

HS Teacher Preparation in the New CUPM Curriculum Guide

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- About MAA
- Membership
- MAA Press
- Meetings
- Competitions
- Programs and Communities
- News

- Programs and Communities
- Curriculum Resources
 - Classroom Capsules and Notes
 - Common Vision
 - Course Communities
- INGenIOuS
- Instructional Practices Guide
- MAA MapleSoft Testing Suite
- META Math
- Progress through Calculus
- Survey and Reports
- Member Communities
- Outreach Initiatives
- Professional Development

Home » Programs and Communities » Curriculum Resources » Committee on the Undergraduate Program in Mathematics

Committee on the Undergraduate Program in Mathematics

The MAA's Committee on the Undergraduate Program in Mathematics (CUPM) is charged with making recommendations to guide mathematics departments in designing curricula for their undergraduate students.



- See the most recent updates for the 2015 CUPM Curriculum Guide.
- View two versions of the 2015 CUPM Curriculum Guide to Majors in the Mathematical Science: The principles contained in the Overview were approved by the board of Governors in August 2014.
 - 2015 CUPM Curriculum Guide (abridged printed version)
 - 2015 CUPM Curriculum Guide (entire Guide - including drafts of all program and course area reports)

CUPM 2015 brochure

Support for this MAA program is provided by the National Science Foundation (grant DUE-1228636).

Related:

- CUPM Recommendations: The First Forty Years
- CUPM Curriculum Guide 2004
- About CUPM
- Illustrative Resources
- Past CUPM Reports
- CUPM Deep History

Key Documents

CBMS, The Mathematical Education of Teachers II

<https://www.cbmsweb.org/the-mathematical-education-of-teachers/>

AMTE, Standards for Preparing Teachers of Mathematics

<https://amte.net/standards>

AMSTAT, The Statistical Education of Teachers

<http://www.amstat.org/asa/files/pdfs/EDU-SET.pdf>

Common Core State Standards

<http://www.corestandards.org/Math/>

“Mathematics for teaching has been described as ‘the body of mathematics that is important for teachers to know in order to be able to successfully manage the mathematical demands of their professional practice, i.e., teaching mathematics to children’ Hyman Bass, Zalman Usiskin, and others call mathematics for teaching ‘applied mathematics’; H. H. Wu calls it ‘mathematical engineering.’ For this reason, this document is written from the point of view that a traditional liberal arts major in mathematics is neither necessary nor sufficient preparation for teaching high school mathematics (unless it is followed by graduate training in mathematics education).”

Andreas J. Stylianides and Gabriel J. Stylianides, *Viewing “Mathematics for Teaching” as a Form of Applied Mathematics: Implications for the Mathematical Preparation of Teachers*, NOTICES of the AMS, 61 (2014), p. 266-276.

www.maa.org/sites/default/files/HighSchoolMathematicsTeachersPASGReport.pdf
**A Professional Program for Preparing Future High School Mathematics
Teachers**

Alan Tucker, Stony Brook University
Elizabeth Burroughs, Montana State University
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“...The recent Mathematical Education of Teachers II report (MET II) from the Conference Board of the Mathematical Sciences has many recommendations to foster .. professionalism among teachers of mathematics (<https://www.cbmsweb.org/the-mathematical-education-of-teachers/>). **As in the first MET report, the high school chapter of the MET II report calls for a greater connection between the college mathematics taken by future teachers and the high school mathematics they will teach.** The MET II recommendations, if widely adopted, will have a powerful impact on the future preparation of high school mathematics teachers.”

“It was long assumed by mathematics faculty that a standard mathematics major, slightly modified for teachers, would be an adequate preparation for teaching high school mathematics, even though most upper-division mathematics courses do not often make explicit connections to high school mathematics. That belief has been questioned and studies have supported this skepticism.

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“Regardless of the titles of the courses or how the mathematical content is organized within courses, our main recommendation for teacher preparation is that the program contain three courses with a purposeful focus on the mathematical content as it connects to high school mathematics.”

Lower Division

- ▶ Calculus → DE
- ▶ Linear Algebra
- ▶ Proofs (possibly discrete mathematics or number theory)
- ▶ Two courses in statistics

One-semester Introductory Statistics course emphasizing data analysis:

- ▶ formulation of statistical questions
- ▶ exploration of univariate data sets and comparisons among multiple univariate data sets
- ▶ introduction to the use of randomization in data production and inferential reasoning
- ▶ inference for means and proportions and differences of means or proportions; notions of p-value and margin of error
- ▶ introduction to probability from relative frequency perspective; additive and multiplicative rules, conditional probability and independence

One-semester Statistical Methods course

- ▶ bivariate categorical data: two-way tables, association, chi-square test
- ▶ bivariate measurement data: scatterplots, association, simple linear regression, correlation
- ▶ exponential and quadratic models; transformations of data (logs, powers)
- ▶ introduction to study design: surveys, experiments and observational studies
- ▶ randomization procedures for data production and inference
- ▶ introduction to one-way ANOVA

Upper Division

- ▶ geometry
- ▶ mathematical structures
- ▶ analysis

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- ▶ abstract algebra
- ▶ Courses including most of the following topics: discrete mathematics, mathematical modeling, number theory, probability, history of mathematics.

Geometry

- ▶ Euclidean geometry, featuring: Laws of Sines and Cosines and trigonometry from a geometric viewpoint, triangle congruence understood from the point of view of rigid motion transformations and similarity from the point of view of rigid motion and dilations.
- ▶ Analytic geometry, including conic sections, to re-prove earlier results in Euclidean geometry.

Mathematical Structures

- ▶ A rigorous look at the rational number system, including place value arithmetic, the unique form of reduced fractions, and the fundamental theorem of arithmetic.
- ▶ Geometry of the plane from a structural point of view.

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- ▶ A rigorous look at the rational number system, including place value arithmetic, the unique form of reduced fractions, and the fundamental theorem of arithmetic.
- ▶ Geometry of the plane from a structural point of view. One approach would be in terms of transformations; the other approach would be in terms of axiomatic systems, briefly surveying some of the other axiomatizations of the plane besides Euclidean geometry and including different formulations of parallelism and perpendicularity.

Analysis for Teachers

- ▶ Rigorous proofs of results about real valued functions, which lay the foundation for high school level material for manipulating polynomials and algebraic equations; then the extensions of the technique for completing the square and the quadratic formula, complex numbers and DeMoivres theorem.
- ▶ Exponential and logarithmic functions, trigonometric and inverse trigonometric functions.
- ▶ Basic limits, including the decimal expansion of a number, and basic theorems about continuous functions, derivatives, Riemann integrals, and the fundamental theorem of calculus

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- ▶ Rings and Fields
- ▶ Rings and Fields and the HS curriculum: complex numbers, DeMoivres theorem, and the Fundamental Theorem of Algebra (without proof), arithmetic in \mathbb{Z} compared with \mathbb{Z}_n , algebra of polynomial rings in base 10 arithmetic of integers, polynomials vs. polynomial functions, brief discussion of compass and straightedge constructibility.
- ▶ Groups.
- ▶ Groups and the HS curriculum: The isomorphism between the additive group of the real numbers and the multiplicative group of the positive real numbers given by the exponential and logarithm functions; congruence and similarity in geometry as equivalence relations defined by groups of transformations; connection between determinants and permutation parity.

Alternative pathways for smaller departments.

"Small departments unable to implement the full recommendations of this report should design a program that includes single and multivariable calculus, a data based statistics course, transition to proofs, and linear algebra. They should include upper division mathematics courses with at least three core mathematics-for-teachers courses, chosen from

- ▶ Abstract algebra (with a syllabus geared towards teachers)
- ▶ Geometry (with a syllabus geared towards teachers)
- ▶ Analysis (with a syllabus geared towards teachers)
- ▶ Capstone course for teachers, focusing on mathematical structure in high school algebra, geometry, and analysis."

“The courses for teachers presented here (other than, perhaps, a geometry course) are currently offered in only a few mathematics departments; UC - Berkeley and UC - Santa Barbara are the best known examples (for information on these courses, see www.ams.sunysb.edu/~tucker/BerkeleyMathEd.pdf and www.ams.sunysb.edu/~tucker/UCSBMathEd.docx).”

“...suggestions for near-term strategies to cover much of the recommended content in geometry, structures, and analysis by modifying existing courses found in many departments...

The geometry material can be incorporated in the geometry course that most departments offer for future high school teachers; the main change would be a decrease in non-Euclidean models and greater attention to the high school topics mentioned above and including a transformational approach...”

“The number systems material in the mathematical structures course and the polynomials and other functions in the analysis course could be covered in a Foundations of High School Mathematics course that many departments offer, or it could be the focus of a capstone course. Alternatively, departments can use (and adjust) an existing number theory course that is geared to the needs of teachers. Readers are referred to the number theory section of the MET II report for a discussion of appropriate topics [3, p. 61]...

The material in the analysis course can be incorporated into an existing analysis course, with particular attention to the topics highlighted in MET II. ”

“...we recognize the constraints on programs in smaller departments. We reiterate that our main recommendation in considering a program for teachers is to carefully address content that underlies, connects to, and is the focus of high school mathematics, either within existing courses required for all mathematics majors or in courses designed specifically for future teachers.”

Preparing Middle School Mathematics Teachers

COMET Subcommittee on Middle School Teacher Preparation Curriculum

Klay Kruczek, Southern Connecticut State University

Kien Lim, University of Texas at El Paso

Steven Morics, University of Redlands

Ayse Sahin, DePaul University

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“The Mathematical Education of Teachers II report (MET II) ... calls for institutions of higher learning to offer and support programs designed specifically to prepare undergraduates to teach mathematics in grades 5 through 8...Currently, the majority of middle school teachers have received their credentials either by completing a program designed for teaching high school mathematics, or by tailoring or supplementing a multiple-subject credential originally designed for elementary school teachers. Given the special needs of middle school students, and the increased demands on their abilities and skills, neither of these approaches directly prepares someone to teach middle school mathematics.”

ABOUT SIGMAA MKT

SIGMAA MKT is a community for all who work on preparation or development for teaching K-12 mathematics. Our members members teach courses or conduct research that may involve examining, designing, developing, piloting, and revising tasks and curricula focused on mathematical knowledge for teaching in grades K-12. These questions guide SIGMAA MKT: What evidence, experience and values guide us in how to prepare and develop K-12 teachers? How do we improve the mathematical education of prospective and practicing K-12 teachers? How do we strengthen the professional community of mathematics faculty working on mathematical preparation or development for teaching K-12 mathematics? How do we strengthen connections and community among such mathematics faculty and practicing K-12 teachers?