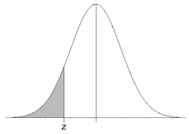
z table

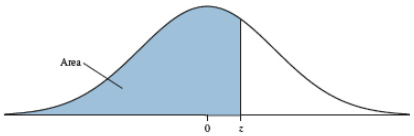
Standard Normal Cumulative Probability Table



Cumulative probabilities for NEGATIVE z-values are shown in the following table:

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
-3.4	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0002
-3.3	0.0005	0.0005	0.0005	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0003
-3.2	0.0007	0.0007	0.0006	0.0006	0.0006	0.0006	0.0006	0.0005	0.0005	0.0005
-3.1	0.0010	0.0009	0.0009	0.0009	0.0008	0.0008	0.0008	0.0008	0.0007	0.0007
-3.0	0.0013	0.0013	0.0013	0.0012	0.0012	0.0011	0.0011	0.0011	0.0010	0.0010
-2.9	0.0019	0.0018	0.0018	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014
-2.8	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019
-2.7	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026
-2.6	0.0047	0.0045	0.0044	0.0043	0.0041	0.0040	0.0039	0.0038	0.0037	0.0036
-2.5	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048
-2.4	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071	0.0069	0.0068	0.0066	0.0064
-2.3	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084
-2.2	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116	0.0113	0.0110
-2.1	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
-2.0	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183
-1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233
-1.8	0.0359	0.0351	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
-1.7	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367
-1.6	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
-1.5	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0559
-1.4	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0721	0.0708	0.0694	0.0681
-1.3	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823
-1.2	0.1151	0.1131	0.1112	0.1093	0.1075	0.1056	0.1038	0.1020	0.1003	0.0985
-1.1	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
-1.0	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379
-0.9	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611
-0.8	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867
-0.7	0.2420	0.2389	0.2358	0.2327	0.2296	0.2266	0.2236	0.2206	0.2177	0.2148
-0.6	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483	0.2451
-0.5	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776
-0.4	0.3446	0.3409	0.3372	0.3336	0.3300	0.3264	0.3228	0.3192	0.3156	0.3121
-0.3	0.3821	0.3783	0.3745	0.3707	0.3669	0.3632	0.3594	0.3557	0.3520	0.3483
-0.2	0.4207	0.4168	0.4129	0.4090	0.4052	0.4013	0.3974	0.3936	0.3897	0.3859
-0.1	0.4602	0.4562	0.4522	0.4483	0.4443	0.4404	0.4364	0.4325	0.4286	0.4247
0.0	0.5000	0.4960	0.4920	0.4880	0.4840	0.4801	0.4761	0.4721	0.4681	0.4641

Table A.2 Cumulative Normal Distribution (continued)



z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998
3.5	.9998	.9998	.9998	.9998	.9998	.9998	.9998	.9998	.9998	.9998
3.6	.9998	.9998	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999
3.7 or more	.9999									

Law of Large Numbers: from chapter 5

Law of large numbers: The sample mean approaches the population mean when the sample size grows.

Density curves

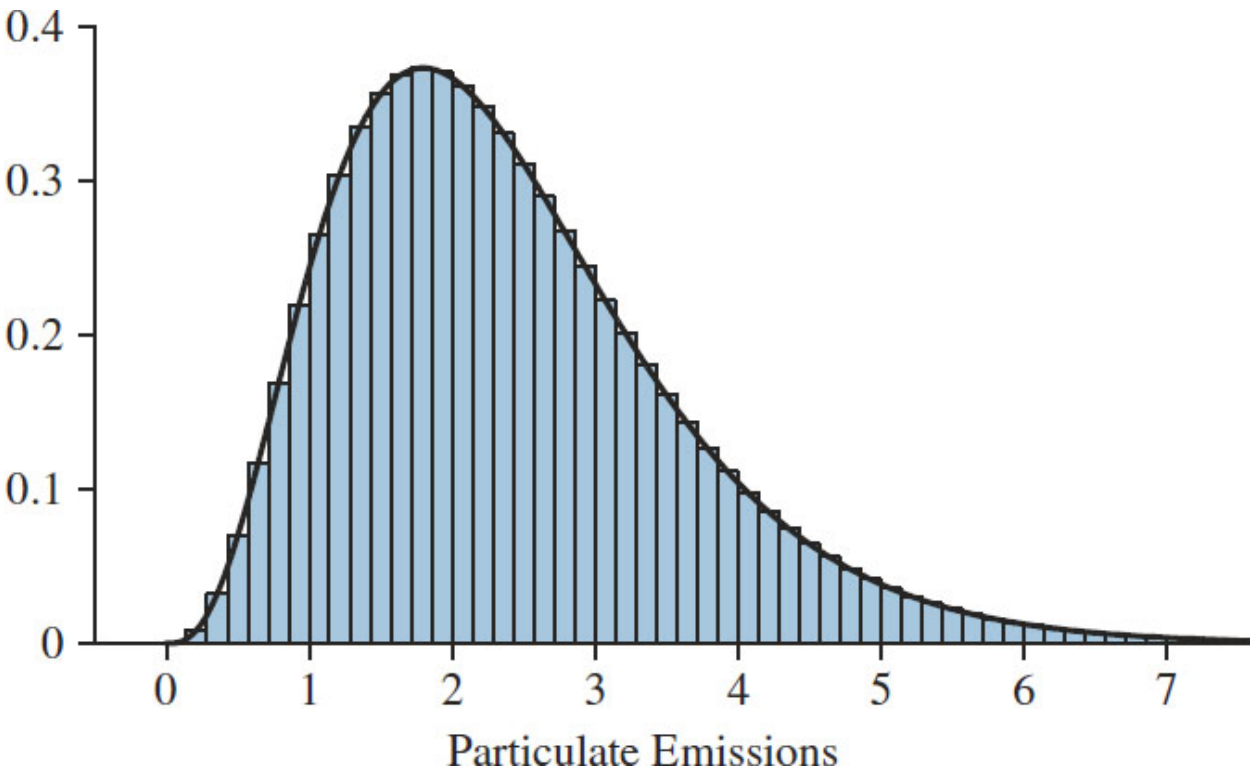
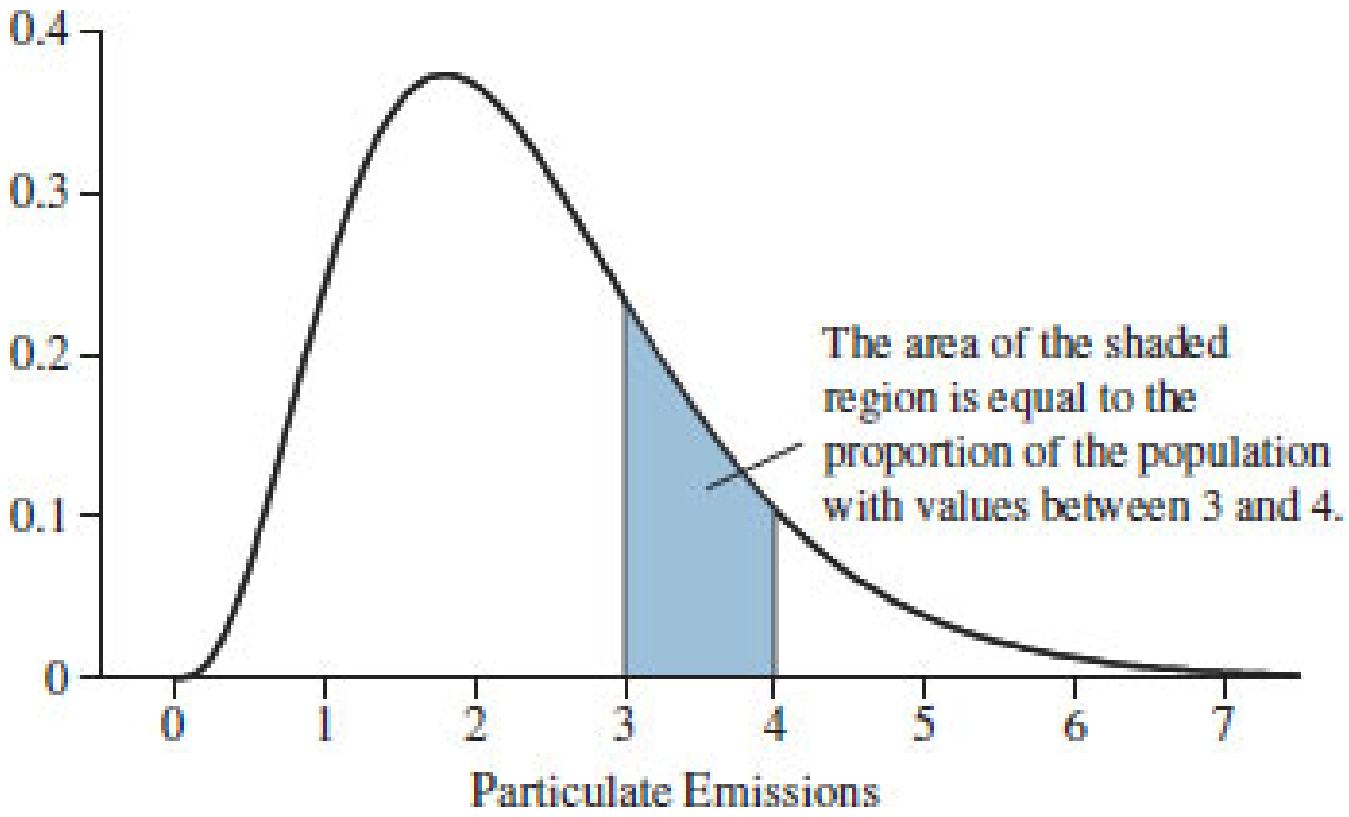


Figure 6.2 p 230.

When a histogram has many narrow rectangles, we can use a smooth curve to approximate the histogram. The curve is called a probability density curve which describes the overall pattern of a distribution. A density curve is always on or above the horizontal axis, and has area exactly 1 underneath it.



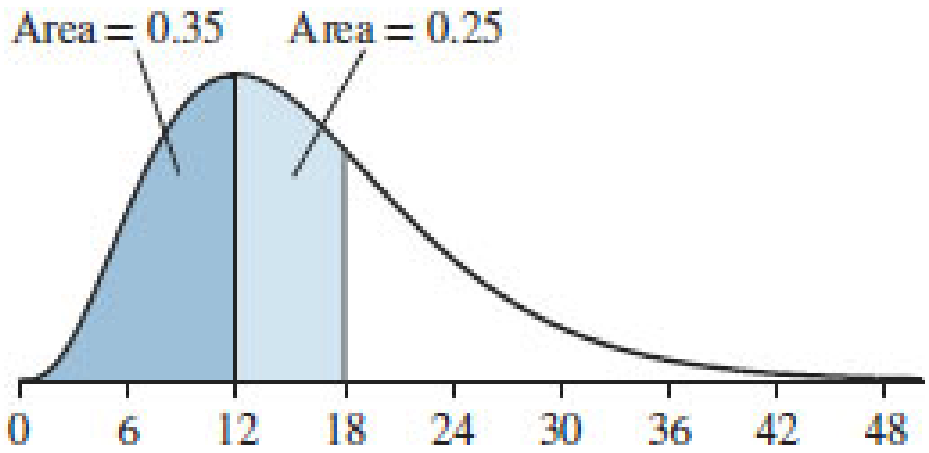
The area under the density curve between two values is the proportion of population values that fall between these two values.

. The following figure is a probability density curve that represents the lifetime, in months, of a certain type of laptop battery.

a. Find the proportion of batteries with lifetimes between 12 and 18 months.

b. Find the proportion of batteries with lifetimes less than 18 months.

c. What is the probability that a randomly chosen battery lasts more than 18 months?

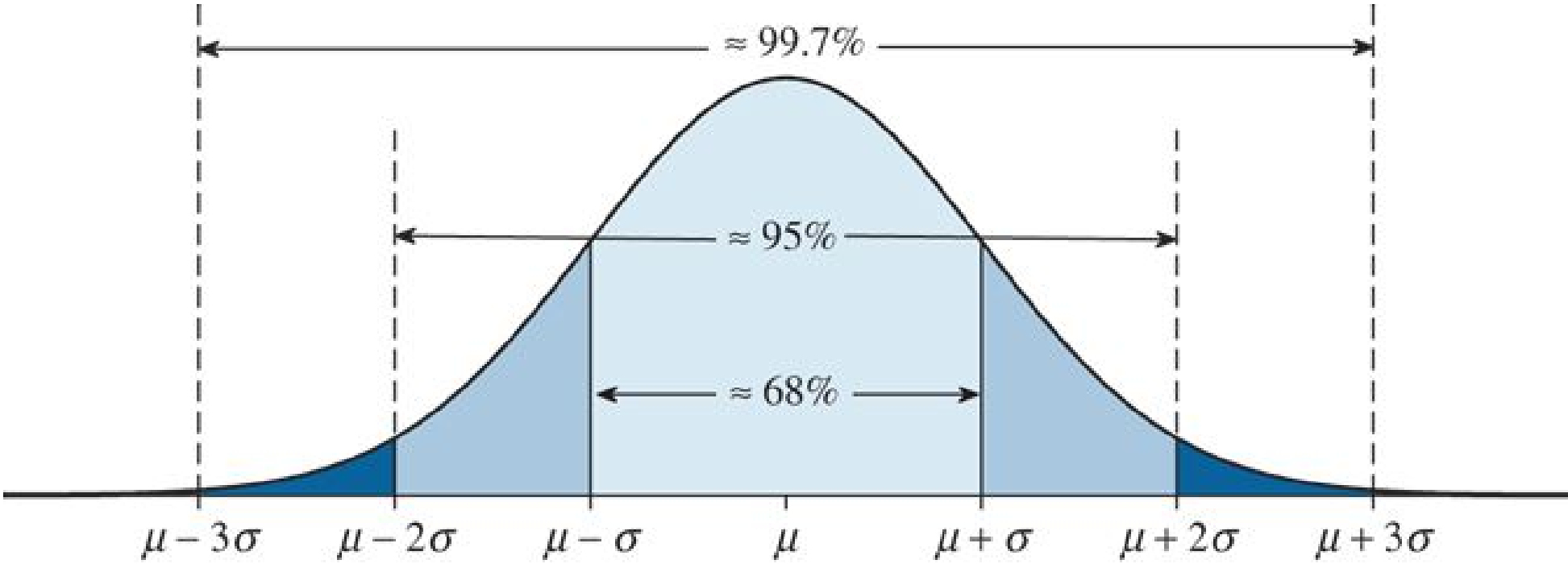


Answer

The answers are 0.25 in a), 0.6 in b) and 0.4 in c).

A **normal distribution** is described by a normal density curve. A normal distribution is completely specified by its mean μ and standard deviation σ.
The Empirical rule:
Approximately 68% of the observations fall within σ of μ.
Approximately 95% of the observations fall within 2σ of μ.
Approximately 99.7% of the observations fall within 3σ of μ.

Figure 6.5.


≈ 99.7%
≈ 95%
≈ 68%
μ − 3σ
μ − 2σ
μ − σ
μ
μ + σ
μ + 2σ
μ + 3σ

example

Adult female heights in North America have approximately a normal distribution with $\mu = 65$ inches and $\sigma = 3.5$ inches.
About 68% of the heights fall between
$[65 - 3.5, 65 + 3.5] = [61.5, 68.5]$ inches.
About 95% of the heights fall between
$[65 - 2 * 3.5, 65 + 2 * 3.5] = [58, 72]$ inches.
About 99.7% of the heights fall between
$[65 - 3 * 3.5, 65 + 3 * 3.5] = [54.5, 75.5]$ inches.

z-score

If x is an observation from a distribution that has mean μ and standard deviation σ, the standardized value of x is
$z = \frac{x-\mu}{\sigma}$.
This standardized value is often called a **z-score**.
A z-score measure how many standard deviations a value is away from the mean.

standard normal distribution

The **standard normal distribution** has mean 0 and standard deviation 1.
If a variable x has any normal distribution $N(\mu, \sigma)$, then $z = \frac{x-\mu}{\sigma}$ has the standard normal distribution.

Standard normal table

Standard Normal Probabilities

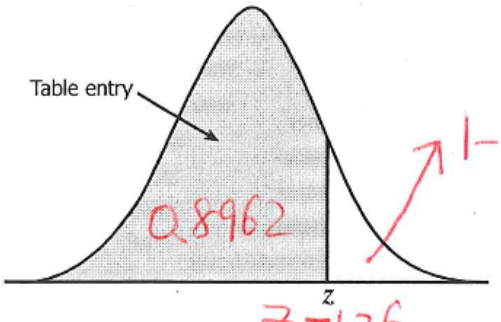
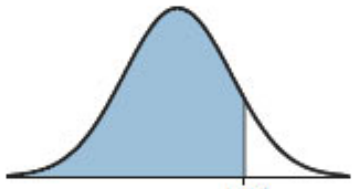


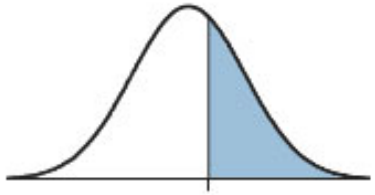
Table entry for z is the area under the standard normal curve to the left of z.

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319

Find area

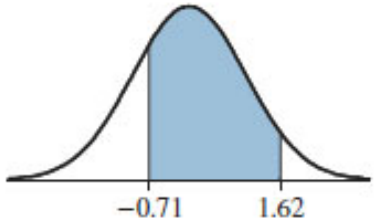
. Find each of the shaded areas under the standard normal curve.





Answer

. Find each of the shaded areas under the standard normal curve.



Answers: 0.8944; 0.3594; 0.7085.

Summary

- Find the area to the left of z: use the area in the table.
- Find the area to the right z: 1 - area to the left of z.
- Find the area between two z scores: find the area to the left of each and use bigger area - smaller area.

More exercise

Find the area to the left of z=-1.96. Answer: 0.0250.
Find the area to the right of z=1.58. Answer: 1-0.9429=0.0571.
Find the area between z=-1.65 and z=1.65. 0.9505-0.0495=0.9010.

Find a z score according to a given area

Standard Normal Probabilities

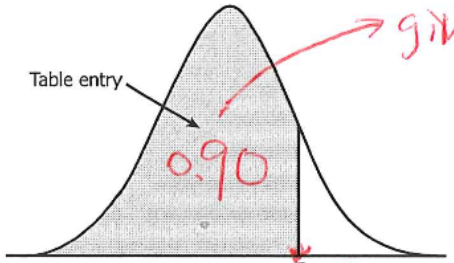


Table entry for z is the area under the standard normal curve to the left of z.

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633

closest to 0.90

Find the z score given an area

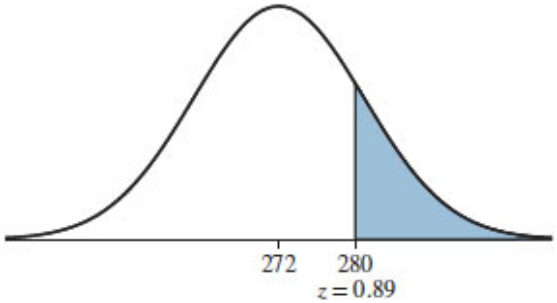
Always use the area to the left of the z score to find the corresponding z score.

Notation z_α : the area to the right of z_α is α.
$z_{0.025} = 1.96, z_{0.01} = 2.33, z_{0.05} = 1.645.$

Applications of the general normal distribution

It is reported the length of pregnancy is approximately normally distributed with $\mu = 272$ and $\sigma = 9$ days. What proportion of pregnancies last longther than 280 days? Between 252 and 298 days?

The z score of 280 is $\frac{280-272}{9} = 0.89$,
$P(x > 280) = P(z > 0.89) = 1 - 0.8133 = 0.1867$.



$P(252 \leq x \leq 298) = P(-2.22 \leq z \leq 2.89) = 0.9981 - 0.0132 = 0.9849$.

exercise

Suppose the test scores follow a normal distribution with $\mu = 82$ and $\sigma = 4$. Find the proportion of test scores that
fall below 88,
fall above 88,
fall below 75,
fall between 75 and 88.

answer: $P(x < 88) = P(z < \frac{88-82}{4}) = P(z < 1.50) = 0.9332$,
$P(x > 88) = 1 - 0.9332 = 0.0668$.
$P(x < 75) = P(z < -1.75) = 0.0401$.
$P(75 < x < 88) = 0.9332 - 0.0401 = 0.8931$.

Find a value given a proportion

Find a value given a proportion.
$x = \mu + z * \sigma$.
example 6.10.
IQ scores $\mu = 100, \sigma = 15$. Find the 90th percentile of the test scores.
We want find x such that 90% of the scores are below x.
Or the area to the left of x is 0.90.
The corresponding $z = 1.28$ and
$x = \mu + z\sigma = 100 + 1.28 * 15 = 119.2$.

Example

Female heights $\mu = 65$ inches, $\sigma = 3.5$ inches.

1). Find x such that 80% of the heights are below this x.
the area to the left of x is 0.80.
$z = 0.84$, and $x = \mu + z\sigma = 65 + 0.84 * 3.5 = 67.94$ inches.

2). Find x such that 5% of the heights are above x.
the area to the right of x is 0.05.
the area to the left of x is 0.95.
$z = 1.645, x = 65 + 1.64 * 3.5 = 70.76$.

exercise

Final exam scores have approximately normal distribution with mean 76 and standard deviation 8. The instructor give a C to scores between 70 and 80.

1). About what proportion of students get a C?

2). Find the upper quartile Q_3 of test scores, i.e., 75% of the test scores are below this value.

1)
$P(70 < x < 80) = P(-0.75 < z < 0.5) = 0.6915 - 0.2266 = 0.4649.$

2) note from p=0.75, we get
$z = 0.67$, and $x = \mu + z\sigma = 76 + 0.67 * 8 = 81.36.$

Exercises

58. The weight of 2-month old male babies is normally distributed with $\mu = 11.5$ pounds and $\sigma = 2.7$ pounds.
a). What proportion of babies weigh more than 13.5 pounds?

59. The diastolic blood pressures of adult women in US are normally distributed with $\mu = 80.5$ and $\sigma = 9.9$.
a). Find the 35th percentile of the blood pressure.

solutions

$P(x > 13.5) = P(z > \frac{13.5-11.5}{2.7}) = P(z > 0.74) = 1 - 0.7704 = 0.2296.$
About 23% of babies weigh more than 13.5 lb.

$P = 0.35, z = -0.39, x = 80.5 - 0.39 * 9.9 = 76.64.$