Sampling distributions

A **parameter** is a number that describes the population.
A **statistic** is a number that can be computed from the sample data. In practice, we often use a statistic to estimate an unknown parameter.
e.g., sample mean $\bar{x}$ is a statistic, a population mean μ is a parameter.

sampling distribution

The **sampling distribution** of a statistic is the distribution of values taken by the statistic in all possible samples of the same size from the same population.

Sampling distribution of $\bar{x}$

Let $\bar{x}$ be the mean of an SRS of size n drawn from a population with mean μ and standard deviation σ. Then the sampling distribution of $\bar{x}$ has mean μ and standard deviation $\sigma/\sqrt{n}$.
Furthermore, if n is large (> 30), the **Central Limit Theorem** says that approximately $\bar{x} \sim N(\mu, \frac{\sigma}{\sqrt{n}})$.

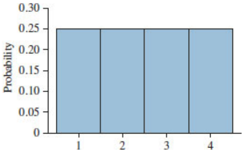


Figure 6.14 Probability histogram for the population

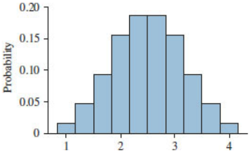


Figure 6.15 Probability histogram for the sampling distribution of $\bar{x}$

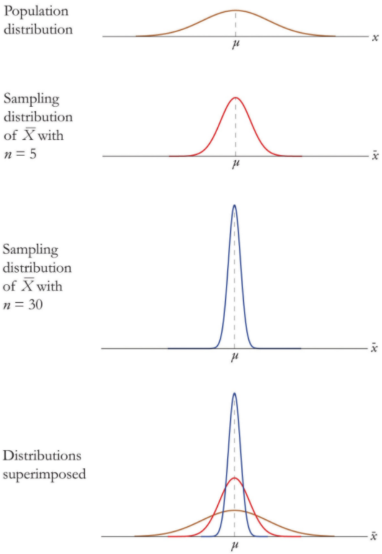
Normal $\bar{x}$

For a random sample from a population:

The sample mean $\bar{x}$ is normally distributed if the population distribution is normal
for any sample size n.

The sample mean $\bar{x}$ is approximately normally distributed if the sample size n is large ($n \geq 30$) regardless of the shape of the population distribution.

Effect of sample size



example

The blood cholesterol level of all men aged 20 to 34 follows the normal distribution with mean $\mu = 188$ mg/dl and standard deviation $\sigma = 41$ mg/dl. A sample survey measures the blood cholesterol level of an SRS of 10 such men.
(a) What are the mean and standard deviation of the sampling distribution of the sample mean $\bar{x}$?
(b). Find $P(185 \leq \bar{x} \leq 191)$.

Answer: (a). $\mu_{\bar{x}} = \mu = 188$, $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{41}{\sqrt{10}} = 12.97$.

(b). Note $\bar{x} \sim N(188, 12.97)$.
$P(185 \leq \bar{x} \leq 191) = P(\frac{185-188}{12.97} \leq z \leq \frac{191-188}{12.97})$
$= P(-0.23 \leq z \leq 0.23) = 0.5910 - 0.4090 = 0.182$.

exercise

Continue with the example. Now change the sample size n to 100.
a). What is the mean and standard deviation of sampling distribution of $\bar{x}$?
b). Find $P(185 \leq \bar{x} \leq 191)$.

section 6.2 exercise 17, 22.

Example

The time that it takes a randomly selected rat to find its way through a maze is normally distributed with $\mu = 1.5$ min and $\sigma = 0.35$ min. Let $\bar{x}$ be the mean time in the maze of 5 randomly selected rats. Then the sample mean $\bar{X}$ is also normal.
Find the probability the sample mean is at most 2 min.

$\mu_{\bar{x}} = 1.5, \sigma_{\bar{x}} = \frac{0.35}{\sqrt{5}} = 0.1565.$
$P(\bar{x} \leq 2) = P(z \leq \frac{2-1.5}{0.1565}) = P(z \leq 3.19) = 0.9993.$

Exercise Prob 21, section 6.2.

The EPA rates the mean highway MPG of 2011 Ford Edge to be $\mu = 27$. Assume $\sigma = 3$. A rental car company buys 60 of these cars.
a). What is the probability that the average mileage of the fleet is bigger than 26.5 MPG?
b). What is the probability that the average mileage of the fleet is between 26 and 26.8 MPG?
c) Find the 20th percentile of the sample mean.

a). $P(\bar{x} > 26.5) = P(z > \frac{26.5-27}{\frac{3}{\sqrt{60}}}) = P(z > -1.29) = 0.9015.$

b). $P(26 \leq \bar{x} \leq 26.8) = P(\frac{26-27}{3/\sqrt{60}} \leq z \leq \frac{26.8-27}{3/\sqrt{60}})$
$= P(-2.58 \leq z \leq -0.52) = 0.3015 - 0.0049 = 0.2966.$

c).
$p = 0.20, z = -0.84, \bar{x} = \mu_{\bar{x}} + z\sigma_{\bar{x}} = \mu + z\frac{\sigma}{\sqrt{n}} = 27 - 0.84 * \frac{3}{\sqrt{60}} = 26.67.$

Exercises

A news report states that the mean distance the commuters in US travel each way to work is 16 miles. Assume the standard deviation is 8 miles. A sample of 75 commuters is taken.
a). What the probability that the sample mean commute distance is greater than 13 miles?
b). What is the probability that the sample mean commute distance is between 18 and 20 miles?
c). Find the 10th percentile of the sample mean.
d). Would it be unusual for the sample mean distance to be greater than 19 miles?
e) Would it be unusual for an individual to have a commute distance greater than 19 mile?

Solutions

$\mu = 16, \sigma = 8, n = 75, 8/\sqrt{75} = 0.9238$

a). $P(\bar{x} > 13) = P(z > \frac{13-16}{0.9238}) = P(z > -3.25) = 0.9994$

b). $P(18 \le \bar{x} \le 20) = P(2.16 \le z \le 4.33) = P(z < 4.33) - P(z < 2.16) = 0.9999 - 0.9846 = 0.0153.$

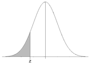
c). $p = 0.10, z = -1.28, \bar{x} = 16 - 1.28 * 0.9238 = 14.82.$

d). Yes. $P(\bar{x} > 19) = P(z > \frac{16-19}{0.9238}) = P(z > 3.25) = 0.0006.$
That is, only about 0.06% of the sample means are greater than 19 miles.

e). No. $P(x > 19) = P(z > \frac{19-16}{8}) = P(z > 0.38) = 0.3519.$
That is, more than 35% individuals commute more than 19 miles.

z table

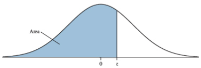
Standard Normal Cumulative Probability Table



Cumulative probabilities for NEGATIVE z-values are shown in the following table:

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
-3.4	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0002
-3.3	0.0005	0.0005	0.0005	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0003
-3.2	0.0007	0.0007	0.0006	0.0006	0.0006	0.0006	0.0006	0.0005	0.0005	0.0005
-3.1	0.0010	0.0009	0.0009	0.0009	0.0008	0.0008	0.0008	0.0008	0.0007	0.0007
-3.0	0.0013	0.0013	0.0013	0.0012	0.0012	0.0011	0.0011	0.0011	0.0010	0.0010
-2.9	0.0019	0.0018	0.0018	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014
-2.8	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019
-2.7	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026
-2.6	0.0047	0.0045	0.0044	0.0043	0.0041	0.0040	0.0039	0.0038	0.0037	0.0036
-2.5	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048
-2.4	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071	0.0069	0.0068	0.0066	0.0064
-2.3	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084
-2.2	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116	0.0113	0.0110
-2.1	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
-2.0	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183
-1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233
-1.8	0.0359	0.0351	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
-1.7	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367
-1.6	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
-1.5	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0559
-1.4	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0721	0.0708	0.0694	0.0681
-1.3	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823
-1.2	0.1151	0.1131	0.1112	0.1093	0.1075	0.1056	0.1038	0.1020	0.1003	0.0985
-1.1	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
-1.0	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379
-0.9	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611
-0.8	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867
-0.7	0.2420	0.2389	0.2358	0.2327	0.2296	0.2266	0.2236	0.2206	0.2177	0.2148
-0.6	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483	0.2451
-0.5	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776
-0.4	0.3446	0.3409	0.3372	0.3336	0.3300	0.3264	0.3228	0.3192	0.3156	0.3121
-0.3	0.3821	0.3783	0.3745	0.3707	0.3669	0.3632	0.3594	0.3557	0.3520	0.3483
-0.2	0.4207	0.4168	0.4129	0.4090	0.4052	0.4013	0.3974	0.3936	0.3897	0.3859
-0.1	0.4602	0.4562	0.4522	0.4483	0.4443	0.4404	0.4364	0.4325	0.4286	0.4247
0.0	0.5000	0.4960	0.4920	0.4880	0.4840	0.4801	0.4761	0.4721	0.4681	0.4641

Table A.2 Cumulative Normal Distribution (continued)



z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998
3.5	.9998	.9998	.9998	.9998	.9998	.9998	.9998	.9998	.9998	.9998
3.6	.9998	.9998	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999
3.7 or more	.9999									