MATH 510, Notes 2

Modern Analysis

James Madison University

Definition: Archimedean understanding

The Archimedean understanding of an infinite series is that it is shorthand for the sequence of finite summations.

Definition: Archimedean understanding

The Archimedean understanding of an infinite series is that it is shorthand for the sequence of finite summations. The *value* of an infinite series, if it exists, is that number T such that given any $L < T$ and any $M > T$, all of the finite sums from some point on will be (strictly) contained between L and M.

Definition: Archimedean understanding
The Archimedean understanding of an infinite series is that it is shorthand for the sequence of finite summations. The *value* of an infinite series, if it exists, is that number T such that given any $L < T$ and any $M > T$, all of the finite sums from some point on will be (strictly) contained between L and M.

More precisely, given $L < T < M$ there is a natural number N whose value depends on the choice of L and M such that every partial sum with at least N terms lies inside the open interval (L, M).

That is, when we talk about the infinite series

$$\sum_{k=0}^{\infty} \frac{1}{2^k} = 1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \frac{1}{32} + \cdots$$

we really mean the sequence of numbers

$$\frac{1}{2^0} = 1$$

That is, when we talk about the infinite series

$$\sum_{k=0}^{\infty} \frac{1}{2^k} = 1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \frac{1}{32} + \cdots$$

we really mean the sequence of numbers

$$\begin{aligned} \frac{1}{2^0} &= 1 \\ 1 + \frac{1}{2} &= \frac{3}{2} \end{aligned}$$

That is, when we talk about the infinite series

$$\sum_{k=0}^{\infty} \frac{1}{2^k} = 1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \frac{1}{32} + \cdots$$

we really mean the sequence of numbers

$$\begin{aligned} \frac{1}{2^0} &= 1 \\ 1 + \frac{1}{2} &= \frac{3}{2} \\ 1 + \frac{1}{2} + \frac{1}{4} &= \end{aligned}$$

That is, when we talk about the infinite series

$$\sum_{k=0}^{\infty} \frac{1}{2^k} = 1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \frac{1}{32} + \cdots$$

we really mean the sequence of numbers

$$\begin{aligned} \frac{1}{2^0} &= 1 \\ 1 + \frac{1}{2} &= \frac{3}{2} \\ 1 + \frac{1}{2} + \frac{1}{4} &= \frac{7}{4} \end{aligned}$$

That is, when we talk about the infinite series

$$\sum_{k=0}^{\infty} \frac{1}{2^k} = 1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \frac{1}{32} + \cdots$$

we really mean the sequence of numbers

$$\begin{aligned} \frac{1}{2^0} &= 1 \\ 1 + \frac{1}{2} &= \frac{3}{2} \\ 1 + \frac{1}{2} + \frac{1}{4} &= \frac{7}{4} \\ 1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} &= \end{aligned}$$

That is, when we talk about the infinite series

$$\sum_{k=0}^{\infty} \frac{1}{2^k} = 1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \frac{1}{32} + \cdots$$

we really mean the sequence of numbers

$$\begin{aligned} \frac{1}{2^0} &= 1 \\ 1 + \frac{1}{2} &= \frac{3}{2} \\ 1 + \frac{1}{2} + \frac{1}{4} &= \frac{7}{4} \\ 1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} &= \frac{15}{8} \end{aligned}$$

That is, when we talk about the infinite series

$$\sum_{k=0}^{\infty} \frac{1}{2^k} = 1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \frac{1}{32} + \cdots$$

we really mean the sequence of numbers

$$\begin{aligned}
\frac{1}{2^0} &= 1 \\
1 + \frac{1}{2} &= \frac{3}{2} \\
1 + \frac{1}{2} + \frac{1}{4} &= \frac{7}{4} \\
1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} &= \frac{15}{8} \\
1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} &=
\end{aligned}$$

That is, when we talk about the infinite series

$$\sum_{k=0}^{\infty} \frac{1}{2^k} = 1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \frac{1}{32} + \cdots$$

we really mean the sequence of numbers

$$\begin{aligned}
\frac{1}{2^0} &= 1 \\
1 + \frac{1}{2} &= \frac{3}{2} \\
1 + \frac{1}{2} + \frac{1}{4} &= \frac{7}{4} \\
1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} &= \frac{15}{8} \\
1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} &= \frac{31}{16}
\end{aligned}$$

That is, when we talk about the infinite series

$$\sum_{k=0}^{\infty} \frac{1}{2^k} = 1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \frac{1}{32} + \cdots$$

we really mean the sequence of numbers

$$\begin{aligned}
\frac{1}{2^0} &= 1 \\
1 + \frac{1}{2} &= \frac{3}{2} \\
1 + \frac{1}{2} + \frac{1}{4} &= \frac{7}{4} \\
1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} &= \frac{15}{8} \\
1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} &= \frac{31}{16} \\
\cdots & \\
1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \cdots + \frac{1}{2^n} &=
\end{aligned}$$

That is, when we talk about the infinite series

$$\sum_{k=0}^{\infty} \frac{1}{2^k} = 1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \frac{1}{32} + \cdots$$

we really mean the sequence of numbers

$$\begin{aligned}
\frac{1}{2^0} &= 1 \\
1 + \frac{1}{2} &= \frac{3}{2} \\
1 + \frac{1}{2} + \frac{1}{4} &= \frac{7}{4} \\
1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} &= \frac{15}{8} \\
1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} &= \frac{31}{16} \\
&\cdots \\
1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \cdots + \frac{1}{2^n} &= \frac{2^{n+1} - 1}{2^n}
\end{aligned}$$

So: we want to think of the series $\sum_{k=0}^{\infty} \frac{1}{2^k}$ as merely a way to represent the sequence of numbers $\{\frac{3}{2}, \frac{7}{4}, \frac{15}{8}, \frac{31}{16}, \cdots\}$ along with a target value, if there is one.

So: we want to think of the series $\sum_{k=0}^{\infty} \frac{1}{2^k}$ as merely a way to represent the sequence of numbers $\{\frac{3}{2}, \frac{7}{4}, \frac{15}{8}, \frac{31}{16}, \cdots\}$ along with a target value, if there is one.

You can probably pretty easily figure out the target value for this series, right?

So: we want to think of the series $\sum_{k=0}^{\infty} \frac{1}{2^k}$ as merely a way to represent the sequence of numbers $\{\frac{3}{2}, \frac{7}{4}, \frac{15}{8}, \frac{31}{16}, \cdots\}$ along with a target value, if there is one.

You can probably pretty easily figure out the target value for this series, right?

$$1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \frac{1}{32} + \cdots = \text{?}$$

So: we want to think of the series $\sum_{k=0}^{\infty} \frac{1}{2^k}$ as merely a way to represent the sequence of numbers $\{\frac{3}{2}, \frac{7}{4}, \frac{15}{8}, \frac{31}{16}, \cdots\}$ along with a target value, if there is one.

You can probably pretty easily figure out the target value for this series, right?

$$1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \frac{1}{32} + \cdots = ?$$

Also important to be able to explain why that number *really is* the target value, in the so-called Archimedean understanding.

We write infinite series using the sum notation and sometimes treat them as if they are just ordinary garden variey sums.

We write infinite series using the sum notation and sometimes treat them as if they are just ordinary garden variey sums.

We need to get a handle on when it is OK to do that and when not. Something worthy of further exploration along these lines are the two series

We write infinite series using the sum notation and sometimes treat them as if they are just ordinary garden variey sums.

We need to get a handle on when it is OK to do that and when not. Something worthy of further exploration along these lines are the two series

$$1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - \frac{1}{6} + \frac{1}{7} - \frac{1}{8} + \frac{1}{9} - \cdots$$

We write infinite series using the sum notation and sometimes treat them as if they are just ordinary garden variey sums.

We need to get a handle on when it is OK to do that and when not. Something worthy of further exploration along these lines are the two series

$$1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - \frac{1}{6} + \frac{1}{7} - \frac{1}{8} + \frac{1}{9} - \cdots$$

and

$$1 + \frac{1}{3} - \frac{1}{2} + \frac{1}{5} + \frac{1}{7} - \frac{1}{4} + \frac{1}{9} + \frac{1}{11} - \frac{1}{6} + \cdots$$

For the function g defined by the infinite series

$$g(x) = \sum_{k=1}^{\infty} \frac{(-1)^{k+1}\sin(kx)}{k} = \sin(x) - \frac{\sin(2x)}{2} + \frac{\sin(3x)}{3} - \frac{\sin(4x)}{4} + \cdots$$

For the function g defined by the infinite series

$$g(x) = \sum_{k=1}^{\infty} \frac{(-1)^{k+1}\sin(kx)}{k} = \sin(x) - \frac{\sin(2x)}{2} + \frac{\sin(3x)}{3} - \frac{\sin(4x)}{4} + \cdots$$

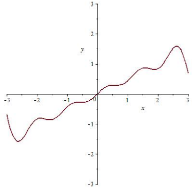
(that is, $g(x)$ is defined to be the target value for the series at any x)

For the function g defined by the infinite series

$$g(x) = \sum_{k=1}^{\infty} \frac{(-1)^{k+1}\sin(kx)}{k} = \sin(x) - \frac{\sin(2x)}{2} + \frac{\sin(3x)}{3} - \frac{\sin(4x)}{4} + \cdots$$

(that is, $g(x)$ is defined to be the target value for the series at any x)

Here are graphs of the partial sums for the first five, ten, and twenty-five terms along with the graph of g itself:

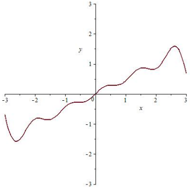


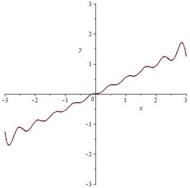
For the function g defined by the infinite series

$$g(x) = \sum_{k=1}^{\infty} \frac{(-1)^{k+1}\sin(kx)}{k} = \sin(x) - \frac{\sin(2x)}{2} + \frac{\sin(3x)}{3} - \frac{\sin(4x)}{4} + \cdots$$

(that is, $g(x)$ is defined to be the target value for the series at any x)

Here are graphs of the partial sums for the first five, ten, and twenty-five terms along with the graph of g itself:



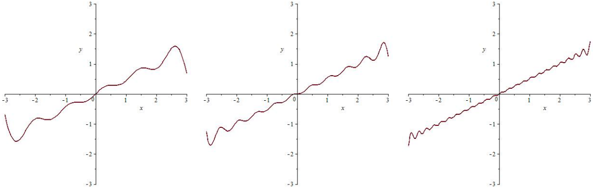


For the function g defined by the infinite series

$$g(x) = \sum_{k=1}^{\infty} \frac{(-1)^{k+1}\sin(kx)}{k} = \sin(x) - \frac{\sin(2x)}{2} + \frac{\sin(3x)}{3} - \frac{\sin(4x)}{4} + \cdots$$

(that is, $g(x)$ is defined to be the target value for the series at any x)

Here are graphs of the partial sums for the first five, ten, and twenty-five terms along with the graph of g itself:

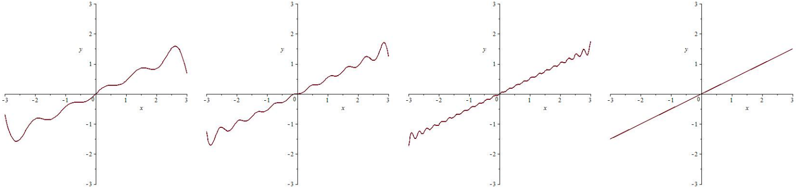


For the function g defined by the infinite series

$$g(x) = \sum_{k=1}^{\infty} \frac{(-1)^{k+1}\sin(kx)}{k} = \sin(x) - \frac{\sin(2x)}{2} + \frac{\sin(3x)}{3} - \frac{\sin(4x)}{4} + \cdots$$

(that is, $g(x)$ is defined to be the target value for the series at any x)

Here are graphs of the partial sums for the first five, ten, and twenty-five terms along with the graph of g itself:

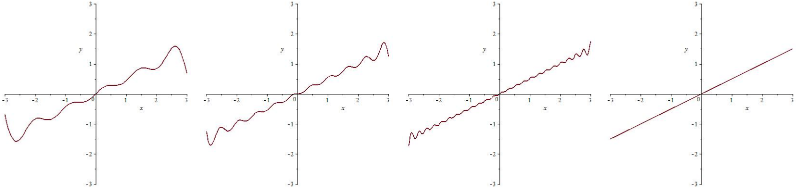


For the function g defined by the infinite series

$$g(x) = \sum_{k=1}^{\infty} \frac{(-1)^{k+1}\sin(kx)}{k} = \sin(x) - \frac{\sin(2x)}{2} + \frac{\sin(3x)}{3} - \frac{\sin(4x)}{4} + \cdots$$

(that is, $g(x)$ is defined to be the target value for the series at any x)

Here are graphs of the partial sums for the first five, ten, and twenty-five terms along with the graph of g itself:



No proof here, but yes - the graph of g is a straight line, at least between -3 and 3.

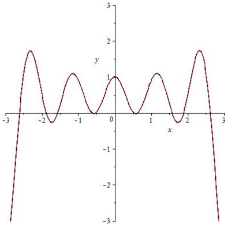
We can easily find the derivative of any of the finite *partial* sums for g. This would eventually give us the series

$$\sum_{k=1}^{\infty}(-1)^{k+1}\cos(kx)$$

We can easily find the derivative of any of the finite *partial* sums for g. This would eventually give us the series

$$\sum_{k=1}^{\infty} (-1)^{k+1} \cos(kx)$$

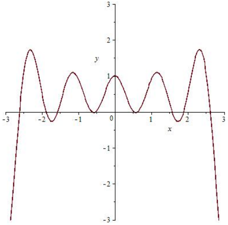
Here are graphs of the partial sums for the first five, ten, and twenty-five terms of the above series (which we are intentionally *not* referring to as $g'(x)$).

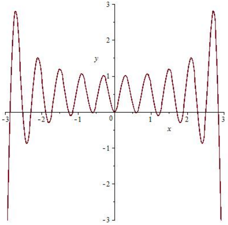


We can easily find the derivative of any of the finite *partial* sums for g. This would eventually give us the series

$$\sum_{k=1}^{\infty}(-1)^{k+1}\cos(kx)$$

Here are graphs of the partial sums for the first five, ten, and twenty-five terms of the above series (which we are intentionally *not* referring to as $g'(x)$).

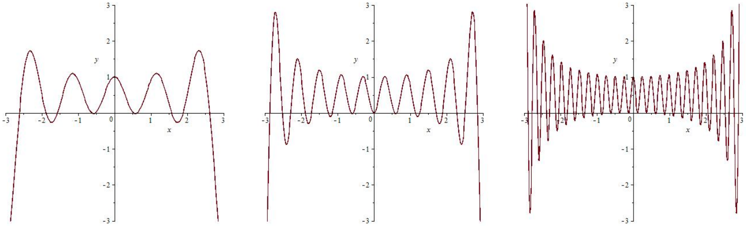




We can easily find the derivative of any of the finite *partial* sums for g. This would eventually give us the series

$$\sum_{k=1}^{\infty}(-1)^{k+1}\cos(kx)$$

Here are graphs of the partial sums for the first five, ten, and twenty-five terms of the above series (which we are intentionally *not* referring to as $g'(x)$).



Why is this a big deal?

Why is this a big deal?

We (like Fourier) want to use infinite series and other sorts of limiting processes to solve problems related to algebra and calculus, especially as they relate to differential equations where many of the applications originate.

Why is this a big deal?

We (like Fourier) want to use infinite series and other sorts of limiting processes to solve problems related to algebra and calculus, especially as they relate to differential equations where many of the applications originate.

Regular addition is commutative, associative, and we have distributive rules with multiplication. Do similar things always work with "infinite sums?"

Also: Some well-known facts from calculus:

Also: Some well-known facts from calculus:

- $\frac{d}{dx}(f(x) + g(x)) = f'(x) + g'(x)$

Also: Some well-known facts from calculus:

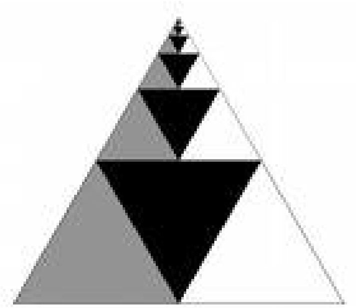
- $\frac{d}{dx}(f(x) + g(x)) = f'(x) + g'(x)$
- $\int_a^b (f + g) = \int_a^b f + \int_a^b g$

Also: Some well-known facts from calculus:

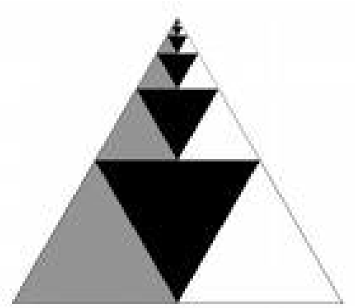
- $\frac{d}{dx}(f(x) + g(x)) = f'(x) + g'(x)$
- $\int_a^b (f + g) = \int_a^b f + \int_a^b g$

These formulas also work for sums of 3 functions (or in fact any finite number of functions). Do they work for "infinite" sums? If not... then what?

But let's look at some situations where things work out a little better:



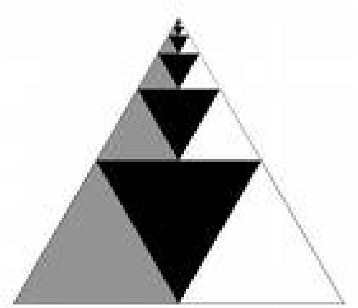
But let's look at some situations where things work out a little better:



Can you use the image to answer the question:

What is the target value of the series $1 + \frac{1}{4} + \frac{1}{16} + \frac{1}{64} + \frac{1}{256} + \cdots$?

But let's look at some situations where things work out a little better:



Can you use the image to answer the question:

What is the target value of the series $1 + \frac{1}{4} + \frac{1}{16} + \frac{1}{64} + \frac{1}{256} + \cdots$?

Or the equivalent $\frac{1}{4} + \frac{1}{16} + \frac{1}{64} + \frac{1}{256} + \cdots$?

Geometric sums, the key identity:

$$1 + x + x^2 + x^3 + \cdots + x^{n-1} = \frac{1}{1-x} - \frac{x^n}{1-x}$$

Geometric sums, the key identity:

$$1 + x + x^2 + x^3 + \cdots + x^{n-1} = \frac{1}{1-x} - \frac{x^n}{1-x}$$

Everyone should know at least a couple of ways to explain why this is true.

Geometric sums, the key identity:

$$1 + x + x^2 + x^3 + \cdots + x^{n-1} = \frac{1}{1-x} - \frac{x^n}{1-x}$$

Everyone should know at least a couple of ways to explain why this is true. (That is, "everyone" in our profession!) Can you?

Let $S = 1 + x + x^2 + x^3 + \cdots + x^{n-1}$. Then

$$xS = x + x^2 + x^3 + \cdots + x^n$$

Let $S = 1 + x + x^2 + x^3 + \cdots + x^{n-1}$. Then

$$\begin{aligned} xS &= x + x^2 + x^3 + \cdots + x^n \\ S - xS &= 1 + x + x^2 + \cdots + x^{n-1} - (x + x^2 + x^3 + \cdots + x^n) \end{aligned}$$

Let $S = 1 + x + x^2 + x^3 + \cdots + x^{n-1}$. Then

$$\begin{aligned} xS &= x + x^2 + x^3 + \cdots + x^n \\ S - xS &= 1 + x + x^2 + \cdots + x^{n-1} - (x + x^2 + x^3 + \cdots + x^n) \\ S(1-x) &= 1 - x^n \end{aligned}$$

Let $S = 1 + x + x^2 + x^3 + \cdots + x^{n-1}$. Then

$$\begin{aligned} xS &= x + x^2 + x^3 + \cdots + x^n \\ S - xS &= 1 + x + x^2 + \cdots + x^{n-1} - (x + x^2 + x^3 + \cdots + x^n) \\ S(1-x) &= 1 - x^n \\ S &= \frac{1}{1-x} - \frac{x^n}{1-x} \end{aligned}$$

Or... to prove

$$1 + x + x^2 + x^3 + \cdots + x^{n-1} = \frac{1}{1-x} - \frac{x^n}{1-x}$$

Or... to prove

$$1 + x + x^2 + x^3 + \cdots + x^{n-1} = \frac{1}{1-x} - \frac{x^n}{1-x}$$

$n = 1$?

$$1 = \frac{1}{1-x} - \frac{x^1}{1-x}$$

Or... to prove

$$1 + x + x^2 + x^3 + \cdots + x^{n-1} = \frac{1}{1-x} - \frac{x^n}{1-x}$$

$n = 1$?

$$1 = \frac{1}{1-x} - \frac{x^1}{1-x}$$

Yes

Or... to prove

$$1 + x + x^2 + x^3 + \cdots + x^{n-1} = \frac{1}{1-x} - \frac{x^n}{1-x}$$

$n = 1$?

$$1 = \frac{1}{1-x} - \frac{x^1}{1-x}$$

Yes

Assume true for $n = K$.

Or... to prove

$$1 + x + x^2 + x^3 + \cdots + x^{n-1} = \frac{1}{1-x} - \frac{x^n}{1-x}$$

$n = 1$?

$$1 = \frac{1}{1-x} - \frac{x^1}{1-x}$$

Yes

Assume true for $n = K$. Then

$$1 + x + x^2 + x^3 + \cdots + x^{K-1} = \frac{1}{1-x} - \frac{x^K}{1-x}$$

Or... to prove

$$1 + x + x^2 + x^3 + \cdots + x^{n-1} = \frac{1}{1-x} - \frac{x^n}{1-x}$$

$n = 1$?

$$1 = \frac{1}{1-x} - \frac{x^1}{1-x}$$

Yes

Assume true for $n = K$. Then

$$1 + x + x^2 + x^3 + \cdots + x^{K-1} = \frac{1}{1-x} - \frac{x^K}{1-x}$$

$$1 + x + x^2 + x^3 + \cdots + x^{K-1} = \frac{1}{1-x} - \frac{x^K}{1-x}$$

$$1 + x + x^2 + x^3 + \cdots + x^{K-1} = \frac{1}{1-x} - \frac{x^K}{1-x}$$

Add x^K to both sides:

$$1 + x + x^2 + x^3 + \cdots + x^{K-1} + x^K = \frac{1}{1-x} - \frac{x^K}{1-x} + x^K$$

$$1 + x + x^2 + x^3 + \cdots + x^{K-1} = \frac{1}{1-x} - \frac{x^K}{1-x}$$

Add x^K to both sides:

$$1 + x + x^2 + x^3 + \cdots + x^{K-1} + x^K = \frac{1}{1-x} - \frac{x^K}{1-x} + x^K$$

$$1 + x + x^2 + x^3 + \cdots + x^{K-1} + x^K = \frac{1}{1-x} - \frac{x^K - x^K + x^{K+1}}{1-x}$$

$$1 + x + x^2 + x^3 + \cdots + x^{K-1} = \frac{1}{1-x} - \frac{x^K}{1-x}$$

Add x^K to both sides:

$$1 + x + x^2 + x^3 + \cdots + x^{K-1} + x^K = \frac{1}{1-x} - \frac{x^K}{1-x} + x^K$$

$$1 + x + x^2 + x^3 + \cdots + x^{K-1} + x^K = \frac{1}{1-x} - \frac{x^K - x^K + x^{K+1}}{1-x}$$

$$= \frac{1}{1-x} - \frac{x^{K+1}}{1-x}$$

QED?

For the series $1 + x + x^2 + x^3 + \cdots$

Using $1 + x + x^2 + x^3 + \cdots + x^{n-1} = \frac{1}{1-x} - \frac{x^n}{1-x}$

For the series $1 + x + x^2 + x^3 + \cdots$

Using $1 + x + x^2 + x^3 + \cdots + x^{n-1} = \frac{1}{1-x} - \frac{x^n}{1-x}$

For which values of x should the series converge?

For the series $1 + x + x^2 + x^3 + \cdots$

Using $1 + x + x^2 + x^3 + \cdots + x^{n-1} = \frac{1}{1-x} - \frac{x^n}{1-x}$

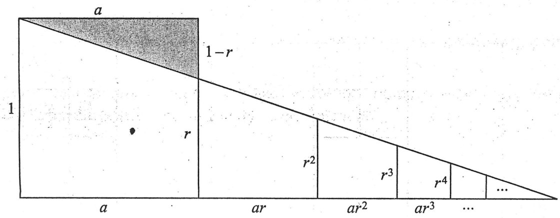
For which values of x should the series converge?

What is target value?

Alternatively, maybe the following is a way to get the series and target value all at once?

Alternatively, maybe the following is a way to get the series and target value all at once?

$$\frac{a + ar + ar^2 + ar^3 + \cdots}{1} = \frac{a}{1-r}.$$



What is the sum:

- $1 + \frac{2}{3} + \frac{4}{9} + \frac{8}{27} + \frac{16}{81} + \cdots$

What is the sum:

- $1 + \frac{2}{3} + \frac{4}{9} + \frac{8}{27} + \frac{16}{81} + \cdots$
- $1 - \frac{2}{3} + \frac{4}{9} - \frac{8}{27} + \frac{16}{81} - \cdots$

What is the sum:

- $1 + \frac{2}{3} + \frac{4}{9} + \frac{8}{27} + \frac{16}{81} + \cdots$
- $1 - \frac{2}{3} + \frac{4}{9} - \frac{8}{27} + \frac{16}{81} - \cdots$
- $\frac{4}{9} + \frac{8}{27} + \frac{16}{81} + \frac{32}{243} + \cdots$

What is the sum:

- $1 + \frac{2}{3} + \frac{4}{9} + \frac{8}{27} + \frac{16}{81} + \cdots$
- $1 - \frac{2}{3} + \frac{4}{9} - \frac{8}{27} + \frac{16}{81} - \cdots$
- $\frac{4}{9} + \frac{8}{27} + \frac{16}{81} + \frac{32}{243} + \cdots$
- $7 + \frac{14}{5} + \frac{28}{25} + \frac{56}{125} + \cdots$

What is the sum:

- $1 + \frac{2}{3} + \frac{4}{9} + \frac{8}{27} + \frac{16}{81} + \cdots$
- $1 - \frac{2}{3} + \frac{4}{9} - \frac{8}{27} + \frac{16}{81} - \cdots$
- $\frac{4}{9} + \frac{8}{27} + \frac{16}{81} + \frac{32}{243} + \cdots$
- $7 + \frac{14}{5} + \frac{28}{25} + \frac{56}{125} + \cdots$
- $1 + \frac{1}{2} - \frac{1}{4} + \frac{1}{8} + \frac{1}{16} - \frac{1}{32} + \frac{1}{64} + \frac{1}{128} - \frac{1}{256} + \cdots$