MATH 231, Chapter 2

Calculus with Functions

James Madison University

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- $a(t) =$ the *acceleration* of the object at time t, i.e. the rate at which the velocity is changing. Typical units are ft/sec per sec (ft/sec^2), miles/hr per hour (mi/hr^2), km/min per minute (km/min^2), etc.

Imagine the following: A bicycle is initially not moving, but beginning to move at time $t = 0$ (so that velocity $v(0) = 0$ but $v(t) > 0$ when $t > 0$). Also let $s(0) = 0$ (recalling that we can define the "coordinates" for our starting position anyway we want).

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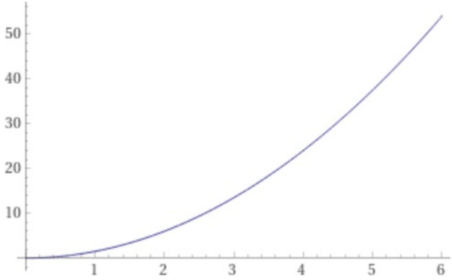

$$s(t) = \frac{3}{2}t^2.$$

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What is the position after 3 seconds (what is $s(3)$)?

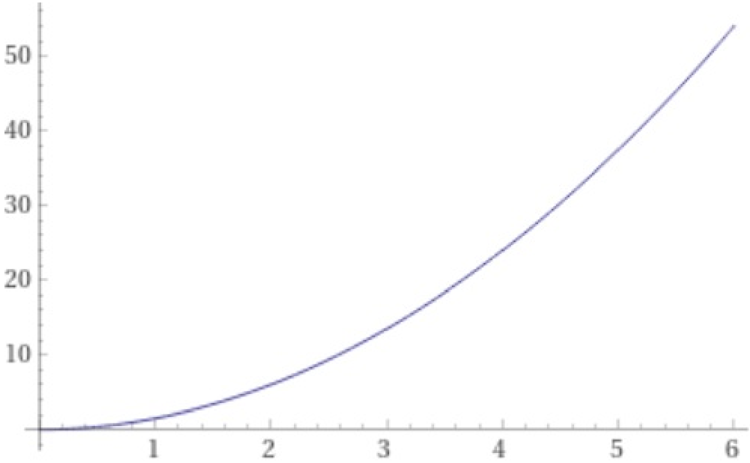
What is $s(5)$?

What is the average velocity of the bicycle between $t = 3$ and $t = 5$?

In other words, starting at $t = 3$, the average velocity over the next 2 seconds is ...?

Starting at $t = 3$ what is the average velocity ...

- ... over the next 1 second?
- ... 0.5 second?
- ... 0.1 second?
- ... 0.01 second?
- ... over the next h seconds? (Where h is an arbitrary but presumably small number.)


50
40
30
20
10
1
2
3
4
5
6

We should be able to guess the exact value for the velocity at three seconds.

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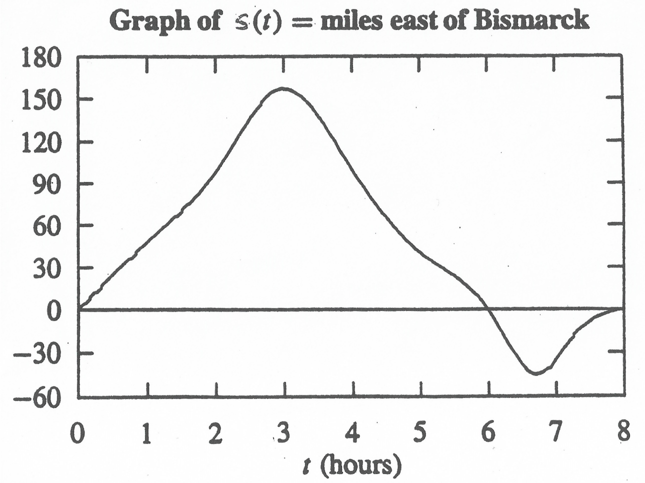
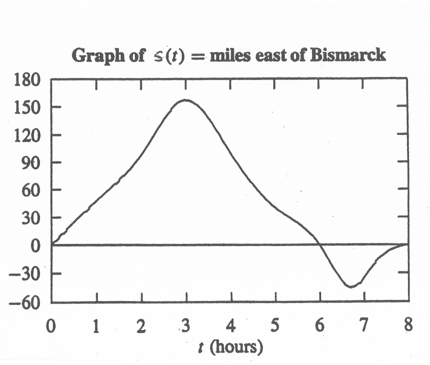
Does it seem reasonable to say the following:
The average velocity between time 3 and time $3+h$ is ...

$$\begin{aligned}
\frac{s(3+h)-s(3)}{h} &= \frac{\frac{3}{2}(3+h)^2-13.5}{h} \\
&= \frac{\frac{3}{2}(9+6h+h^2)-13.5}{h} \\
&= \frac{9h+\frac{3}{2}h^2}{h} \\
&= 9+\frac{3}{2}h
\end{aligned}$$

Another example? Below is the graph of “position” for a little 8 hour automobile trek.

Another example? Below is the graph of "position" for a little 8 hour automobile trek. Think about how to describe the trip. What we really want is a graph of velocity (obviously "approximate"). Bismarck ...North Dakota.




Graph of s(t) = miles east of Bismarck
180
150
120
90
60
30
0
−30
−60
0
1
2
3
4
5
6
7
8
t (hours)

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$$v(3) = \lim_{h \to 0} \frac{s(3+h) - s(3)}{h}.$$

If t is any particular time (t could be 2 or 3 or 1.67 or whatever) then the velocity at time t is

$$v(t) = \lim_{h \to 0} \frac{s(t+h) - s(t)}{h}.$$

The rate of change of s (or any function, not necessarily representing *position*) is also called the *derivative* of s and denoted s'. So we could write

$$s'(t) = \lim_{h \to 0} \frac{s(t+h) - s(t)}{h}.$$

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There is nothing special about position functions. If f is *any* function, then the derivative f' (or rate of change) of f is given by

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Note that there is no guarantee that the limit above actually exists. Sometimes limits exists, sometimes not. When and where *this* limit does exist (and thus the derivative exists) we say that the function is *differentiable*.

The Derivative of a Function at a Point

The ***derivative at*** $x = c$ of a function f is the number

$$f'(c) = \lim_{h \to 0} \frac{f(c+h) - f(c)}{h}, \quad \text{or equivalently,} \quad f'(c) = \lim_{z \to c} \frac{f(z) - f(c)}{z - c},$$

provided that this limit exists.

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Differentiability at a Point

A function f is ***differentiable at*** $x = c$ if $\lim_{h \to 0} \frac{f(c+h) - f(c)}{h}$ exists.

One-sided Differentiability at a Point

The ***left derivative*** and ***right derivative*** of a function f at a point $x = c$ are, respectively, equal to the following, if they exist:

$$f'_-(c) = \lim_{h \to 0^-} \frac{f(c+h) - f(c)}{h}, \qquad f'_+(c) = \lim_{h \to 0^+} \frac{f(c+h) - f(c)}{h}.$$

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Equation of the Tangent Line to a Function at a Point

The ***tangent line*** to the graph of a function f at a point $x = c$ is defined to be the line passing through $(c, f(c))$ with slope $f'(c)$, provided that the derivative $f'(c)$ exists. This line has equation

$$y = f(c) + f'(c)(x - c).$$

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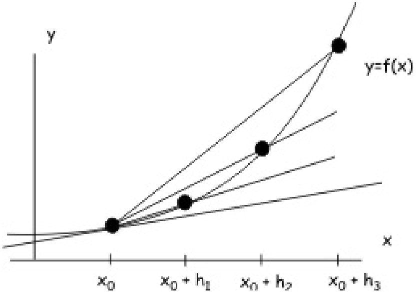
Local Linearity

If f has a well-defined derivative $f'(c)$ at the point $x = c$, then, for values of x near c, the function $f(x)$ can be approximated by the tangent line to f at $x = c$ with the ***linearization of f around*** $x = c$ given by

$$f(x) \approx f(c) + f'(c)(x - c).$$

The derivative *graphically*:



The **average** rate of change between point x_0 and point $x_0 + h$ is the slope of the line joining two points $(x_0, f(x_0))$ and $(x_0 + h, f(x_0 + h))$. This is called a *secant* line.

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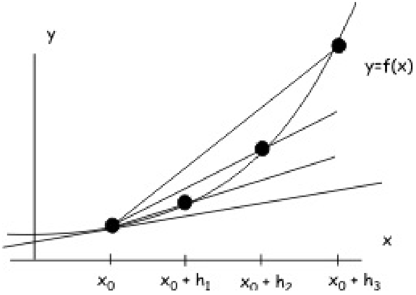

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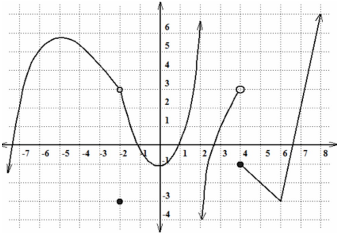
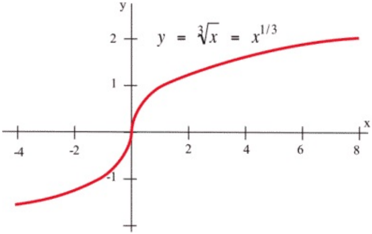
The actual (instantaneous) rate of change at a point is the slope of the line *tangent* to the curve at that point. The slope of the tangent line is given by f', the limit of the slopes of the secant lines:

$$f'(x_0) = \lim_{h \to 0} \frac{f(x_0 + h) - f(x_0)}{h}.$$

More on this later, especially how this fits with the algebra and formulas, but we should consider what it means if a function does *not* have a derivative at a point, at least graphically. Here are a couple examples. Where do the functions in the graph fail to have a derivative (where are they not *differentiable*)?





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Uh oh, lots of alternative symbols:
$f'(x), \frac{dy}{dx}, y', y'(x), \frac{df}{dx}, D_x(y), \frac{d}{dx}(f(x)), \frac{d}{dx}(y), \dot{y}, \ldots$

Other variables: If W is a function of z, for example, then derivative is $\frac{dW}{dz}$ or $W'(z)$.

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Try another?

$$g(x) = \frac{1}{x+3}$$

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Derivatives of Constant, Identity, and Linear Functions

For any real numbers k, m, and b,

(a) $\frac{d}{dx}(k) = 0$ **(b)** $\frac{d}{dx}(x) = 1$ **(c)** $\frac{d}{dx}(mx+b) = m$

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\end{aligned}
$$

What about x^4??

$$
\begin{aligned}
(x+h)^0 &= 1 \\
(x+h)^1 &= x+h \\
(x+h)^2 &= x^2+2xh+h^2 \\
(x+h)^3 &= x^3+3x^2h+3xh^2+h^3 \\
(x+h)^4 &= ?
\end{aligned}
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(x+h)^4 &= x^4+4x^3h+6x^2h^2+4xh^3+h^4 \\
(x+h)^5 &= x^5+5x^4h+10x^3h^2+10x^2h^3+5xh^4+h^5
\end{aligned}
$$

$$\cdots$$

$$(x+h)^n = \;??$$

$$(x+h)^n = x^n + nx^{n-1}h + \frac{n(n-1)}{2}x^{n-2}h^2 + \cdots + h^n$$

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So

$$\begin{aligned}
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&= \lim_{h\to 0} \frac{x^n + nx^{n-1}h + \frac{n(n-1)}{2}x^{n-2}h^2 + \cdots + h^n - x^n}{h} \\
&= \lim_{h\to 0} \frac{nx^{n-1}h + \frac{n(n-1)}{2}x^{n-2}h^2 + \cdots + h^n}{h} \\
&= \lim_{h\to 0} nx^{n-1} + \frac{n(n-1)}{2}x^{n-2}h + \cdots + h^{n-1} \\
&= ??
\end{aligned}$$

The Power Rule

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Derivatives of Constant Multiples and Sums of Functions

If f and g are functions and k is a constant, then for all x where the functions involved are differentiable, we have the following differentiation formulas:

Constant Multiple Rule: $(kf)'(x) = kf'(x)$

Sum Rule: $(f+g)'(x) = f'(x) + g'(x)$

Difference Rule: $(f-g)'(x) = f'(x) - g'(x)$

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4. $y = \sqrt[5]{x^7}$ (assumes more than we have explained ...)

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5. $y = \frac{2x^7 - 3x^3}{x^5}$
6. $y = \sqrt[3]{5x} + x^5$

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7. $y = \frac{\sqrt{x} - 3x^3}{x^2}$

Derivatives of Products and Quotients of Functions

If f and g are functions, then for all x such that both f and g are differentiable, we have the following differentiation formulas:

$$\textbf{\textit{Product Rule:}}\quad (fg)'(x) = f'(x)\,g(x) + f(x)\,g'(x)$$

$$\textbf{\textit{Quotient Rule:}}\quad \left(\frac{f}{g}\right)'(x) = \frac{f'(x)\,g(x) - f(x)g'(x)}{(g(x))^2}$$

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The Chain Rule

Suppose $f(u(x))$ is a composition of functions. Then for all values of x at which u is differentiable at x and f is differentiable at $u(x)$, the derivative of f with respect to x is equal to the product of the derivative of f with respect to u and the derivative of u with respect to x.

In Leibniz notation, we write this as

$$\frac{df}{dx} = \frac{df}{du}\,\frac{du}{dx}.$$

In "prime" notation, we write it as

$$(f \circ u)'(x) = f'(u(x))\,u'(x).$$

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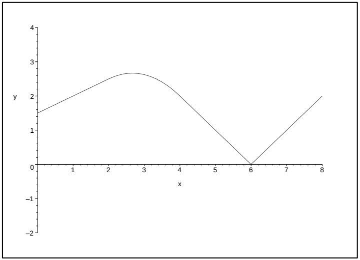
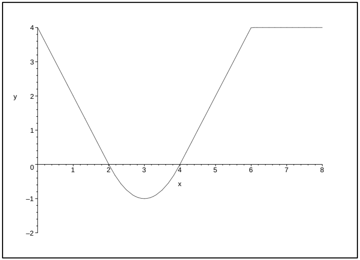
- $r(x) = (x^3 + \sqrt{3x^2 + 2})^8$





Graphs of f (left) and g (right) are above. Let $P(x) = f(x)g(x)$, $Q(x) = f(x)/g(x)$ and $H(x) = g(f(x))$. Evaluate the following:

$f(1) =$ $\qquad$ $g(1) =$ $\qquad$ $f(5) =$ $\qquad$ $g(5) =$

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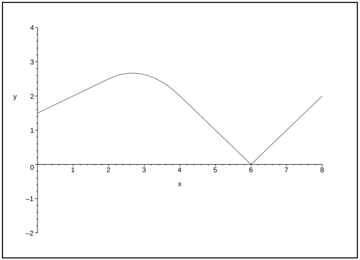

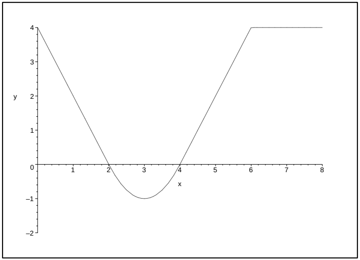

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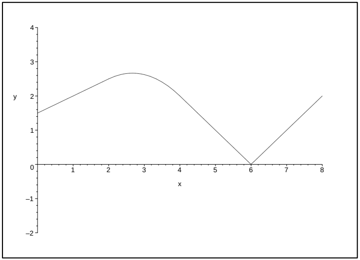

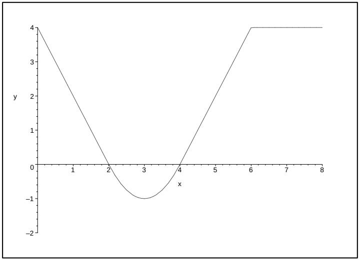

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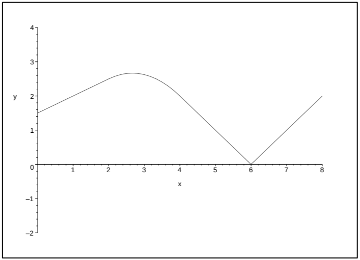

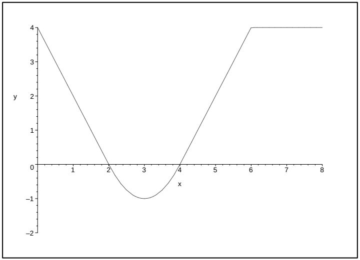

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