisition and retention of customers, a company can focus on maximizing a variety of indicators such as customer equity, profit or sales growth. In this article we use standard techniques from multivariable calculus to maximize market share through analysis of retention and switching ratios both in the absence and presence of a budget constraint. In the process, we identify firm theoretical foundations for some common intuitions, and give new insights on optimal competitive strategy. We include practical numerical examples that show marketing practitioners how to apply our analysis.
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INTRODUCTION

Investments related to the demand generation process represent one of the largest discretionary expenses for marketing managers. Deciding how to allocate such funds to best achieve marketing objectives is therefore of critical importance. While there are many decisions that must be made in designing targeted demand generation marketing programs, one of the most fundamental decisions is deciding how to balance investments in acquiring new customers with investments in retaining current customers.

Blattberg and Deighton (1996) developed a decision calculus approach first pioneered by Little (1970) to find the optimal budget values for acquisition and retention spending. Berger and Nasr-Bechwati (2001) extended this budget optimization approach to determine the optimal allocation of budget between acquisition and retention spending to maximize customer equity. Reinartz *et al* (2005) use data for a high-tech, high-consideration product to empirically model the interaction between marketing-mix variables and determine the optimal allocation of resources between acquisition and retention.

It is often difficult to make a distinction between marketing activities whose primary goal is to acquire a customer from those whose primary goal is to retain a customer. For example, a company may offer technical webinars to generate leads, with the goal of capturing new customers. However, the webinars can also serve to reinforce a vendor's image as a domain expert with current customers, which builds brand loyalty and increases customer retention. Practically speaking, however, the marketing practitioner typically has a specific marketing objective that classifies specific tactics in a marketing campaign as primarily serving either an acquisition strategy or a retention strategy. Table 1 lists some common examples of each for a business-to-business marketing scenario.

The models found in the literature to date, which provide marketing guidance on acquisition and retention spending have relied largely on customer equity or customer lifetime value (CLV) as the objective function to be maximized. One of the first accounts of the CLV concept appears in Shaw and Stone (1988). CLV is an appropriate metric for many situations, in particular those involving direct marketing. However, it suffers from a number of specific drawbacks that limit its broad applicability in practice:

- As McManus and Guilding (2008) point out, a close examination of CLV models reveals that they focus on customer retention as the most important attribute of customer management. As such, they overemphasize the importance of relationship marketing at the expense of competitive warfare in shaping marketing strategy.
- CLV models make unrealistic assumptions. For example, (Singh and Jain, 2010) some models assume that a customer is retained forever, or that a customer who is not retained is lost forever. However, customers who make sequential purchase decisions over time for a particular type of product can be won and lost multiple times.
- For most companies, financial systems are designed to report product costs rather than customer-specific costs.
- In many, if not most, marketing organizations the person primarily responsible for making tactical decisions on acquisition and retention spending is not accountable for profit. Marketing's primary accountabilities are typically sales growth and market share.

In this article, we take a different approach. We focus on market share as the objective function to be maximized. First, we develop an expression for market share based on a simple Markov brand switching model that takes into account both customer retention as well as customer acquisition as variables. Second, we use a vector calculus approach to develop an expression for the optimal ratio of acquisition and retention efforts. We then build on the vector calculus expression to provide guidance on the combinations of acquisition and retention rates that result in the steepest path of ascent for market share,

Table 1: Common marketing tactics

Acquisition marketing tactics	*Retention marketing tactics*
Competitive print advertising	Welcome/onboarding program
Online pay for placement	On-site customer events, seminars
Competitive trade-in offer	Customer newsletter
Trade shows, seminars, Webcasts	Executive briefing
Search engine optimization	Account-specific Web portal
Educational offers	Current customer VIP seminar
Promotional discounts	Product upgrade promotion
Success stories, ROI calculators	Company-hosted Website communities/blogs
Teleprospecting	Product usage videos, application notes
Pre-sales video demos	Cross-sell programs

in the absence of a budget constraint. Finally, we use the method of Lagrange multipliers to show how to maximize market share subject to a budget constraint, and provide a numerical example based on the decision calculus approach of Blattberg and Deighton (1996).

EXPRESSION FOR MARKET SHARE BASED ON A SIMPLE MARKOV CHAIN MODEL

Since the first survey on the use of Markov brand switching models in marketing by Ehrenberg (1965), both research and practice have advanced considerably. Practical applications can be found in consumer beverages (Awogbemi *et al*, 2012), durables (Colombo and Morrison, 1989), telecommunication services (Datong, 2011), insurance (Rösch and Schmidbauer, 2009) and Internet services (Lee *et al*, 2003).

While complex Markov models have been developed to describe switching behavior (for example, Pfeiffer and Carraway, 2000), here we derive an expression for market share based on the simple two-state Markov model shown in Figure 1. Following Hernitor and Magee (1961), one state represents the pool of customers who have purchased from 'Company' while the other state represents the pool of all customers who have purchased from its competitors.

To simplify, we assume no growth in the population of customers, and that every customer makes a purchase once per year. Let r be the per cent of Company's customers retained each year and let s be the per cent of those who switch from a competing product to Company's product each year. Let X_i represent the number of Company's customers after i years and let Y_i represent the number of customers using a competitor's product after i years. Given these assumptions, we have

$$X_{i+1} = rX_i + sY_i,$$
$$Y_{i+1} = (1-r)X_i + (1-s)Y_i. \qquad (1)$$

Let T be the transition matrix for this Markov model. Then the model satisfies the matrix equation

$$\begin{pmatrix} X_{i+1} & Y_{i+1} \end{pmatrix} = \begin{pmatrix} X_i & Y_i \end{pmatrix} T = \begin{pmatrix} X_i & Y_i \end{pmatrix} \begin{pmatrix} r & 1-r \\ s & 1-s \end{pmatrix}.$$

It is well known that this Markov process will approach an equilibrium state E where $E = ET$. This equilibrium is characteristically independent of the initial conditions of the system. Let m represent Company's market share at equilibrium, then $1-m$ is its competitors' market share. At equilibrium $E = \begin{pmatrix} m & 1-m \end{pmatrix}$ the matrix equation $E = ET$ is given by

$$\begin{pmatrix} m & 1-m \end{pmatrix} = \begin{pmatrix} m & 1-m \end{pmatrix} \begin{pmatrix} r & 1-r \\ s & 1-s \end{pmatrix} = \begin{pmatrix} mr + (1-m)s & m(1-r) + (1-m)(1-s) \end{pmatrix}.$$

Then Company's market share m is given by

$$m = \frac{s}{1-r+s}. \qquad (2)$$

Note that this derivation does not depend on the length of the interpurchase interval; if that interval was a fixed number of years n (where n could be greater or less than 1), then we could replace X_i and Y_i with X_i/n and Y_i/n in equation (1). Doing so still produces equation (2) for market share, so this model depends only on the fact that every customer makes a single purchase in a given time period.

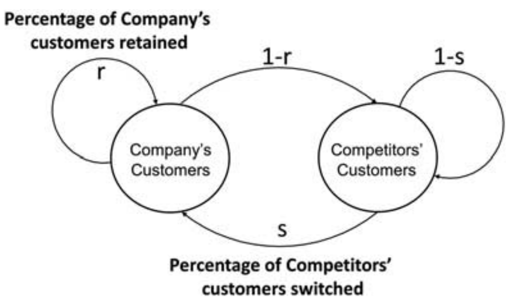

Figure 1: A Markov model for brand switching.

Equation (2) gives Company's market share explicitly as a function of its retention and switching ratios. We can visualize this as a surface in three dimensions, as shown in Figure 2. Notice that many different combinations of retention ratio and switching ratio yield equivalent market share.

There are two approaches to estimating the values of *m, r* and *s* in equation (2). In consumer marketing, market share can be estimated using switching and retention ratios determined with the help of survey panel data. However, in business-to-business marketing, survey panel data is rarely available. An alternative approach in such cases is to take estimated market share as an input, either from syndicated third-party market research or directly from the company's own marketing department. Since the company's customer retention ratio can be estimated based on internal sales information, it is then possible to derive the competitive switching ratio using equation (2).

From Figure 2, a number of insights are intuitively apparent. First, at low values of the retention ratio, market share is fundamentally limited, regardless of one's success at inducing customers to switch. Second, for situations in which switching costs are high (as is the case for capital equipment), and therefore switching ratios are inherently low, increases in market share can be achieved by emphasizing customer retention, but significant increases in market share do not occur until the retention ratio exceeds 50–60 per cent.

Contrary to the often-quoted viewpoint (Anderson and Narus, 2004; Walsh *et al*, 2005; Gee *et al*, 2008) that customer retention should precede customer acquisition as a priority, it should not be concluded that customer retention is the key to market share growth. Notice that the quantity $1-r$ represents the percentage of Company's customers who switch to the competition. Then Company's market share (given by equation (2)) is the proportion of customers who switch to its brand out of the total number of customers who switch brands. For Company, this means that increasing market share is as much about marketing warfare to

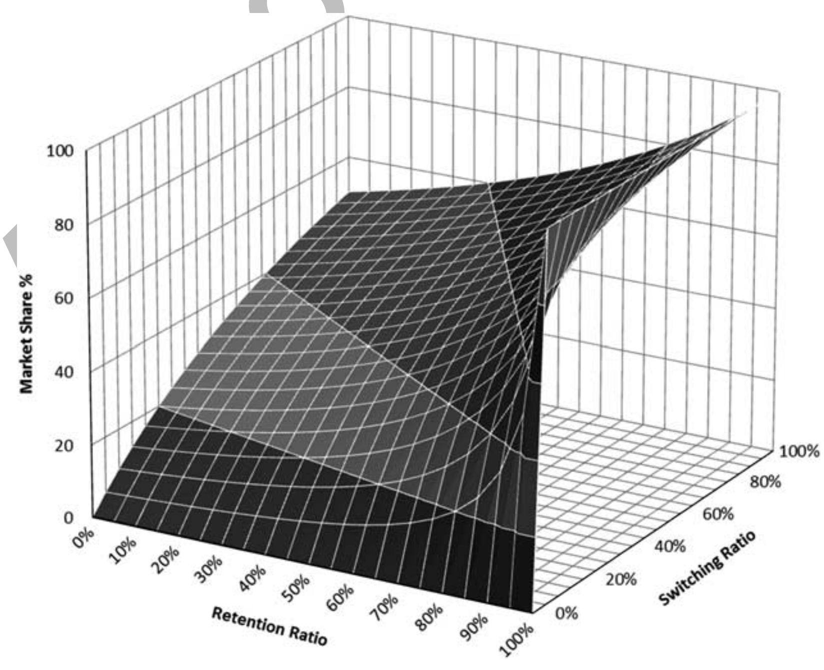

Figure 2: Market share *m* as a function of switching and retention ratios.

capture competitors' customers as it is about maintaining its own customer relationships. Whether to put more emphasis on acquisition or retention is the topic of the next section.

Before moving on, however, we explore the impact on the model of relaxing the assumption that all buyers in the market make a purchase during each year. Equation (2) expresses the steady-state value to which market share converges over time, given that assumption. In many business-to-business and business-to-consumer scenarios, however, the interpurchase interval varies by customer. For example, consumers typically buy a new printer every several years, and scientists and engineers purchase new measurement equipment such as oscilloscopes every 3–5 years. Practically speaking, during any given year a portion of the total market makes a purchase. Suppose we relax the assumption of a fixed interpurchase interval for the entire customer population, and instead assume that the random variable p_i describes the distribution of interpurchase intervals among the customer base. Then $1/p_i$ describes the proportion of the population making a purchase in any given time period and equation (1) can be rewritten as

$$\begin{aligned} X_{i+1} &= X_i - (1-r)\left(\frac{1}{p_i}X_i\right) + s\left(\frac{1}{p_i}Y_i\right) \\ Y_{i+1} &= Y_i + (1-r)\left(\frac{1}{p_i}X_i\right) - s\left(\frac{1}{p_i}Y_i\right) \end{aligned} \tag{3}$$

Equation (3) defines a time-inhomogeneous Markov process; however one can show that it is equivalent to the homogeneous Markov process with transition matrix

$$\begin{pmatrix} 1-\mathcal{E}(1-r) & \mathcal{E}(1-r) \\ \mathcal{E}s & 1-\mathcal{E}s \end{pmatrix}$$

where $\mathcal{E}$ is the expected value of the random variable $1/p_i$. Then the model defined by equation (3) has a steady state, and it is straightforward to show that the steady state is in fact still given by equation (2) (though the convergence time will be affected by $\mathcal{E}$).

PATH OF STEEPEST ASCENT FOR MARKET SHARE

Although equation (2) assumes that the retention and switching ratios are stable over time, one of the primary goals of a business is to increase its market share. Thus, the goal is to alter the values of the retention and switching ratios such that market share increases. The business question we now investigate is: What is the optimal policy for changing these ratios so as to result in the greatest possible increase in market share?

Equation (2) is a function $m(r, s)$ representing a scalar field of the market share associated to the pair (r, s) of retention and switching ratios. In vector calculus, the gradient of a scalar field is a vector field where the vector at any point is in the direction of greatest increase in the scalar field. In other words, evaluating the gradient of $m(r, s)$ at a point (r_0, s_0) gives the direction of the steepest path up the market share 'hill' when $r = r_0$ and $s = s_0$. From the graph of the surface m (Figure 2) it is clear that market share is increased only by increasing at least one of s and r, so we expect all gradient vectors to point away from the origin.

Recall that

$$\partial_x z := \frac{\partial z}{\partial x}$$

is the partial derivative of a function $z(x, y)$ with respect to the variable x. The gradient of $m(r, s)$ is given by

$$\begin{aligned} \nabla m &= (\partial_r m)\vec{r} + (\partial_s m)\vec{s} \\ &= \left(\frac{s}{(1-r+s)^2}\right)\vec{r} + \left(\frac{1}{1-r+s} - \frac{s}{(1-r+s)^2}\right)\vec{s} \end{aligned} \tag{4}$$

where $\vec{r}$ and $\vec{s}$ are unit vectors in the two coordinate directions.

Figure 3 gives the vector field defined by equation (4). Here, the vectors point in the direction of the path of the greatest increase in market share from a given point. The length of the vectors describes the rate of increase in market share at each point.

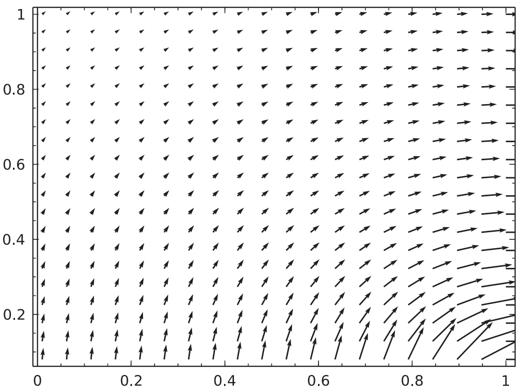

Figure 3: The gradient of *m*.

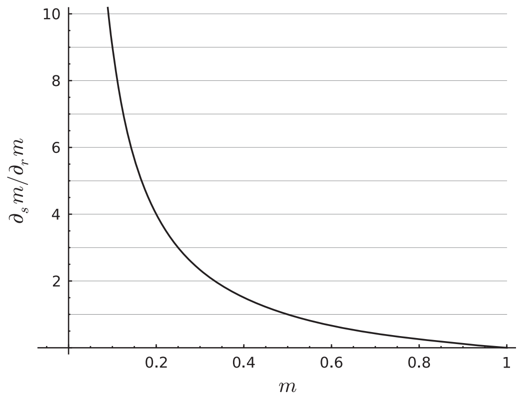

Figure 4: Graph of equation (6).

Using equation (2) to rewrite equation (4) gives

$$\begin{aligned}\nabla m &= (\partial_r m)\vec{r} + (\partial_s m)\vec{s} \\ &= \left(\frac{1}{1-r+s}m\right)\vec{r} + \left(\frac{1}{1-r+s}(1-m)\right)\vec{s} \end{aligned} \quad (5)$$

and therefore

$$\frac{\partial_s m}{\partial_r m} = \frac{1-m}{m}. \quad (6)$$

Figure 4 (the graph of equation (6)) visually confirms the common intuition of many marketing and sales professionals that at low values of market share, incremental marketing and sales investments should be weighted toward acquiring new customers. In contrast, when a firm has a high market share, incremental investments in marketing and sales should be weighted toward improving customer retention.

In the absence of a budget constraint, we can determine the specific *path* on the market share surface that results in the steepest gains in market share. This path is a curve in *r* and *s* that starts at the current pair of ratios (r_0, s_0) and is always tangent to the gradient vector, so its slope ds/dr is given by

$$\frac{ds}{dr} = \frac{\partial_s m}{\partial_r m} = \frac{\frac{(1-r)}{(1-r+s)^2}}{\frac{s}{(1-r+s)^2}} = \frac{1-r}{s}. \quad (7)$$

This is a separable differential equation, and we solve by integration. The path of

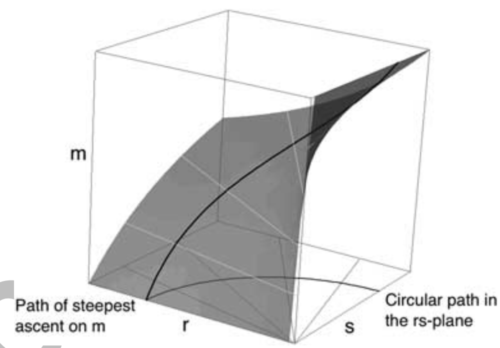

Figure 5: The path of steepest ascent on *m*.

steepest ascent from (r_0, s_0) is then given by

$$s^2 + (r-1)^2 = 1 + C \quad (8)$$

where $C = s_0^2 + r_0^2 - 2r_0$ is a constant. Substituting C into the previous equation yields

$$s^2 + (r-1)^2 = s_0^2 + (r_0 - 1)^2. \quad (9)$$

This curve is a circle in the *rs*-plane centered at $(r, s) = (1, 0)$ with radius determined by the initial pair (r_0, s_0).

The fact that only the radius of this path changes as (r_0, s_0) changes is a result of the high level of symmetry in the surface *m*. In Figure 5 we show this circular path in the *rs*-plane as well as projected onto the surface itself.

For marketing practitioners, this result provides a simple graphical approach as an alternative to using equation (9), as shown in Figure 6. Assume the marketing practitioner has estimated the current value

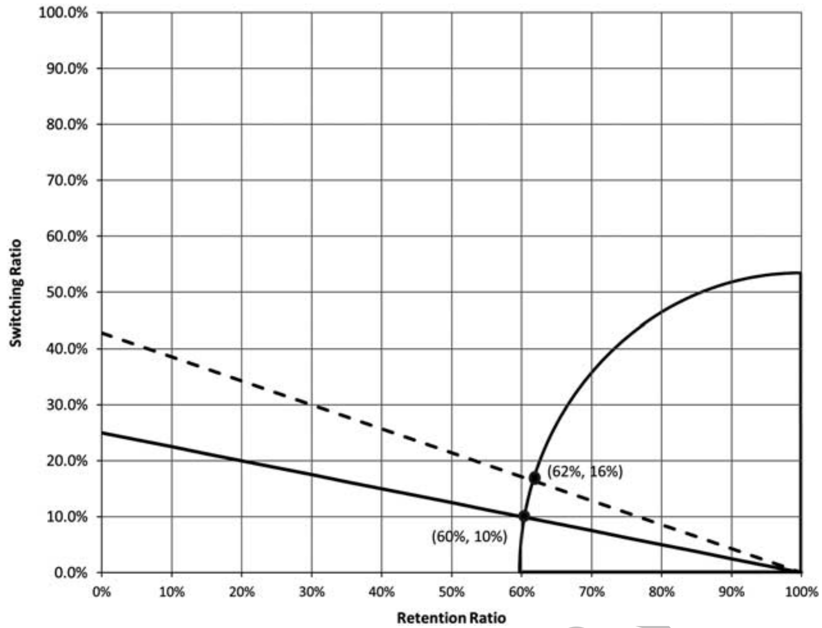

Figure 6: Practical example: The path of steepest ascent from 20 to 30 per cent market share.

of (r_0, s_0) as (60 per cent, 10 per cent), corresponding to a market share of 20 per cent. Suppose the goal is to increase market share from 20 to 30 per cent. For any market share m, the line $s=(m)/(1-m)(1-r)$ on the rs-plane represents all possible points (r, s) corresponding to a market share of m, so it is a simple matter to draw the line corresponding to a 30 per cent market share. Then, using a compass anchored at (1, 0) with its radius set at the point (60 per cent, 10 per cent), the practitioner can trace out a circular arc in the direction of the 30 per cent share line. The points along this arc contain the values of (r, s), which represent the shortest 'path' to a 30 per cent market share from the current market share.

A close examination of Figure 6 visually reinforces the implications of Figure 4. Notice that following the circular 'steepest path' always calls for increasing both the retention ratio and the switching ratio. However, at low values of market share, following this path in a clockwise direction calls for much larger changes in the switching ratio than the retention ratio. Conversely, at high values of market share, following the path calls for much larger changes in the retention ratio than the switching ratio.

OPTIMIZING MARKET SHARE GIVEN A BUDGET CONSTRAINT

In the previous section, we derived a strategy for changing Company's retention and switching ratios to achieve the fastest possible increase in market share in the absence of a budget constraint. This was an ideal situation; however, in most cases marketing practitioners face the practical reality of a limited budget. Now we mathematically derive the conditions for optimizing market share subject to a budget constraint, and provide a numerical example in which marketing spending to achieve target switching and retention ratios is based on the decision calculus approach of Blattberg and Deighton (1996).

Let N be the number of consumers in the market, so that Nm is the number of consumers purchasing from Company and $N(1-m)$ is the number of consumers purchasing from Company's competitors.

Let D_r be an arbitrary function of r that describes the relationship between marketing spending per installed base customer per year and the retention ratio r. Let D_s be an arbitrary function of s that describes the relationship between marketing spending per prospect per year and the competitive switching ratio s.

If B is the size of Company's annual demand generation budget, we have the constraint

$$NmD_r + N(1-m)D_s \leq B. \tag{10}$$

Let $g(r, s) = B - NmD_r - N(1-m)D_s$; then equation (10) is equivalent to the constraint

$$g(r,\ s) \geq 0. \tag{11}$$

In order to maximize market share $m(r, s)$ as given by equation (2) subject to the budget constraint in (11), we apply the method of Lagrange multipliers, a standard optimization technique in multivariable calculus. In this method, we consider the vector equation

$$\nabla m(r,\ s) = \lambda \nabla g(r,\ s)$$

where λ is some real number constant (called the *Lagrange multiplier*). This relation gives a system of equations

$$\left\{ \begin{array}{l} \partial_r m = \lambda \partial_r g \\ \partial_s m = \lambda \partial_s g \end{array} \right\} = \left\{ \begin{array}{l} \frac{1-r}{(1-r+s)^2} = \lambda \left[-N \frac{1-r}{(1-r+s)^2} (D_r - D_s + (1-r+s)\partial_s D_s) \right] \\ \frac{s}{(1-r+s)^2} = \lambda \left[-N \frac{s}{(1-r+s)^2} (D_r - D_s + (1-r+s)\partial_r D_r) \right] \end{array} \right\} \tag{12}$$

Solving this system of equations, we find that to maximize market share with respect to the budget constraint given by condition (10) requires

$$\partial_r D_r = \partial_s D_s. \tag{13}$$

Recall that these derivatives represent the marginal cost of retaining or acquiring a customer, respectively, so this solution implies that Company should equalize these marginal costs to optimize market share. Now we consider a numerical application of (10) in which the functions D_r and D_s are adapted from Blattberg and Deighton (1996).

A NUMERICAL EXAMPLE

The contribution of Blattberg and Deighton (1996) was to employ a decision calculus approach to build a simple model describing the impact of marketing spending on customer retention and customer acquisition rates. They start with three reasonable assumptions: zero spending results in zero acquisition or retention rates; marketing spending has strictly diminishing returns; and there exists a maximum 'ceiling rate' for acquisition or retention, regardless of the amount of marketing spending. These assumptions are combined with managerial judgments to create a formal model to help answer a complex business question about optimal marketing spending. In the following example, we employ the same decision calculus approach, with the adaptation that the switching and retention ratios drive market share according to (2).

Let us consider the following example, in which we assume there are 1 000 000 target customers. From historical data, the firm knows that it retains $r_0 = 60$ per cent of its installed base, and marketing indicates that the firm succeeds in acquiring $s_0 = 7$ per cent of its competitors' customers each period. As a result, according to equation (2), it enjoys a steady state market share of 148 936 customers, or 14.9 per cent. Thus, 60 per cent of the firm's customer base (89 362) is composed of customers who are retained from the prior period and 40 per cent (59 574) are those who switched from its competitors' customer base of 851 064 during the prior period.

As suggested by Blattberg–Deighton, we model the relationship between r, the retention ratio, and D_r, the retention spending per customer per period, as

$$r = CR_r[(1 - exp(-k_r D_r))] \tag{14}$$

where CR_r is the manager's judgment of the maximum retention rate regardless of marketing spending and k_r is a real parameter. For analogous definitions of s, D_s, CR_s and k_s, we use the model

$$s = CR_s[(1 - exp(-k_s D_s)]. \qquad (15)$$

Analysis of the firm's US$5 million annual demand generation budget shows that the firm is currently spending $D_r = \$5$ per installed base customer per year ($744 681) attempting to retain their revenue stream, while spending $D_s = \$5$ per prospect per year ($4 255 319) to induce its competitors' customers to switch to the firm's product. The Product Manager's judgment is that the maximum retention rate CR_r is 80 per cent regardless of per-customer promotional spending, and the maximum competitive switching rate CR_s is 20 per cent regardless of per-prospect promotional spending.

Substituting these values into equations (14) and (15), we can calculate the parameters k_r and k_s as

$$k_r = \frac{-ln\left(1 - \left(\frac{r_0}{CR_r}\right)\right)}{D_r} = \frac{-ln\left(1 - \left(\frac{0.6}{0.8}\right)\right)}{\$5} = 0.27726,$$

$$k_s = \frac{-ln\left(1 - \left(\frac{s_0}{CR_s}\right)\right)}{D_s} = \frac{-ln\left(1 - \left(\frac{0.07}{0.2}\right)\right)}{\$5} = 0.08616.$$

Following Pfeiffer's (2005) approach to take the inverse of the Blattberg–Deighton equations, the Product Manager derives the following profiles for retention spending and competitive switching spending,

$$\begin{aligned} D_r &= \frac{-1}{k_r} ln\left(\frac{CR_r - r}{CR_r}\right) = \frac{-1}{0.27726} ln\left(\frac{0.8 - r}{0.8}\right) \\ D_s &= \frac{-1}{k_s} ln\left(\frac{CR_s - s}{CR_s}\right) = \frac{-1}{0.08616} ln\left(\frac{0.2 - s}{0.2}\right). \end{aligned} \qquad (16)$$

Note that when we apply the optimality condition (13) to the above expressions for D_r and D_s, we find that it occurs when

$$k_r(CR_r - r) = k_s(CR_s - s), \qquad (17)$$

which is identical to that predicted by Pfeiffer (2005).

To find the marketing spending that maximizes market share subject to the budget constraint (10), equation (17) would typically be solved for either r or s and substituted back into equation (10) to complete the solution. Unfortunately, this substitution yields a highly non-linear equation that cannot be solved algebraically. However, it is possible to numerically solve for the values of r and s, which satisfy the budget constraint using Excel's Goal Seek function. To do so, we first solve equation (17) for r to get

$$r = CR_r \left(\frac{k_s}{k_r}\right)(CR_s - s). \qquad (18)$$

We create a cell formula for equation (18) using the above example values for CR_r, CR_s, k_s and k_r, while leaving s, the switching ratio, as a cell reference with an arbitrary value between 0 and 1. We then set up additional cells in Excel corresponding to the three terms in the budget equation (10). It is then a simple matter to invoke the Goal Seek function, in which the cell value of the switching ratio is varied until a budget value of $5 million is achieved. The resulting solution is given in Table 2.

The revised spending allocation of the $5 million budget results in a predicted market share of 18.5 per cent, which is 24 per cent higher than the current state. Table 3 compares the current and optimal spending

Table 2: Current versus optimal conditions

	Current	*Optimal*
Retention ratio	60%	75.5%
Switching ratio	7%	5.6%
Retention budget	$0.74 million	$1.92 million
Switching budget	$4.26 million	$3.08 million
Market share	14.9%	18.5%

Table 3: Optimal versus current spending

	Current	*Optimal*
Spending per installed base customer	$5.00	$10.39
Spending per prospect	$5.00	$3.78

Table 4: Optimal versus current conversion costs

	Current	*Optimal*
Spending per *retained* installed base customer	\$8.33	\$13.76
Spending per *acquired* prospect	\$71.43	\$67.99

patterns per installed base customer and per prospect.

What a good Marketing Manager will care about, however, is the *net* conversion costs for won deals. Dividing the above numbers by the respective retention ratios and switching ratios in Table 2, the net conversion costs per customer won each period are given in Table 4.

As the analysis above shows, at low market share levels, it is easy for Marketing and Sales to get caught up in a 'beat the competition' mindset that leads to over-investing in programs aimed at winning share from the competition while under-investing in programs designed to retain current customers. Understanding how current market share is driven by the specific dynamics of one's switching and retention ratios is critical to making sound demand generation investment decisions. Practically speaking, optimizing market share in the face of a limited budget requires taking into account exactly where you are on the market share 'hill' while meeting the condition specified by equation (13).

MODEL LIMITATIONS AND DIRECTIONS FOR FUTURE RESEARCH

A number of simplifying assumptions have been made in order to make the mathematics tractable, but which may be problematic in practice. Here we identify the key assumptions, and suggest directions for future research.

- *Assuming all buyers have a fixed interpurchase interval*: We showed that if each buyer has a fixed interpurchase interval (so that the population has an interpurchase interval described by a random variable) then the model converges to equation (2) for market share. In reality, consumers and businesses change their habits because of growth or new technologies; one direction for future research would be to investigate the model when every individual's interpurchase interval is a random variable.
- *Assuming that the switching and retention ratios remain constant over time*: In reality, competitors will aggressively adjust their marketing strategies to combat perceived threats to market share, so the steady-state market share predicted by equation (2) may not occur. However, as long as revised estimates of the retention and switching ratios are made regularly, marketing practitioners can still take advantage of the policies suggested by equations (9) and (13). One could apply Bayesian methods to update the values of the switching and retention ratios and determine their effect on market share.
- *Assuming the values of the switching and retention ratios represent the behavior of all of the company's buyers*: More typically, some customers exhibit significantly more brand loyalty than others. A three-state model could be used to investigate the effects of this assumption, in which one's customers are segmented into Loyals and Switchers, with correspondingly different switching and retention ratios. This approach would provide more finely tuned policies for marketing practitioners toward spending on each segment.
- *Assuming the values of the switching and retention ratios are sufficiently accurate to drive decision making*: The values of the switching and retention ratios must be estimated. An additional research direction would be to determine how sensitive the optimal solutions given by equations (9) and (13) are to variations in the values of the estimated switching and retention ratios.
- *Assuming no growth in the market*: If the market is growing, equation (2) is valid

only if market growth is evenly distributed across all competitors. A direction for research would be to modify equation (1) to add a growth component, and determine revised expressions for equations (9) and (13). A further area of investigation involves finding how the optimal solutions vary as a function of market growth rate.

- *Assuming that marketing spending is the sole driver of market share*: We make this assumption in the numerical example in the section 'Optimizing market share given a budget constraint'. In fact, both marketing and the sales function generate leads. In principle, equation (2) does not depend on whether the variables that lead to changes in switching or retention behavior are related to spending on marketing, R&D, channels of distribution or a combination of all three. For example, Dong *et al* (2007) explore how channel quality affects optimal acquisition and retention spending. A research direction would be to theoretically and empirically investigate switching and retention ratios in the context of a broader competitive strategy.

In conclusion, we have combined the mathematics for Markov switching models with vector calculus in a novel way to show how marketing practitioners can optimize market share through an analysis of their switching and retention ratios. Applying standard principles of vector calculus allowed us to visualize and validate the conditions for maximizing market share, both in the absence and the presence of a budget constraint. In doing so, we have provided a straightforward mathematical framework that provides practical insights to marketing practitioners as they try to achieve their primary goal of maximizing market share.

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