Mathematics Standards of Learning

for
Virginia
Public Schools

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Preface

The Standards of Learning in this publication represent a significant development in public education in Virginia. These standards focus on the mathematical knowledge and skills all students need for the future, and they have been aligned with national expectations for postsecondary success.

The Standards of Learning provide a framework for instructional programs designed to raise the academic achievement of all students in Virginia and are an important part of Virginia's efforts to provide challenging educational programs in the public schools.

The Standards of Learning set reasonable targets and expectations for what teachers need to teach and students need to learn. The standards are not intended to encompass the entire curriculum for a given grade level or course or to prescribe how the content should be taught; the standards are to be incorporated into a broader, locally designed curriculum. Teachers are encouraged to go beyond the standards and select instructional strategies and assessment methods appropriate for their students.

The Standards of Learning are recognized as a model for other states. They were developed through a series of public hearings and the efforts of parents, teachers, representatives from higher education, and business and industry leaders. The standards set clear, concise, and measurable academic expectations for young people. Parents are encouraged to work with their children to help them achieve these academic standards.

A major objective of Virginia's educational agenda is to give the citizens of Virginia a program of public education that is among the best in the nation and that meets the needs of all young people in Virginia. These Standards of Learning chart the course for achieving that objective.

Introduction

The Standards of Learning for mathematics identify academic content for essential components of the mathematics curriculum at different grade levels for Virginia's public schools. Recommendations and reports from Achieve, the College Board, and ACT, as well as the National Assessment of Educational Progress (NAEP) Frameworks, the *Curriculum Focal Points* from the National Council of Teachers of Mathematics (NCTM), *Principles and Standards for School Mathematics* from NCTM, the Singapore Curricula, the *Guidelines for Assessment and Instruction in Statistics Education (GAISE) Report* from the American Statistical Association, and the *Report of the President's National Mathematics Advisory Panel* were considered in identifying mathematics content necessary for success for all students in postsecondary pursuits.

Standards are identified for kindergarten through grade eight and for a core set of high school courses. Throughout a student's mathematics schooling from kindergarten through grade eight, specific content strands or topics are included. These content strands are Number and Number Sense; Computation and Estimation; Measurement; Geometry; Probability and Statistics; and Patterns, Functions, and Algebra. The Standards of Learning for each strand progress in complexity at each grade level and throughout the high school courses.

The Mathematics Standards of Learning Curriculum Framework is a companion document to the Mathematics Standards of Learning that amplifies the Mathematics Standards of Learning and defines the content knowledge, skills, and understandings that are measured by the Standards of Learning assessments. The Curriculum Framework provides additional guidance to school divisions and their teachers as they develop an instructional program appropriate for their students. It assists teachers in their lesson planning by identifying essential understandings, defining essential content knowledge, and describing the intellectual skills students need to use. This supplemental framework delineates in greater specificity the minimum content that all teachers should teach and all students should learn.

The Standards of Learning are not intended to encompass the entire curriculum for a given grade level or course or to prescribe how the content should be taught. Teachers are encouraged to go beyond the standards and select instructional strategies and assessment methods appropriate for their students.

Goals

Students today require more rigorous mathematical knowledge and skills to pursue higher education, to compete in a technologically sophisticated work force, and to be informed citizens. Students must gain an understanding of fundamental ideas in arithmetic, measurement, geometry, probability, data analysis and statistics, and algebra and functions, and they must develop proficiency in mathematical skills. In addition, students must learn to use a variety of methods and tools to compute, including paper and pencil, mental arithmetic, estimation, and calculators. Graphing utilities, spreadsheets, calculators, computers, and other forms of electronic information technology are now standard tools for mathematical problem solving in science, engineering, business and industry, government, and practical affairs. Hence, the use of technology must be an integral part of teaching, learning, and assessment. However, facility in the use of technology shall not be regarded as a substitute for a student's understanding of quantitative concepts and relationships or for proficiency in basic computations. The teaching of computer/technology skills should be the shared responsibility of teachers of all disciplines.

The content of the mathematics standards is intended to support the following five goals for students: becoming mathematical problem solvers, communicating mathematically, reasoning mathematically, making mathematical connections, and using mathematical representations to model and interpret practical situations.

Mathematical Problem Solving

Students will apply mathematical concepts and skills and the relationships among them to solve problem situations of varying complexities. Students also will recognize and create problems from real-life data and situations within and outside mathematics and then apply appropriate strategies to find acceptable solutions. To accomplish this goal, students will need to develop a repertoire of skills and strategies for solving a variety of problem types. A major goal of the mathematics program is to help students become competent mathematical problem solvers.

Mathematical Communication

Students will use the language of mathematics, including specialized vocabulary and symbols, to express mathematical ideas precisely. Representing, discussing, reading, writing, and listening to mathematics will help students to clarify their thinking and deepen their understanding of the mathematics being studied.

Mathematical Reasoning

Students will recognize reasoning and proof as fundamental aspects of mathematics. Students will learn and apply inductive and deductive reasoning skills to make, test, and evaluate mathematical statements and to justify steps in mathematical procedures. Students will use logical reasoning to analyze an argument and to determine whether conclusions are valid. In addition, students will learn to apply proportional and spatial reasoning and to reason from a variety of representations such as graphs, tables, and charts.

Mathematical Connections

Students will relate concepts and procedures from different topics in mathematics to one another and see mathematics as an integrated field of study. Through the application of content and process skills, students will make connections between different areas of mathematics and between mathematics and other disciplines, especially science. Science and mathematics teachers and curriculum writers are encouraged to develop mathematics and science curricula that reinforce each other.

Mathematical Representations

Students will represent and describe mathematical ideas, generalizations, and relationships with a variety of methods. Students will understand that representations of mathematical ideas are an essential part of learning, doing, and communicating mathematics. Students should move easily among different representations—graphical, numerical, algebraic, verbal, and physical—and recognize that representation is both a process and a product.

Kindergarten

The kindergarten standards place emphasis on developing the concept of number by counting; combining, sorting, and comparing sets of objects; recognizing and describing simple repeating patterns; and recognizing shapes and sizes of figures and objects. Students will investigate nonstandard measurement, collect data, and create graphs. The idea of fractions will be introduced.

While learning mathematics, students will be actively engaged, using concrete materials and appropriate technologies such as calculators and computers. However, facility in the use of technology shall not be regarded as a substitute for a student's understanding of quantitative concepts and relationships or for proficiency in basic computations.

Mathematics has its own language, and the acquisition of specialized vocabulary and language patterns is crucial to a student's understanding and appreciation of the subject. Students should be encouraged to use correctly the concepts, skills, symbols, and vocabulary identified in the following set of standards.

Problem solving has been integrated throughout the six content strands. The development of problem-solving skills should be a major goal of the mathematics program at every grade level. Instruction in the process of problem solving will need to be integrated early and continuously into each student's mathematics education. Students must be helped to develop a wide range of skills and strategies for solving a variety of problem types.

Number and Number Sense
Focus: Whole Number Concepts

K.1   The student, given two sets, each containing 10 or fewer concrete objects, will identify and describe one set as having more, fewer, or the same number of members as the other set, using the concept of one-to-one correspondence.

K.2   The student, given a set containing 15 or fewer concrete objects, will
   a) tell how many are in the set by counting the number of objects orally;
   b) write the numeral to tell how many are in the set; and
   c) select the corresponding numeral from a given set of numerals.

K.3   The student, given an ordered set of ten objects and/or pictures, will indicate the ordinal position of each object, first through tenth, and the ordered position of each object.

K.4   The student will
   a) count forward to 100 and backward from 10;
   b) identify one more than a number and one less than a number; and
   c) count by fives and tens to 100.

K.5   The student will identify the parts of a set and/or region that represent fractions for halves and fourths.

Computation and Estimation
Focus: Whole Number Operations

K.6   The student will model adding and subtracting whole numbers, using up to 10 concrete objects.

Measurement
Focus: Instruments and Attributes

K.7   The student will recognize a penny, nickel, dime, and quarter and will determine the value of a collection of pennies and/or nickels whose total value is 10 cents or less.

K.8   The student will identify the instruments used to measure length (ruler), weight (scale), time (clock: digital and analog; calendar: day, month, and season), and temperature (thermometer).

K.9   The student will tell time to the hour, using analog and digital clocks.

K.10   The student will compare two objects or events, using direct comparisons or nonstandard units of measure, according to one or more of the following attributes: length (shorter, longer), height (taller, shorter), weight (heavier, lighter), temperature (hotter, colder). Examples of nonstandard units include foot length, hand span, new pencil, paper clip, and block.

Geometry
Focus: Plane Figures

K.11   The student will
      a) identify, describe, and trace plane geometric figures (circle, triangle, square, and rectangle); and
      b) compare the size (larger, smaller) and shape of plane geometric figures (circle, triangle, square, and rectangle).

K.12   The student will describe the location of one object relative to another (above, below, next to) and identify representations of plane geometric figures (circle, triangle, square, and rectangle) regardless of their positions and orientations in space.

Probability and Statistics
Focus: Data Collection and Display

K.13   The student will gather data by counting and tallying.

K.14   The student will display gathered data in object graphs, picture graphs, and tables, and will answer questions related to the data.

Patterns, Functions, and Algebra
Focus: Attributes and Patterning

K.15   The student will sort and classify objects according to attributes.

K.16   The student will identify, describe, and extend repeating patterns.

Grade One

The first-grade standards place emphasis on counting, sorting, and comparing sets of up to 100 objects; recognizing and describing simple repeating and growing patterns; and tracing, describing, and sorting plane geometric figures. Students' understanding of number will be expanded through learning and applying the basic addition facts through the nines table and the corresponding subtraction facts; using nonstandard units to measure; and organizing and interpreting data. Fractional concepts will be expanded.

While learning mathematics, students will be actively engaged, using concrete materials and appropriate technologies such as calculators and computers. However, facility in the use of technology shall not be regarded as a substitute for a student's understanding of quantitative concepts and relationships or for proficiency in basic computations.

Mathematics has its own language, and the acquisition of specialized vocabulary and language patterns is crucial to a student's understanding and appreciation of the subject. Students should be encouraged to use correctly the concepts, skills, symbols, and vocabulary identified in the following set of standards.

Problem solving has been integrated throughout the six content strands. The development of problem-solving skills should be a major goal of the mathematics program at every grade level. Instruction in the process of problem solving will need to be integrated early and continuously into each student's mathematics education. Students must be helped to develop a wide range of skills and strategies for solving a variety of problem types.

Number and Number Sense
Focus: Place Value and Fraction Concepts

1.1   The student will
   a) count from 0 to 100 and write the corresponding numerals; and
   b) group a collection of up to 100 objects into tens and ones and write the corresponding numeral to develop an understanding of place value.

1.2   The student will count forward by ones, twos, fives, and tens to 100 and backward by ones from 30.

1.3   The student will identify the parts of a set and/or region that represent fractions for halves, thirds, and fourths and write the fractions.

Computation and Estimation
Focus: Whole Number Operations

1.4   The student, given a familiar problem situation involving magnitude, will
   a) select a reasonable order of magnitude from three given quantities: a one-digit numeral, a two-digit numeral, and a three-digit numeral (e.g., 5, 50, 500); and
   b) explain the reasonableness of the choice.

1.5   The student will recall basic addition facts with sums to 18 or less and the corresponding subtraction facts.

1.6   The student will create and solve one-step story and picture problems using basic addition facts with sums to 18 or less and the corresponding subtraction facts.

Measurement
Focus: Time and Nonstandard Measurement

1.7   The student will
   a) identify the number of pennies equivalent to a nickel, a dime, and a quarter; and
   b) determine the value of a collection of pennies, nickels, and dimes whose total value is 100 cents or less.

1.8   The student will tell time to the half-hour, using analog and digital clocks.

1.9   The student will use nonstandard units to measure length, weight/mass, and volume.

1.10   The student will compare, using the concepts of more, less, and equivalent,
        a) the volumes of two given containers; and
        b) the weight/mass of two objects, using a balance scale.

1.11   The student will use calendar language appropriately (e.g., names of the months, *today, yesterday, next week, last week*).

Geometry
Focus: Characteristics of Plane Figures

1.12   The student will identify and trace, describe, and sort plane geometric figures (triangle, square, rectangle, and circle) according to number of sides, vertices, and right angles.

1.13   The student will construct, model, and describe objects in the environment as geometric shapes (triangle, rectangle, square, and circle) and explain the reasonableness of each choice.

Probability and Statistics
Focus: Data Collection and Interpretation

1.14   The student will investigate, identify, and describe various forms of data collection (e.g., recording daily temperature, lunch count, attendance, favorite ice cream), using tables, picture graphs, and object graphs.

1.15   The student will interpret information displayed in a picture or object graph, using the vocabulary *more, less, fewer, greater than, less than,* and *equal to*.

Patterns, Functions, and Algebra
Focus: Patterning and Equivalence

1.16   The student will sort and classify concrete objects according to one or more attributes, including color, size, shape, and thickness.

1.17   The student will recognize, describe, extend, and create a wide variety of growing and repeating patterns.

1.18   The student will demonstrate an understanding of equality through the use of the equal sign.

Grade Two

The second-grade standards extend the study of number and spatial sense to include three-digit whole numbers and solid geometric figures. Students will continue to learn, use, and gain proficiency in the basic addition facts through the tens table and the corresponding subtraction facts. Students will begin to use U.S. Customary and metric units of measure; predict, using simple probability; and create and interpret picture and bar graphs. Students will work with a variety of patterns and will develop knowledge of equality by identifying missing numbers in addition and subtraction facts.

While learning mathematics, students will be actively engaged, using concrete materials and appropriate technologies such as calculators and computers. However, facility in the use of technology shall not be regarded as a substitute for a student's understanding of quantitative concepts and relationships or for proficiency in basic computations.

Mathematics has its own language, and the acquisition of specialized vocabulary and language patterns is crucial to a student's understanding and appreciation of the subject. Students should be encouraged to use correctly the concepts, skills, symbols, and vocabulary identified in the following set of standards.

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Number and Number Sense
Focus: Place Value, Number Patterns, and Fraction Concepts

2.1 The student will
  a) read, write, and identify the place value of each digit in a three-digit numeral, using numeration models;
  b) round two-digit numbers to the nearest ten; and
  c) compare two whole numbers between 0 and 999, using symbols (>, <, or =) and words (*greater than, less than,* or *equal to*).

2.2 The student will
  a) identify the ordinal positions first through twentieth, using an ordered set of objects; and
  b) write the ordinal numbers.

2.3 The student will
  a) identify the parts of a set and/or region that represent fractions for halves, thirds, fourths, sixths, eighths, and tenths;
  b) write the fractions; and
  c) compare the unit fractions for halves, thirds, fourths, sixths, eighths, and tenths.

2.4 The student will
  a) count forward by twos, fives, and tens to 100, starting at various multiples of 2, 5, or 10;
  b) count backward by tens from 100; and
  c) recognize even and odd numbers.

Computation and Estimation
Focus: Number Relationships and Operations

2.5 The student will recall addition facts with sums to 20 or less and the corresponding subtraction facts.

2.6   The student, given two whole numbers whose sum is 99 or less, will
     a)   estimate the sum; and
     b)   find the sum, using various methods of calculation.

2.7   The student, given two whole numbers, each of which is 99 or less, will
     a)   estimate the difference; and
     b)   find the difference, using various methods of calculation.

2.8   The student will create and solve one- and two-step addition and subtraction problems, using data from simple tables, picture graphs, and bar graphs.

2.9   The student will recognize and describe the related facts that represent and describe the inverse relationship between addition and subtraction.

Measurement
Focus: Money, Linear Measurement, Weight/Mass, and Volume

2.10   The student will
     a)   count and compare a collection of pennies, nickels, dimes, and quarters whose total value is $2.00 or less; and
     b)   correctly use the cent symbol (¢), dollar symbol ($), and decimal point (.).

2.11   The student will estimate and measure
     a)   length to the nearest centimeter and inch;
     b)   weight/mass of objects in pounds/ounces and kilograms/grams, using a scale; and
     c)   liquid volume in cups, pints, quarts, gallons, and liters.

2.12   The student will tell and write time to the nearest five minutes, using analog and digital clocks.

2.13   The student will
     a)   determine past and future days of the week; and
     b)   identify specific days and dates on a given calendar.

2.14   The student will read the temperature on a Celsius and/or Fahrenheit thermometer to the nearest 10 degrees.

Geometry
Focus: Symmetry and Plane and Solid Figures

2.15   The student will
     a)   draw a line of symmetry in a figure; and
     b)   identify and create figures with at least one line of symmetry.

2.16   The student will identify, describe, compare, and contrast plane and solid geometric figures (circle/sphere, square/cube, and rectangle/rectangular prism).

Probability and Statistics
Focus: Applications of Data

2.17   The student will use data from experiments to construct picture graphs, pictographs, and bar graphs.

2.18   The student will use data from experiments to predict outcomes when the experiment is repeated.

2.19   The student will analyze data displayed in picture graphs, pictographs, and bar graphs.

Patterns, Functions, and Algebra
Focus: Patterning and Numerical Sentences

2.20   The student will identify, create, and extend a wide variety of patterns.

2.21   The student will solve problems by completing numerical sentences involving the basic facts for addition and subtraction. The student will create story problems, using the numerical sentences.

2.22   The student will demonstrate an understanding of equality by recognizing that the symbol = in an equation indicates equivalent quantities and the symbol ≠ indicates that quantities are not equivalent.

Grade Three

The third-grade standards place emphasis on learning multiplication and division facts through the twelves table. Students will be fluent in the basic addition facts through the tens table and the corresponding subtraction facts. Concrete materials and two-dimensional representations will be used to introduce addition and subtraction with fractions and the concept of probability as chance. Students will use standard units (U.S. Customary and metric) to measure temperature, length, liquid volume, and weight and identify relevant properties of shapes, points, line segments, rays, angles, vertices, and lines. Students will investigate and describe the identity and commutative properties for addition and multiplication.

While learning mathematics, students will be actively engaged, using concrete materials and appropriate technologies such as calculators and computers. However, facility in the use of technology shall not be regarded as a substitute for a student's understanding of quantitative concepts and relationships or for proficiency in basic computations.

Mathematics has its own language, and the acquisition of specialized vocabulary and language patterns is crucial to a student's understanding and appreciation of the subject. Students should be encouraged to use correctly the concepts, skills, symbols, and vocabulary identified in the following set of standards.

Problem solving has been integrated throughout the six content strands. The development of problem-solving skills should be a major goal of the mathematics program at every grade level. Instruction in the process of problem solving will need to be integrated early and continuously into each student's mathematics education. Students must be helped to develop a wide range of skills and strategies for solving a variety of problem types.

Number and Number Sense
Focus: Place Value and Fractions

3.1 The student will
   a) read and write six-digit numerals and identify the place value and value of each digit;
   b) round whole numbers, 9,999 or less, to the nearest ten, hundred, and thousand; and
   c) compare two whole numbers between 0 and 9,999, using symbols (>, <, or =) and words (*greater than, less than*, or *equal to*).

3.2 The student will recognize and use the inverse relationships between addition/subtraction and multiplication/division to complete basic fact sentences. The student will use these relationships to solve problems.

3.3 The student will
   a) name and write fractions (including mixed numbers) represented by a model;
   b) model fractions (including mixed numbers) and write the fractions' names; and
   c) compare fractions having like and unlike denominators, using words and symbols (>, <, or =).

Computation and Estimation
Focus: Computation and Fraction Operations

3.4 The student will estimate solutions to and solve single-step and multistep problems involving the sum or difference of two whole numbers, each 9,999 or less, with or without regrouping.

3.5 The student will recall multiplication facts through the twelves table, and the corresponding division facts.

3.6 The student will represent multiplication and division, using area, set, and number line models, and create and solve problems that involve multiplication of two whole numbers, one factor 99 or less and the second factor 5 or less.

3.7   The student will add and subtract proper fractions having like denominators of 12 or less.

Measurement
Focus: U.S. Customary and Metric Units, Area and Perimeter, and Time

3.8   The student will determine, by counting, the value of a collection of bills and coins whose total value is $5.00 or less, compare the value of the bills and coins, and make change.

3.9   The student will estimate and use U.S. Customary and metric units to measure

    a) length to the nearest $\frac{1}{2}$-inch, inch, foot, yard, centimeter, and meter;

    b) liquid volume in cups, pints, quarts, gallons, and liters;
    c) weight/mass in ounces, pounds, grams, and kilograms; and
    d) area and perimeter.

3.10   The student will
    a) measure the distance around a polygon in order to determine perimeter; and
    b) count the number of square units needed to cover a given surface in order to determine area.

3.11   The student will
    a) tell time to the nearest minute, using analog and digital clocks; and
    b) determine elapsed time in one-hour increments over a 12-hour period.

3.12   The student will identify equivalent periods of time, including relationships among days, months, and years, as well as minutes and hours.

3.13   The student will read temperature to the nearest degree from a Celsius thermometer and a Fahrenheit thermometer. Real thermometers and physical models of thermometers will be used.

Geometry
Focus: Properties and Congruence Characteristics of Plane and Solid Figures

3.14   The student will identify, describe, compare, and contrast characteristics of plane and solid geometric figures (circle, square, rectangle, triangle, cube, rectangular prism, square pyramid, sphere, cone, and cylinder) by identifying relevant characteristics, including the number of angles, vertices, and edges, and the number and shape of faces, using concrete models.

3.15   The student will identify and draw representations of points, line segments, rays, angles, and lines.

3.16   The student will identify and describe congruent and noncongruent plane figures.

Probability and Statistics
Focus: Applications of Data and Chance

3.17   The student will
    a) collect and organize data, using observations, measurements, surveys, or experiments;
    b) construct a line plot, a picture graph, or a bar graph to represent the data; and
    c) read and interpret the data represented in line plots, bar graphs, and picture graphs and write a sentence analyzing the data.

3.18   The student will investigate and describe the concept of probability as chance and list possible results of a given situation.

Patterns, Functions, and Algebra
Focus: Patterns and Property Concepts

3.19   The student will recognize and describe a variety of patterns formed using numbers, tables, and pictures, and extend the patterns, using the same or different forms.

3.20   The student will
   a) investigate the identity and the commutative properties for addition and multiplication; and
   b) identify examples of the identity and commutative properties for addition and multiplication.

Grade Four

The fourth-grade standards place emphasis on multiplication and division with whole numbers and solving problems involving addition and subtraction of fractions and decimals by finding common multiples and factors. Students will be fluent in the basic multiplication facts through the twelves table and the corresponding division facts as they become proficient in multiplying larger numbers. Students also will refine their estimation skills for computations and measurements. Students will identify and describe representations of points, lines, line segments, rays, and angles, including endpoints and vertices. Concrete materials and two-dimensional representations will be used to solve problems involving perimeter, patterns, probability, and equivalence of fractions and decimals. Students will recognize images of figures resulting from geometric transformations, such as reflection, translation, and rotation. Students will investigate and describe the associative property for addition and multiplication.

While learning mathematics, students will be actively engaged, using concrete materials and appropriate technologies such as calculators and computers. However, facility in the use of technology shall not be regarded as a substitute for a student's understanding of quantitative concepts and relationships or for proficiency in basic computations.

Mathematics has its own language, and the acquisition of specialized vocabulary and language patterns is crucial to a student's understanding and appreciation of the subject. Students should be encouraged to use correctly the concepts, skills, symbols, and vocabulary identified in the following set of standards.

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Number and Number Sense
Focus: Place Value, Fractions, and Decimals

4.1 The student will
   a) identify orally and in writing the place value for each digit in a whole number expressed through millions;
   b) compare two whole numbers expressed through millions, using symbols (>, <, or =); and
   c) round whole numbers expressed through millions to the nearest thousand, ten thousand, and hundred thousand.

4.2 The student will
   a) compare and order fractions and mixed numbers;
   b) represent equivalent fractions; and
   c) identify the division statement that represents a fraction.

4.3 The student will
   a) read, write, represent, and identify decimals expressed through thousandths;
   b) round decimals to the nearest whole number, tenth, and hundredth;
   c) compare and order decimals; and
   d) given a model, write the decimal and fraction equivalents.

Computation and Estimation
Focus: Factors and Multiples, and Fraction and Decimal Operations

4.4   The student will
   a) estimate sums, differences, products, and quotients of whole numbers;
   b) add, subtract, and multiply whole numbers;
   c) divide whole numbers, finding quotients with and without remainders; and
   d) solve single-step and multistep addition, subtraction, and multiplication problems with whole numbers.

4.5   The student will
   a) determine common multiples and factors, including least common multiple and greatest common factor;
   b) add and subtract fractions having like and unlike denominators that are limited to 2, 3, 4, 5, 6, 8, 10, and 12, and simplify the resulting fractions, using common multiples and factors;
   c) add and subtract with decimals; and
   d) solve single-step and multistep practical problems involving addition and subtraction with fractions and with decimals.

Measurement
Focus: Equivalence within U.S. Customary and Metric Systems

4.6   The student will
   a) estimate and measure weight/mass and describe the results in U.S. Customary and metric units as appropriate; and
   b) identify equivalent measurements between units within the U.S. Customary system (ounces, pounds, and tons) and between units within the metric system (grams and kilograms).

4.7   The student will
   a) estimate and measure length, and describe the result in both metric and U.S. Customary units; and
   b) identify equivalent measurements between units within the U.S. Customary system (inches and feet; feet and yards; inches and yards; yards and miles) and between units within the metric system (millimeters and centimeters; centimeters and meters; and millimeters and meters).

4.8   The student will
   a) estimate and measure liquid volume and describe the results in U.S. Customary units; and
   b) identify equivalent measurements between units within the U.S. Customary system (cups, pints, quarts, and gallons).

4.9   The student will determine elapsed time in hours and minutes within a 12-hour period.

Geometry
Focus: Representations and Polygons

4.10   The student will
   a) identify and describe representations of points, lines, line segments, rays, and angles, including endpoints and vertices; and
   b) identify representations of lines that illustrate intersection, parallelism, and perpendicularity.

4.11   The student will
   a) investigate congruence of plane figures after geometric transformations, such as reflection, translation, and rotation, using mirrors, paper folding, and tracing; and
   b) recognize the images of figures resulting from geometric transformations, such as translation, reflection, and rotation.

4.12   The student will
   a) define *polygon*; and
   b) identify polygons with 10 or fewer sides.

Probability and Statistics
Focus: Outcomes and Data

4.13   The student will
   a) predict the likelihood of an outcome of a simple event; and
   b) represent probability as a number between 0 and 1, inclusive.

4.14   The student will collect, organize, display, and interpret data from a variety of graphs.

Patterns, Functions, and Algebra
Focus: Geometric Patterns, Equality, and Properties

4.15   The student will recognize, create, and extend numerical and geometric patterns.

4.16   The student will
   a) recognize and demonstrate the meaning of equality in an equation; and
   b) investigate and describe the associative property for addition and multiplication.

Grade Five

The fifth-grade standards place emphasis on number sense with whole numbers, fractions, and decimals. This focus includes concepts of prime and composite numbers, identifying even and odd numbers, and solving problems using order of operations for positive whole numbers. Students will develop proficiency in the use of fractions and decimals to solve problems. Students will collect, display, and analyze data in a variety of ways and solve probability problems, using a sample space or tree diagram. Students also will solve problems involving volume, area, and perimeter. Students will be introduced to variable expressions and open sentences, and will model one-step linear equations in one variable, using addition and subtraction. Students will investigate and recognize the distributive property. All of these skills assist in the development of the algebraic concepts needed for success in the middle grades.

While learning mathematics, students will be actively engaged, using concrete materials and appropriate technologies such as calculators and computers. However, facility in the use of technology shall not be regarded as a substitute for a student's understanding of quantitative concepts and relationships or for proficiency in basic computations.

Mathematics has its own language, and the acquisition of specialized vocabulary and language patterns is crucial to a student's understanding and appreciation of the subject. Students should be encouraged to use correctly the concepts, skills, symbols, and vocabulary identified in the following set of standards.

Problem solving has been integrated throughout the six content strands. The development of problem-solving skills should be a major goal of the mathematics program at every grade level. Instruction in the process of problem solving will need to be integrated early and continuously into each student's mathematics education. Students must be helped to develop a wide range of skills and strategies for solving a variety of problem types.

Number and Number Sense
Focus: Prime and Composite Numbers and Rounding Decimals

5.1   The student, given a decimal through thousandths, will round to the nearest whole number, tenth, or hundredth.

5.2   The student will
      a) recognize and name fractions in their equivalent decimal form and vice versa; and
      b) compare and order fractions and decimals in a given set from least to greatest and greatest to least.

5.3   The student will
      a) identify and describe the characteristics of prime and composite numbers; and
      b) identify and describe the characteristics of even and odd numbers.

Computation and Estimation
Focus: Multistep Applications and Order of Operations

5.4   The student will create and solve single-step and multistep practical problems involving addition, subtraction, multiplication, and division with and without remainders of whole numbers.

5.5   The student will
      a) find the sum, difference, product, and quotient of two numbers expressed as decimals through thousandths (divisors with only one nonzero digit); and
      b) create and solve single-step and multistep practical problems involving decimals.

5.6   The student will solve single-step and multistep practical problems involving addition and subtraction with fractions and mixed numbers and express answers in simplest form.

5.7   The student will evaluate whole number numerical expressions, using the order of operations limited to parentheses, addition, subtraction, multiplication, and division.

Measurement
Focus: Perimeter, Area, Volume, and Equivalent Measures

5.8   The student will
  a) find perimeter, area, and volume in standard units of measure;
  b) differentiate among perimeter, area, and volume and identify whether the application of the concept of perimeter, area, or volume is appropriate for a given situation;
  c) identify equivalent measurements within the metric system;
  d) estimate and then measure to solve problems, using U.S. Customary and metric units; and
  e) choose an appropriate unit of measure for a given situation involving measurement using U.S. Customary and metric units.

5.9   The student will identify and describe the diameter, radius, chord, and circumference of a circle.

5.10   The student will determine an amount of elapsed time in hours and minutes within a 24-hour period.

5.11   The student will measure right, acute, obtuse, and straight angles.

Geometry
Focus: Classification and Subdividing

5.12   The student will classify
  a) angles as right, acute, obtuse, or straight; and
  b) triangles as right, acute, obtuse, equilateral, scalene, or isosceles.

5.13   The student, using plane figures (square, rectangle, triangle, parallelogram, rhombus, and trapezoid), will
  a) develop definitions of these plane figures; and
  b) investigate and describe the results of combining and subdividing plane figures.

Probability and Statistics
Focus: Outcomes and Measures of Center

5.14   The student will make predictions and determine the probability of an outcome by constructing a sample space.

5.15   The student, given a problem situation, will collect, organize, and interpret data in a variety of forms, using stem-and-leaf plots and line graphs.

5.16   The student will
  a) describe mean, median, and mode as measures of center;
  b) describe mean as fair share;
  c) find the mean, median, mode, and range of a set of data; and
  d) describe the range of a set of data as a measure of variation.

Patterns, Functions, and Algebra
Focus: Equations and Properties

5.17   The student will describe the relationship found in a number pattern and express the relationship.

5.18   The student will
   a) investigate and describe the concept of variable;
   b) write an open sentence to represent a given mathematical relationship, using a variable;
   c) model one-step linear equations in one variable, using addition and subtraction; and
   d) create a problem situation based on a given open sentence, using a single variable.

5.19   The student will investigate and recognize the distributive property of multiplication over addition.

Grade Six

The sixth-grade standards are a transition from the emphasis placed on whole number arithmetic in the elementary grades to foundations of algebra. The standards emphasize rational numbers. Students will use ratios to compare data sets; recognize decimals, fractions, and percents as ratios; solve single-step and multistep problems, using rational numbers; and gain a foundation in the understanding of integers. Students will solve linear equations and use algebraic terminology. Students will solve problems involving area, perimeter, and surface area, work with π (pi), and focus on the relationships among the properties of quadrilaterals. In addition, students will focus on applications of probability and statistics.

While learning mathematics, students will be actively engaged, using concrete materials and appropriate technology such as calculators, computers, and spreadsheets. However, facility in the use of technology shall not be regarded as a substitute for a student's understanding of quantitative concepts and relationships or for proficiency in basic computations. Students will also identify real-life applications of the mathematical principles they are learning and apply these to science and other disciplines they are studying.

Mathematics has its own language, and the acquisition of specialized vocabulary and language patterns is crucial to a student's understanding and appreciation of the subject. Students should be encouraged to use correctly the concepts, skills, symbols, and vocabulary identified in the following set of standards.

Problem solving has been integrated throughout the six content strands. The development of problem-solving skills should be a major goal of the mathematics program at every grade level. Instruction in the process of problem solving will need to be integrated early and continuously into each student's mathematics education. Students must be helped to develop a wide range of skills and strategies for solving a variety of problem types.

Number and Number Sense
Focus: Relationships among Fractions, Decimals, and Percents

6.1   The student will describe and compare data, using ratios, and will use appropriate notations,

such as $\frac{a}{b}$, *a* to *b*, and *a:b*.

6.2   The student will
  a) investigate and describe fractions, decimals, and percents as ratios;
  b) identify a given fraction, decimal, or percent from a representation;
  c) demonstrate equivalent relationships among fractions, decimals, and percents; and
  d) compare and order fractions, decimals, and percents.

6.3   The student will
  a) identify and represent integers;
  b) order and compare integers; and
  c) identify and describe absolute value of integers.

6.4   The student will demonstrate multiple representations of multiplication and division of fractions.

6.5   The student will investigate and describe concepts of positive exponents and perfect squares.

Computation and Estimation
Focus: Applications of Operations with Rational Numbers

6.6   The student will
  a) multiply and divide fractions and mixed numbers; and
  b) estimate solutions and then solve single-step and multistep practical problems involving addition, subtraction, multiplication, and division of fractions.

6.7   The student will solve single-step and multistep practical problems involving addition, subtraction, multiplication, and division of decimals.

6.8   The student will evaluate whole number numerical expressions, using the order of operations.

Measurement
Focus: Problem Solving with Area, Perimeter, Volume, and Surface Area

6.9   The student will make ballpark comparisons between measurements in the U.S. Customary System of measurement and measurements in the metric system.

6.10   The student will
   a) define π (pi) as the ratio of the circumference of a circle to its diameter;
   b) solve practical problems involving circumference and area of a circle, given the diameter or radius;
   c) solve practical problems involving area and perimeter; and
   d) describe and determine the volume and surface area of a rectangular prism.

Geometry
Focus: Properties and Relationships

6.11   The student will
   a) identify the coordinates of a point in a coordinate plane; and
   b) graph ordered pairs in a coordinate plane.

6.12   The student will determine congruence of segments, angles, and polygons.

6.13   The student will describe and identify properties of quadrilaterals.

Probability and Statistics
Focus: Practical Applications of Statistics

6.14   The student, given a problem situation, will
   a) construct circle graphs;
   b) draw conclusions and make predictions, using circle graphs; and
   c) compare and contrast graphs that present information from the same data set.

6.15   The student will
   a) describe mean as balance point; and
   b) decide which measure of center is appropriate for a given purpose.

6.16   The student will
   a) compare and contrast dependent and independent events; and
   b) determine probabilities for dependent and independent events.

Patterns, Functions, and Algebra
Focus: Variable Equations and Properties

6.17   The student will identify and extend geometric and arithmetic sequences.

6.18   The student will solve one-step linear equations in one variable involving whole number coefficients and positive rational solutions.

6.19   The student will investigate and recognize
   a) the identity properties for addition and multiplication;
   b) the multiplicative property of zero; and
   c) the inverse property for multiplication.

6.20   The student will graph inequalities on a number line.

Grade Seven

The seventh-grade standards continue to emphasize the foundations of algebra. Students who successfully complete the seventh-grade standards should be prepared to study Algebra I in grade eight. Topics in grade seven include proportional reasoning, integer computation, solving two-step linear equations, and recognizing different representations for relationships. Students will apply the properties of real numbers in solving equations, solve inequalities, and use data analysis techniques to make inferences, conjectures, and predictions.

While learning mathematics, students will be actively engaged, using concrete materials and appropriate technology such as calculators, computers, and spreadsheets. However, facility in the use of technology shall not be regarded as a substitute for a student's understanding of quantitative concepts and relationships or for proficiency in basic computations. Students will also identify real-life applications of the mathematical principles they are learning and apply these to science and other disciplines they are studying.

Mathematics has its own language, and the acquisition of specialized vocabulary and language patterns is crucial to a student's understanding and appreciation of the subject. Students should be encouraged to use correctly the concepts, skills, symbols, and vocabulary identified in the following set of standards.

Problem solving has been integrated throughout the six content strands. The development of problem-solving skills should be a major goal of the mathematics program at every grade level. Instruction in the process of problem solving will need to be integrated early and continuously into each student's mathematics education. Students must be helped to develop a wide range of skills and strategies for solving a variety of problem types.

Number and Number Sense
Focus: Proportional Reasoning

7.1   The student will
         a)   investigate and describe the concept of negative exponents for powers of ten;
         b)   determine scientific notation for numbers greater than zero;
         c)   compare and order fractions, decimals, percents, and numbers written in scientific notation;
         d)   determine square roots; and
         e)   identify and describe absolute value for rational numbers.

7.2   The student will describe and represent arithmetic and geometric sequences, using variable expressions.

Computation and Estimation
Focus: Integer Operations and Proportional Reasoning

7.3   The student will
         a)   model addition, subtraction, multiplication, and division of integers; and
         b)   add, subtract, multiply, and divide integers.

7.4   The student will solve single-step and multistep practical problems, using proportional reasoning.

Measurement
Focus: Proportional Reasoning

7.5   The student will
   a) describe volume and surface area of cylinders;
   b) solve practical problems involving the volume and surface area of rectangular prisms and cylinders; and
   c) describe how changing one measured attribute of a rectangular prism affects its volume and surface area.

7.6   The student will determine whether plane figures—quadrilaterals and triangles—are similar and write proportions to express the relationships between corresponding sides of similar figures.

Geometry
Focus: Relationships between Figures

7.7   The student will compare and contrast the following quadrilaterals based on properties: parallelogram, rectangle, square, rhombus, and trapezoid.

7.8   The student, given a polygon in the coordinate plane, will represent transformations (reflections, dilations, rotations, and translations) by graphing in the coordinate plane.

Probability and Statistics
Focus: Applications of Statistics and Probability

7.9   The student will investigate and describe the difference between the experimental probability and theoretical probability of an event.

7.10   The student will determine the probability of compound events, using the Fundamental (Basic) Counting Principle.

7.11   The student, given data for a practical situation, will
   a) construct and analyze histograms; and
   b) compare and contrast histograms with other types of graphs presenting information from the same data set.

Patterns, Functions, and Algebra
Focus: Linear Equations

7.12   The student will represent relationships with tables, graphs, rules, and words.

7.13   The student will
   a) write verbal expressions as algebraic expressions and sentences as equations and vice versa; and
   b) evaluate algebraic expressions for given replacement values of the variables.

7.14   The student will
   a) solve one- and two-step linear equations in one variable; and
   b) solve practical problems requiring the solution of one- and two-step linear equations.

7.15   The student will
   a) solve one-step inequalities in one variable; and
   b) graph solutions to inequalities on the number line.

7.16   The student will apply the following properties of operations with real numbers:
     a)   the commutative and associative properties for addition and multiplication;
     b)   the distributive property;
     c)   the additive and multiplicative identity properties;
     d)   the additive and multiplicative inverse properties; and
     e)   the multiplicative property of zero.

Grade Eight

The eighth-grade standards are intended to serve two purposes. First, the standards contain content that reviews or extends concepts and skills learned in previous grades. Second, they contain new content that prepares students for more abstract concepts in algebra and geometry. The eighth-grade standards provide students additional instruction and time to acquire the concepts and skills necessary for success in Algebra I. Students will gain proficiency in computation with rational numbers and will use proportions to solve a variety of problems. New concepts include solving multistep equations and inequalities, graphing linear equations, visualizing three-dimensional shapes represented in two-dimensional drawings, and applying transformations to geometric shapes in the coordinate plane. Students will verify and apply the Pythagorean Theorem and represent relations and functions, using tables, graphs, and rules. The eighth-grade standards provide a more solid foundation in Algebra I for those students not ready for Algebra I in grade eight.

While learning mathematics, students will be actively engaged, using concrete materials and appropriate technologies. However, facility in the use of technology shall not be regarded as a substitute for a student's understanding of quantitative concepts and relationships or for proficiency in basic computations. Students will also identify real-life applications of the mathematical principles they are learning that can be applied to science and other disciplines they are studying.

Mathematics has its own language, and the acquisition of specialized vocabulary and language patterns is crucial to a student's understanding and appreciation of the subject. Students should be encouraged to use correctly the concepts, skills, symbols, and vocabulary identified in the following set of standards.

Problem solving has been integrated throughout the six content strands. The development of problem-solving skills should be a major goal of the mathematics program at every grade level. Instruction in the process of problem solving will need to be integrated early and continuously into each student's mathematics education. Students must be helped to develop a wide range of skills and strategies for solving a variety of problem types.

Number and Number Sense
Focus: Relationships within the Real Number System

8.1   The student will
    a) simplify numerical expressions involving positive exponents, using rational numbers, order of operations, and properties of operations with real numbers; and
    b) compare and order decimals, fractions, percents, and numbers written in scientific notation.

8.2   The student will describe orally and in writing the relationships between the subsets of the real number system.

Computation and Estimation
Focus: Practical Applications of Operations with Real Numbers

8.3   The student will
    a) solve practical problems involving rational numbers, percents, ratios, and proportions; and
    b) determine the percent increase or decrease for a given situation.

8.4   The student will apply the order of operations to evaluate algebraic expressions for given replacement values of the variables.

8.5   The student will
    a) determine whether a given number is a perfect square; and
    b) find the two consecutive whole numbers between which a square root lies.

Measurement
Focus: Problem Solving

8.6 The student will
   a) verify by measuring and describe the relationships among vertical angles, adjacent angles, supplementary angles, and complementary angles; and
   b) measure angles of less than 360°.

8.7 The student will
   a) investigate and solve practical problems involving volume and surface area of prisms, cylinders, cones, and pyramids; and
   b) describe how changing one measured attribute of a figure affects the volume and surface area.

Geometry
Focus: Problem Solving with 2- and 3-Dimensional Figures

8.8 The student will
   a) apply transformations to plane figures; and
   b) identify applications of transformations.

8.9 The student will construct a three-dimensional model, given the top or bottom, side, and front views.

8.10 The student will
   a) verify the Pythagorean Theorem; and
   b) apply the Pythagorean Theorem.

8.11 The student will solve practical area and perimeter problems involving composite plane figures.

Probability and Statistics
Focus: Statistical Analysis of Graphs and Problem Situations

8.12 The student will determine the probability of independent and dependent events with and without replacement.

8.13 The student will
   a) make comparisons, predictions, and inferences, using information displayed in graphs; and
   b) construct and analyze scatterplots.

Patterns, Functions, and Algebra
Focus: Linear Relationships

8.14 The student will make connections between any two representations (tables, graphs, words, and rules) of a given relationship.

8.15 The student will
   a) solve multistep linear equations in one variable with the variable on one and two sides of the equation;
   b) solve two-step linear inequalities and graph the results on a number line; and
   c) identify properties of operations used to solve an equation.

8.16 The student will graph a linear equation in two variables.

8.17 The student will identify the domain, range, independent variable, or dependent variable in a given situation.

Algebra I

The standards below outline the content for a one-year course in Algebra I. All students are expected to achieve the Algebra I standards. When planning for instruction, consideration will be given to the sequential development of concepts and skills by using concrete materials to assist students in making the transition from the arithmetic to the symbolic. Students should be helped to make connections and build relationships between algebra and arithmetic, geometry, and probability and statistics. Connections also should be made to other subject areas through practical applications. This approach to teaching algebra should help students attach meaning to the abstract concepts of algebra.

These standards require students to use algebra as a tool for representing and solving a variety of practical problems. Tables and graphs will be used to interpret algebraic expressions, equations, and inequalities and to analyze behaviors of functions.

Graphing calculators, computers, and other appropriate technology tools will be used to assist in teaching and learning. Graphing utilities enhance the understanding of functions; they provide a powerful tool for solving and verifying solutions to equations and inequalities.

Throughout the course, students should be encouraged to engage in discourse about mathematics with teachers and other students, use the language and symbols of mathematics in representations and communication, discuss problems and problem solving, and develop confidence in themselves as mathematics students.

Expressions and Operations

A.1   The student will represent verbal quantitative situations algebraically and evaluate these expressions for given replacement values of the variables.

A.2   The student will perform operations on polynomials, including
   a) applying the laws of exponents to perform operations on expressions;
   b) adding, subtracting, multiplying, and dividing polynomials; and
   c) factoring completely first- and second-degree binomials and trinomials in one or two variables. Graphing calculators will be used as a tool for factoring and for confirming algebraic factorizations.

A.3   The student will express the square roots and cube roots of whole numbers and the square root of a monomial algebraic expression in simplest radical form.

Equations and Inequalities

A.4   The student will solve multistep linear and quadratic equations in two variables, including
   a) solving literal equations (formulas) for a given variable;
   b) justifying steps used in simplifying expressions and solving equations, using field properties and axioms of equality that are valid for the set of real numbers and its subsets;
   c) solving quadratic equations algebraically and graphically;
   d) solving multistep linear equations algebraically and graphically;
   e) solving systems of two linear equations in two variables algebraically and graphically; and
   f) solving real-world problems involving equations and systems of equations.
   Graphing calculators will be used both as a primary tool in solving problems and to verify algebraic solutions.

A.5   The student will solve multistep linear inequalities in two variables, including
- a) solving multistep linear inequalities algebraically and graphically;
- b) justifying steps used in solving inequalities, using axioms of inequality and properties of order that are valid for the set of real numbers and its subsets;
- c) solving real-world problems involving inequalities; and
- d) solving systems of inequalities.

A.6   The student will graph linear equations and linear inequalities in two variables, including
- a) determining the slope of a line when given an equation of the line, the graph of the line, or two points on the line. Slope will be described as rate of change and will be positive, negative, zero, or undefined; and
- b) writing the equation of a line when given the graph of the line, two points on the line, or the slope and a point on the line.

Functions

A.7   The student will investigate and analyze function (linear and quadratic) families and their characteristics both algebraically and graphically, including
- a) determining whether a relation is a function;
- b) domain and range;
- c) zeros of a function;
- d) *x*- and *y*-intercepts;
- e) finding the values of a function for elements in its domain; and
- f) making connections between and among multiple representations of functions including concrete, verbal, numeric, graphic, and algebraic.

A.8   The student, given a situation in a real-world context, will analyze a relation to determine whether a direct or inverse variation exists, and represent a direct variation algebraically and graphically and an inverse variation algebraically.

Statistics

A.9   The student, given a set of data, will interpret variation in real-world contexts and calculate and interpret mean absolute deviation, standard deviation, and z-scores.

A.10   The student will compare and contrast multiple univariate data sets, using box-and-whisker plots.

A.11   The student will collect and analyze data, determine the equation of the curve of best fit in order to make predictions, and solve real-world problems, using mathematical models. Mathematical models will include linear and quadratic functions.

Geometry

This course is designed for students who have successfully completed the standards for Algebra I. All students are expected to achieve the Geometry standards. The course includes, among other things, properties of geometric figures, trigonometric relationships, and reasoning to justify conclusions. Methods of justification will include paragraph proofs, two-column proofs, indirect proofs, coordinate proofs, algebraic methods, and verbal arguments. A gradual development of formal proof will be encouraged. Inductive and intuitive approaches to proof as well as deductive axiomatic methods should be used.

This set of standards includes emphasis on two- and three-dimensional reasoning skills, coordinate and transformational geometry, and the use of geometric models to solve problems. A variety of applications and some general problem-solving techniques, including algebraic skills, should be used to implement these standards. Calculators, computers, graphing utilities (graphing calculators or computer graphing simulators), dynamic geometry software, and other appropriate technology tools will be used to assist in teaching and learning. Any technology that will enhance student learning should be used.

Reasoning, Lines, and Transformations

G.1   The student will construct and judge the validity of a logical argument consisting of a set of premises and a conclusion. This will include
    a)   identifying the converse, inverse, and contrapositive of a conditional statement;
    b)   translating a short verbal argument into symbolic form;
    c)   using Venn diagrams to represent set relationships; and
    d)   using deductive reasoning.

G.2   The student will use the relationships between angles formed by two lines cut by a transversal to
    a)   determine whether two lines are parallel;
    b)   verify the parallelism, using algebraic and coordinate methods as well as deductive proofs; and
    c)   solve real-world problems involving angles formed when parallel lines are cut by a transversal.

G.3   The student will use pictorial representations, including computer software, constructions, and coordinate methods, to solve problems involving symmetry and transformation. This will include
    a)   investigating and using formulas for finding distance, midpoint, and slope;
    b)   applying slope to verify and determine whether lines are parallel or perpendicular;
    c)   investigating symmetry and determining whether a figure is symmetric with respect to a line or a point; and
    d)   determining whether a figure has been translated, reflected, rotated, or dilated, using coordinate methods.

G.4   The student will construct and justify the constructions of
    a)   a line segment congruent to a given line segment;
    b)   the perpendicular bisector of a line segment;
    c)   a perpendicular to a given line from a point not on the line;
    d)   a perpendicular to a given line at a given point on the line;
    e)   the bisector of a given angle,
    f)   an angle congruent to a given angle; and
    g)   a line parallel to a given line through a point not on the given line.

Triangles

G.5   The student, given information concerning the lengths of sides and/or measures of angles in triangles, will
   a)   order the sides by length, given the angle measures;
   b)   order the angles by degree measure, given the side lengths;
   c)   determine whether a triangle exists; and
   d)   determine the range in which the length of the third side must lie.
   These concepts will be considered in the context of real-world situations.

G.6   The student, given information in the form of a figure or statement, will prove two triangles are congruent, using algebraic and coordinate methods as well as deductive proofs.

G.7   The student, given information in the form of a figure or statement, will prove two triangles are similar, using algebraic and coordinate methods as well as deductive proofs.

G.8   The student will solve real-world problems involving right triangles by using the Pythagorean Theorem and its converse, properties of special right triangles, and right triangle trigonometry.

Polygons and Circles

G.9   The student will verify characteristics of quadrilaterals and use properties of quadrilaterals to solve real-world problems.

G.10   The student will solve real-world problems involving angles of polygons.

G.11   The student will use angles, arcs, chords, tangents, and secants to
   a)   investigate, verify, and apply properties of circles;
   b)   solve real-world problems involving properties of circles; and
   c)   find arc lengths and areas of sectors in circles.

G.12   The student, given the coordinates of the center of a circle and a point on the circle, will write the equation of the circle.

Three-Dimensional Figures

G.13   The student will use formulas for surface area and volume of three-dimensional objects to solve real-world problems.

G.14   The student will use similar geometric objects in two- or three-dimensions to
   a)   compare ratios between side lengths, perimeters, areas, and volumes;
   b)   determine how changes in one or more dimensions of an object affect area and/or volume of the object;
   c)   determine how changes in area and/or volume of an object affect one or more dimensions of the object; and
   d)   solve real-world problems about similar geometric objects.

Algebra, Functions, and Data Analysis

The following standards outline the content for a one-year course in Algebra, Functions, and Data Analysis. This course is designed for students who have successfully completed the standards for Algebra I. Within the context of mathematical modeling and data analysis, students will study functions and their behaviors, systems of inequalities, probability, experimental design and implementation, and analysis of data. Data will be generated by practical applications arising from science, business, and finance. Students will solve problems that require the formulation of linear, quadratic, exponential, or logarithmic equations or a system of equations.

Through the investigation of mathematical models and interpretation/analysis of data from real life situations, students will strengthen conceptual understandings in mathematics and further develop connections between algebra and statistics. Students should use the language and symbols of mathematics in representations and communication throughout the course.

These standards include a transformational approach to graphing functions and writing equations when given the graph of the equation. Transformational graphing builds a strong connection between algebraic and graphic representations of functions.

The infusion of technology (graphing calculator and/or computer software) in this course will assist in modeling and investigating functions and data analysis.

Algebra and Functions

AFDA.1 The student will investigate and analyze function (linear, quadratic, exponential, and logarithmic) families and their characteristics. Key concepts include
   a) continuity;
   b) local and absolute maxima and minima;
   c) domain and range;
   d) zeros;
   e) intercepts;
   f) intervals in which the function is increasing/decreasing;
   g) end behaviors; and
   h) asymptotes.

AFDA.2 The student will use knowledge of transformations to write an equation, given the graph of a function (linear, quadratic, exponential, and logarithmic).

AFDA.3 The student will collect data and generate an equation for the curve (linear, quadratic, exponential, and logarithmic) of best fit to model real-world problems or applications. Students will use the best fit equation to interpolate function values, make decisions, and justify conclusions with algebraic and/or graphical models.

AFDA.4 The student will transfer between and analyze multiple representations of functions, including algebraic formulas, graphs, tables, and words. Students will select and use appropriate representations for analysis, interpretation, and prediction.

AFDA.5 The student will determine optimal values in problem situations by identifying constraints and using linear programming techniques.

Data Analysis

AFDA.6 The student will calculate probabilities. Key concepts include
    a) conditional probability;
    b) dependent and independent events;
    c) addition and multiplication rules;
    d) counting techniques (permutations and combinations); and
    e) Law of Large Numbers.

AFDA.7 The student will analyze the normal distribution. Key concepts include
    a) characteristics of normally distributed data;
    b) percentiles;
    c) normalizing data, using z-scores; and
    d) area under the standard normal curve and probability.

AFDA.8 The student will design and conduct an experiment/survey. Key concepts include
    a) sample size;
    b) sampling technique;
    c) controlling sources of bias and experimental error;
    d) data collection; and
    e) data analysis and reporting.

Algebra II

The standards below outline the content for a one-year course in Algebra II. Students enrolled in Algebra II are assumed to have mastered those concepts outlined in the Algebra I standards. All students preparing for postsecondary and advanced technical studies are expected to achieve the Algebra II standards. A thorough treatment of advanced algebraic concepts will be provided through the study of functions, "families of functions," equations, inequalities, systems of equations and inequalities, polynomials, rational and radical equations, complex numbers, and sequences and series. Emphasis will be placed on practical applications and modeling throughout the course of study. Oral and written communication concerning the language of algebra, logic of procedures, and interpretation of results should also permeate the course.

These standards include a transformational approach to graphing functions. Transformational graphing uses translation, reflection, dilation, and rotation to generate a "family of graphs" from a given graph and builds a strong connection between algebraic and graphic representations of functions. Students will vary the coefficients and constants of an equation, observe the changes in the graph of the equation, and make generalizations that can be applied to many graphs.

Graphing utilities (graphing calculators or computer graphing simulators), computers, spreadsheets, and other appropriate technology tools will be used to assist in teaching and learning. Graphing utilities enhance the understanding of realistic applications through mathematical modeling and aid in the investigation and study of functions. They also provide an effective tool for solving and verifying solutions to equations and inequalities. Any other available technology that will enhance student learning should be used.

Expressions and Operations

AII.1   The student, given rational, radical, or polynomial expressions, will
- a) add, subtract, multiply, divide, and simplify rational algebraic expressions;
- b) add, subtract, multiply, divide, and simplify radical expressions containing rational numbers and variables, and expressions containing rational exponents;
- c) write radical expressions as expressions containing rational exponents and vice versa; and
- d) factor polynomials completely.

AII.2*   The student will investigate and apply the properties of arithmetic and geometric sequences and series to solve real-world problems, including writing the first n terms, finding the nth term, and evaluating summation formulas. Notation will include Σ and a_n.
*Standard AII.2 will be assessed in the Functions and Statistics reporting category. (Revised March 2011)

AII.3   The student will perform operations on complex numbers, express the results in simplest form using patterns of the powers of i, and identify field properties that are valid for the complex numbers.

Equations and Inequalities

AII.4   The student will solve, algebraically and graphically,
- a) absolute value equations and inequalities;
- b) quadratic equations over the set of complex numbers;
- c) equations containing rational algebraic expressions; and
- d) equations containing radical expressions.

Graphing calculators will be used for solving and for confirming the algebraic solutions.

AII.5   The student will solve nonlinear systems of equations, including linear-quadratic and quadratic-quadratic, algebraically and graphically. Graphing calculators will be used as a tool to visualize graphs and predict the number of solutions.

Functions

AII.6   The student will recognize the general shape of function (absolute value, square root, cube root, rational, polynomial, exponential, and logarithmic) families and will convert between graphic and symbolic forms of functions. A transformational approach to graphing will be employed. Graphing calculators will be used as a tool to investigate the shapes and behaviors of these functions.

AII.7   The student will investigate and analyze functions algebraically and graphically. Key concepts include
a)   domain and range, including limited and discontinuous domains and ranges;
b)   zeros;
c)   *x*- and *y*-intercepts;
d)   intervals in which a function is increasing or decreasing;
e)   asymptotes;
f)   end behavior;
g)   inverse of a function; and
h)   composition of multiple functions.
Graphing calculators will be used as a tool to assist in investigation of functions.

AII.8   The student will investigate and describe the relationships among solutions of an equation, zeros of a function, *x*-intercepts of a graph, and factors of a polynomial expression.

Statistics

AII.9   The student will collect and analyze data, determine the equation of the curve of best fit, make predictions, and solve real-world problems, using mathematical models. Mathematical models will include polynomial, exponential, and logarithmic functions.

AII.10   The student will identify, create, and solve real-world problems involving inverse variation, joint variation, and a combination of direct and inverse variations.

AII.11   The student will identify properties of a normal distribution and apply those properties to determine probabilities associated with areas under the standard normal curve.

AII.12   The student will compute and distinguish between permutations and combinations and use technology for applications.

Trigonometry

The standards below outline the content for a one-semester course in trigonometry. Students enrolled in trigonometry are assumed to have mastered those concepts outlined in the Algebra II standards. A thorough treatment of trigonometry will be provided through the study of trigonometric definitions, applications, graphing, and solving trigonometric equations and inequalities. Emphasis should also be placed on using connections between right triangle ratios, trigonometric functions, and circular functions. In addition, applications and modeling should be included throughout the course of study. Emphasis should also be placed on oral and written communication concerning the language of mathematics, logic of procedure, and interpretation of results.

Graphing calculators, computers, and other appropriate technology tools will be used to assist in teaching and learning. Graphing utilities enhance the understanding of realistic applications through modeling and aid in the investigation of trigonometric functions and their inverses. They also provide a powerful tool for solving and verifying solutions to trigonometric equations and inequalities.

T.1   The student, given a point other than the origin on the terminal side of an angle, will use the definitions of the six trigonometric functions to find the sine, cosine, tangent, cotangent, secant, and cosecant of the angle in standard position. Trigonometric functions defined on the unit circle will be related to trigonometric functions defined in right triangles.

T.2   The student, given the value of one trigonometric function, will find the values of the other trigonometric functions, using the definitions and properties of the trigonometric functions.

T.3   The student will find, without the aid of a calculator, the values of the trigonometric functions of the special angles and their related angles as found in the unit circle. This will include converting angle measures from radians to degrees and vice versa.

T.4   The student will find, with the aid of a calculator, the value of any trigonometric function and inverse trigonometric function.

T.5   The student will verify basic trigonometric identities and make substitutions, using the basic identities.

T.6   The student, given one of the six trigonometric functions in standard form, will
        a) state the domain and the range of the function;
        b) determine the amplitude, period, phase shift, vertical shift, and asymptotes;
        c) sketch the graph of the function by using transformations for at least a two-period interval; and
        d) investigate the effect of changing the parameters in a trigonometric function on the graph of the function.

T.7   The student will identify the domain and range of the inverse trigonometric functions and recognize the graphs of these functions. Restrictions on the domains of the inverse trigonometric functions will be included.

T.8   The student will solve trigonometric equations that include both infinite solutions and restricted domain solutions and solve basic trigonometric inequalities.

T.9   The student will identify, create, and solve real-world problems involving triangles. Techniques will include using the trigonometric functions, the Pythagorean Theorem, the Law of Sines, and the Law of Cosines.

Algebra II and Trigonometry

The standards for this combined course in Algebra II and Trigonometry include all of the standards listed for Algebra II and Trigonometry. This course is designed for advanced students who are capable of a more rigorous course at an accelerated pace. The standards listed for this course provide the foundation for students to pursue a sequence of advanced mathematical studies from Mathematical Analysis to Advanced Placement Calculus.

Expressions and Operations

AII/T.1   The student, given rational, radical, or polynomial expressions, will
   a) add, subtract, multiply, divide, and simplify rational algebraic expressions;
   b) add, subtract, multiply, divide, and simplify radical expressions containing rational numbers and variables, and expressions containing rational exponents;
   c) write radical expressions as expressions containing rational exponents and vice versa; and
   d) factor polynomials completely.

AII/T.2   The student will investigate and apply the properties of arithmetic and geometric sequences and series to solve real-world problems, including writing the first n terms, finding the nth term, and evaluating summation formulas. Notation will include Σ and a_n.

AII/T.3   The student will perform operations on complex numbers, express the results in simplest form using patterns of the powers of i, and identify field properties that are valid for the complex numbers.

Equations and Inequalities

AII/T.4   The student will solve, algebraically and graphically,
   a) absolute value equations and inequalities;
   b) quadratic equations over the set of complex numbers;
   c) equations containing rational algebraic expressions; and
   d) equations containing radical expressions.
   Graphing calculators will be used for solving and for confirming the algebraic solutions.

AII/T.5   The student will solve nonlinear systems of equations, including linear-quadratic and quadratic-quadratic, algebraically and graphically. Graphing calculators will be used as a tool to visualize graphs and predict the number of solutions.

Functions

AII/T.6   The student will recognize the general shape of function (absolute value, square root, cube root, rational, polynomial, exponential, and logarithmic) families and will convert between graphic and symbolic forms of functions. A transformational approach to graphing will be employed. Graphing calculators will be used as a tool to investigate the shapes and behaviors of these functions.

AII/T.7   The student will investigate and analyze functions algebraically and graphically. Key concepts include
   a) domain and range, including limited and discontinuous domains and ranges;
   b) zeros;
   c) x- and y-intercepts;
   d) intervals in which a function is increasing or decreasing;
   e) asymptotes;
   f) end behavior;
   g) inverse of a function; and
   h) composition of multiple functions.
   Graphing calculators will be used as a tool to assist in the investigation of functions.

AII/T.8   The student will investigate and describe the relationships among solutions of an equation, zeros of a function, *x*-intercepts of a graph, and factors of a polynomial expression.

Statistics

AII/T.9   The student will collect and analyze data, determine the equation of the curve of best fit, make predictions, and solve real-world problems, using mathematical models. Mathematical models will include polynomial, exponential, and logarithmic functions.

AII/T.10 The student will identify, create, and solve real-world problems involving inverse variation, joint variation, and a combination of direct and inverse variations.

AII/T.11 The student will identify properties of a normal distribution and apply those properties to determine probabilities associated with areas under the standard normal curve.

AII/T.12 The student will compute and distinguish between permutations and combinations and use technology for applications.

Trigonometry

AII/T.13 The student, given a point other than the origin on the terminal side of an angle, will use the definitions of the six trigonometric functions to find the sine, cosine, tangent, cotangent, secant, and cosecant of the angle in standard position. Trigonometric functions defined on the unit circle will be related to trigonometric functions defined in right triangles.

AII/T.14 The student, given the value of one trigonometric function, will find the values of the other trigonometric functions, using the definitions and properties of the trigonometric functions.

AII/T.15 The student will find, without the aid of a calculator, the values of the trigonometric functions of the special angles and their related angles as found in the unit circle. This will include converting angle measures from radians to degrees and vice versa.

AII/T.16 The student will find, with the aid of a calculator, the value of any trigonometric function and inverse trigonometric function.

AII/T.17 The student will verify basic trigonometric identities and make substitutions, using the basic identities.

AII/T.18 The student, given one of the six trigonometric functions in standard form, will
   a) state the domain and the range of the function;
   b) determine the amplitude, period, phase shift, vertical shift, and asymptotes;
   c) sketch the graph of the function by using transformations for at least a two-period interval; and
   d) investigate the effect of changing the parameters in a trigonometric function on the graph of the function.

AII/T.19 The student will identify the domain and range of the inverse trigonometric functions and recognize the graphs of these functions. Restrictions on the domains of the inverse trigonometric functions will be included.

AII/T.20 The student will solve trigonometric equations that include both infinite solutions and restricted domain solutions and solve basic trigonometric inequalities.

AII/T.21 The student will identify, create, and solve real-world problems involving triangles. Techniques will include using the trigonometric functions, the Pythagorean Theorem, the Law of Sines, and the Law of Cosines.

Computer Mathematics

This course is intended to provide students with experiences in using computer programming techniques and skills to solve problems that can be set up as mathematical models. Students enrolled in Computer Mathematics are assumed to have studied the concepts and skills in Algebra I and beginning geometry. Students who successfully complete the standards for this course may earn credit toward meeting the mathematics graduation requirement. It is recognized that many students will gain computer skills in other mathematics courses or in a separate curriculum outside of mathematics and prior to high school. In such cases, the standards indicated by an asterisk (*) should be included in the student's course of study and treated as a review.

Even though computer ideas should be introduced in the context of mathematical concepts, problem solving per se should be developed in the most general sense, making the techniques applicable by students in many other environments. Strategies include defining the problem; developing, refining, and implementing a plan; and testing and revising the solution. Programming, ranging from simple programs involving only a few lines to complex programs involving subprograms, should permeate the entire course and may include programming a graphing calculator or scripting a problem solution in a database or spreadsheet. Programming concepts, problem-solving strategies, and mathematical applications should be integrated throughout the course.

These standards identify fundamental principles and concepts in the field of computer science that will be used within the context of mathematical problem solving in a variety of applications. As students develop and refine skills in logic, organization, and precise expression, they will apply those skills to enhance learning in all disciplines.

COM.1   The student will apply programming techniques and skills to solve practical real-world problems in mathematics arising from consumer, business, and other applications in mathematics. Problems will include opportunities for students to analyze data in charts, graphs, and tables and to use their knowledge of equations, formulas, and functions to solve these problems.

*COM.2   The student will design, write, test, debug, and document a program. Programming documentation will include preconditions and postconditions of program segments, input/output specifications, the step-by-step plan, the test data, a sample run, and the program listing with appropriately placed comments.

*COM.3   The student will write program specifications that define the constraints of a given problem. These specifications will include descriptions of preconditions, postconditions, the desired output, analysis of the available input, and an indication as to whether or not the problem is solvable under the given conditions.

*COM.4   The student will design a step-by-step plan (algorithm) to solve a given problem. The plan will be in the form of a program flowchart, pseudo code, hierarchy chart, and/or data-flow diagram.

*COM.5   The student will divide a given problem into manageable sections (modules) by task and implement the solution. The modules will include an appropriate user-defined function, subroutines, and procedures. Enrichment topics might include user-defined libraries (units) and object-oriented programming.

*COM.6   The student will design and implement the input phase of a program, which will include designing screen layout and getting information into the program by way of user interaction, data statements, and/or file input. The input phase will also include methods of filtering out invalid data (error trapping).

*COM.7   The student will design and implement the output phase of a computer program, which will include designing output layout, accessing a variety of output devices, using output statements, and labeling results.

COM.8 The student will design and implement computer graphics, which will include topics appropriate for the available programming environment as well as student background. Students will use graphics as an end in itself, as an enhancement to other output, and as a vehicle for reinforcing programming techniques.

COM.9 The student will define simple variable data types that include integer, real (fixed and scientific notation), character, string, and Boolean.

COM.10 The student will use appropriate variable data types, including integer, real (fixed and scientific notation), character, string, and Boolean. This will also include variables representing structured data types.

COM.11 The student will describe the way the computer stores, accesses, and processes variables, including the following topics: the use of variables versus constants, variables' addresses, pointers, parameter passing, scope of variables, and local versus global variables.

*COM.12 The student will translate a mathematical expression into a computer statement, which involves writing assignment statements and using the order of operations.

COM.13 The student will select and implement built-in (library) functions in processing data.

COM.14 The student will implement conditional statements that include "if/then" statements, "if/then/else" statements, case statements, and Boolean logic.

COM.15 The student will implement loops, including iterative loops. Other topics will include single entry point, single exit point, preconditions, and postconditions.

COM.16 The student will select and implement appropriate data structures, including arrays (one-dimensional and/or multidimensional), files, and records. Implementation will include creating the data structure, putting information into the structure, and retrieving information from the structure.

*COM.17 The student will implement pre-existing algorithms, including sort routines, search routines, and simple animation routines.

COM.18 The student will test a program, using an appropriate set of data. The set of test data should be appropriate and complete for the type of program being tested.

COM.19 The student will debug a program, using appropriate techniques (e.g., appropriately placed controlled breaks, the printing of intermediate results, other debugging tools available in the programming environment), and identify the difference between syntax errors and logic errors.

COM.20 The student will design, write, test, debug, and document a complete structured program that requires the synthesis of many of the concepts contained in previous standards.

Probability and Statistics

The following standards outline the content of a one-year course in Probability and Statistics. If a one-semester course is desired, the standards with an asterisk (*) would apply. Students enrolled in this course are assumed to have mastered the concepts identified in the Standards of Learning for Algebra II. The purpose of the course is to present basic concepts and techniques for collecting and analyzing data, drawing conclusions, and making predictions.

A graphing calculator is essential for every student taking the Probability and Statistics course and is required for the Advanced Placement Statistics Examination. The calculator may not fully substitute for a computer, however. In the absence of a computer for student use, teachers may provide students with examples of computer output generated by a statistical software package.

*PS.1   The student will analyze graphical displays of univariate data, including dotplots, stemplots, and histograms, to identify and describe patterns and departures from patterns, using central tendency, spread, clusters, gaps, and outliers. Appropriate technology will be used to create graphical displays.

*PS.2   The student will analyze numerical characteristics of univariate data sets to describe patterns and departures from patterns, using mean, median, mode, variance, standard deviation, interquartile range, range, and outliers.

*PS.3   The student will compare distributions of two or more univariate data sets, analyzing center and spread (within group and between group variations), clusters and gaps, shapes, outliers, or other unusual features.

*PS.4   The student will analyze scatterplots to identify and describe the relationship between two variables, using shape; strength of relationship; clusters; positive, negative, or no association; outliers; and influential points.

PS.5   The student will find and interpret linear correlation, use the method of least squares regression to model the linear relationship between two variables, and use the residual plots to assess linearity.

PS.6   The student will make logarithmic and power transformations to achieve linearity.

PS.7   The student, using two-way tables, will analyze categorical data to describe patterns and departure from patterns and to find marginal frequency and relative frequencies, including conditional frequencies.

*PS.8   The student will describe the methods of data collection in a census, sample survey, experiment, and observational study and identify an appropriate method of solution for a given problem setting.

*PS.9   The student will plan and conduct a survey. The plan will address sampling techniques (e.g., simple random, stratified) and methods to reduce bias.

PS.10   The student will plan and conduct an experiment. The plan will address control, randomization, and measurement of experimental error.

*PS.11   The student will identify and describe two or more events as complementary, dependent, independent, and/or mutually exclusive.

*PS.12   The student will find probabilities (relative frequency and theoretical), including conditional probabilities for events that are either dependent or independent, by applying the Law of Large Numbers concept, the addition rule, and the multiplication rule.

*PS.13   The student will develop, interpret, and apply the binomial probability distribution for discrete random variables, including computing the mean and standard deviation for the binomial variable.

PS.14   The student will simulate probability distributions, including binomial and geometric.

PS.15   The student will identify random variables as independent or dependent and find the mean and standard deviations for sums and differences of independent random variables.

*PS.16   The student will identify properties of a normal distribution and apply the normal distribution to determine probabilities, using a table or graphing calculator.

*PS.17   The student, given data from a large sample, will find and interpret point estimates and confidence intervals for parameters. The parameters will include proportion and mean, difference between two proportions, and difference between two means (independent and paired).

PS.18   The student will apply and interpret the logic of a hypothesis-testing procedure. Tests will include large sample tests for proportion, mean, difference between two proportions, and difference between two means (independent and paired) and Chi-squared tests for goodness of fit, homogeneity of proportions, and independence.

PS.19   The student will identify the meaning of sampling distribution with reference to random variable, sampling statistic, and parameter and explain the Central Limit Theorem. This will include sampling distribution of a sample proportion, a sample mean, a difference between two sample proportions, and a difference between two sample means.

PS.20   The student will identify properties of a t-distribution and apply t-distributions to single-sample and two-sample (independent and matched pairs) t-procedures, using tables or graphing calculators.

Discrete Mathematics

The following standards outline the content of a one-year course in Discrete Mathematics. If a one-semester course is desired, the standards with an asterisk (*) would apply. Students enrolled in Discrete Mathematics are assumed to have mastered the concepts outlined in the Standards of Learning for Algebra II.

Discrete mathematics may be described as the study of mathematical properties of sets and systems that have a countable (discrete) number of elements. With the advent of modern technology, discrete (discontinuous) models have become as important as continuous models. In this course, the main focus is problem solving in a discrete setting. Techniques that are not considered in the current traditional courses of algebra, geometry, and calculus will be utilized. As students solve problems, they will analyze and determine whether or not a solution exists (existence problems), investigate how many solutions exist (counting problems), and focus on finding the best solution (optimization problems). Connections will be made to other disciplines. The importance of discrete mathematics has been influenced by computers. Modern technology (graphing calculators and/or computers) will be an integral component of this course.

*DM.1   The student will model problems, using vertex-edge graphs. The concepts of valence, connectedness, paths, planarity, and directed graphs will be investigated. Adjacency matrices and matrix operations will be used to solve problems (e.g., food chains, number of paths).

*DM.2   The student will solve problems through investigation and application of circuits, cycles, Euler Paths, Euler Circuits, Hamilton Paths, and Hamilton Circuits. Optimal solutions will be sought using existing algorithms and student-created algorithms.

*DM.3   The student will apply graphs to conflict-resolution problems, such as map coloring, scheduling, matching, and optimization. Graph coloring and chromatic number will be used.

*DM.4   The student will apply algorithms, such as Kruskal's, Prim's, or Dijkstra's, relating to trees, networks, and paths. Appropriate technology will be used to determine the number of possible solutions and generate solutions when a feasible number exists.

*DM.5   The student will use algorithms to schedule tasks in order to determine a minimum project time. The algorithms will include critical path analysis, the list-processing algorithm, and student-created algorithms.

*DM.6   The student will solve linear programming problems. Appropriate technology will be used to facilitate the use of matrices, graphing techniques, and the Simplex method of determining solutions.

DM.7   The student will analyze and describe the issue of fair division (e.g., cake cutting, estate division). Algorithms for continuous and discrete cases will be applied.

DM.8   The student will investigate and describe weighted voting and the results of various election methods. These may include approval and preference voting as well as plurality, majority, run-off, sequential run-off, Borda count, and Condorcet winners.

DM.9   The student will identify apportionment inconsistencies that apply to issues such as salary caps in sports and allocation of representatives to Congress. Historical and current methods will be compared.

DM.10   The student will use the recursive process and difference equations with the aid of appropriate technology to generate
a)   compound interest;
b)   sequences and series;
c)   fractals;
d)   population growth models; and
e)   the Fibonacci sequence.

DM.11   The student will describe and apply sorting algorithms and coding algorithms used in sorting, processing, and communicating information. These will include
  a) bubble sort, merge sort, and network sort; and
  b) ISBN, UPC, zip, and banking codes.

DM.12   The student will select, justify, and apply an appropriate technique to solve a logic problem. Techniques will include Venn diagrams, truth tables, and matrices.

DM.13   The student will apply the formulas of combinatorics in the areas of
  a) the Fundamental (Basic) Counting Principle;
  b) knapsack and bin-packing problems;
  c) permutations and combinations; and
  d) the pigeonhole principle.

Mathematical Analysis

The standards below outline the content for a one-year course in Mathematical Analysis. Students enrolled in Mathematical Analysis are assumed to have mastered Algebra II concepts and have some exposure to trigonometry. Mathematical Analysis develops students' understanding of algebraic and transcendental functions, parametric and polar equations, sequences and series, and vectors. The content of this course serves as appropriate preparation for a calculus course.

Graphing calculators, computers, and other appropriate technology tools will be used to assist in teaching and learning. Graphing utilities enhance the understanding of realistic applications through modeling and aid in the investigation of functions and their inverses. They also provide a powerful tool for solving and verifying solutions to equations and inequalities.

MA.1   The student will investigate and identify the characteristics of polynomial and rational functions and use these to sketch the graphs of the functions. This will include determining zeros, upper and lower bounds, *y*-intercepts, symmetry, asymptotes, intervals for which the function is increasing or decreasing, and maximum or minimum points. Graphing utilities will be used to investigate and verify these characteristics.

MA.2   The student will apply compositions of functions and inverses of functions to real-world situations. Analytical methods and graphing utilities will be used to investigate and verify the domain and range of resulting functions.

MA.3   The student will investigate and describe the continuity of functions, using graphs and algebraic methods.

MA.4   The student will expand binomials having positive integral exponents through the use of the Binomial Theorem, the formula for combinations, and Pascal's Triangle.

MA.5   The student will find the sum (sigma notation included) of finite and infinite convergent series, which will lead to an intuitive approach to a limit.

MA.6   The student will use mathematical induction to prove formulas and mathematical statements.

MA.7   The student will find the limit of an algebraic function, if it exists, as the variable approaches either a finite number or infinity. A graphing utility will be used to verify intuitive reasoning, algebraic methods, and numerical substitution.

MA.8   The student will investigate and identify the characteristics of conic section equations in (*h, k*) and standard forms. Transformations in the coordinate plane will be used to graph conic sections.

MA.9   The student will investigate and identify the characteristics of exponential and logarithmic functions in order to graph these functions and solve equations and real-world problems. This will include the role of e, natural and common logarithms, laws of exponents and logarithms, and the solution of logarithmic and exponential equations.

MA.10   The student will investigate and identify the characteristics of the graphs of polar equations, using graphing utilities. This will include classification of polar equations, the effects of changes in the parameters in polar equations, conversion of complex numbers from rectangular form to polar form and vice versa, and the intersection of the graphs of polar equations.

MA.11   The student will perform operations with vectors in the coordinate plane and solve real-world problems, using vectors. This will include the following topics: operations of addition, subtraction, scalar multiplication, and inner (dot) product; norm of a vector; unit vector; graphing; properties; simple proofs; complex numbers (as vectors); and perpendicular components.

MA.12   The student will use parametric equations to model and solve application problems.

MA.13 The student will identify, create, and solve real-world problems involving triangles. Techniques will include using the trigonometric functions, the Pythagorean Theorem, the Law of Sines, and the Law of Cosines.

MA.14 The student will use matrices to organize data and will add and subtract matrices, multiply matrices, multiply matrices by a scalar, and use matrices to solve systems of equations.