MATH 237: Vector Calculus

PRACTICE EXAM III

Answer each of the following problems. When in doubt, err on the side of more explanation rather than less–the goal of your answer is to convince the reader of your knowledge. All work must be your own.

Caveat: This is a practice exam. It is intended to give a sense of the format and type of question that will be on the real exam. It may be slightly longer than the true exam; problems may seem more or less difficult than those on the true exam. Moreover, this exam is not vetted as carefully as the true exam. Expect typos.

(1) (a) Give a *very careful* explanation of $\int f(x,y)\ dA$ as a limit of Riemann sums. There is more than one acceptable formalism for this, but whichever you use, explain what every term means.

(b) Generalize your answer above to describe the integral of $f(x_1,\ldots,x_n)$, a function whose domain is a subset of $\mathbb{R}^n$. What information does this integral give?

(c) What does the change of variables factor in spherical co-ordinates, $\rho^2 \sin\phi$, account for? When and why is it used?

(2) (a) Evaluate

$$\int_0^4 \int_0^1 \int_{2y}^2 \frac{4\cos(x^2)}{2\sqrt{z}}\, dxdydz$$

(b) Evaluate the triple integral of $g(x,y,z) = x^4 + x^2y^2 + x^2z^2$ over the unit ball $x^2 + y^2 + z^2 \leq 1$.
(*A trig identity for* $\sin^2 \alpha$ *in terms of* $\cos(2\alpha)$ *might be useful here.*)

(3) Find the average value of $f(x, y) = e^{x^2+y^2}$ on the region in the xy-plan bounded by the annuli $x^2 + y^2 = 4$ and $x^2 + y^2 = 9$.

(4) Find the volume of the solid that lies inside the sphere of radius 3 centered at the origin and outside the cylinder $y^2 + z^2 = 1$.

(5) (a) Find the mass of the solid bounded by the hyperboloid $z^2 = a^2+x^2+y^2$ and the portion of the cone $z^2 = 2(x^2 + y^2)$ that lies above the xy-plane, if the density of the solid varies proportionately with distance from the origin.

(b) Recall that the center of mass of a solid with mass M is given by $(\bar{x}, \bar{y}, \bar{z})$, where

$$\bar{x} = \frac{1}{M}\iiint_R x\delta(x, y, z)\, dV, \quad \bar{y} = \frac{1}{M}\iiint_R y\delta(x, y, z)\, dV,$$

$$\bar{z} = \frac{1}{M}\iiint_R z\delta(x, y, z)\, dV,$$

Write, but do not evaluate, integral formulas for the center of mass of the solid above.

(6) Evaluate

$$\iint_{\Omega} xy^2 \, dA,$$

where Ω is the region bounded by $y = 3x, y = \frac{1}{3}x, xy = 27$ and $xy = 3$, using a suitable change of variables.