Introduction to Inference

Statistical **inference** draws conclusions about a population from sample data.

Conditions about estimating a population mean μ:
1. We have a SRS from the population.
2. The variable has a normal distribution $N(\mu, \sigma)$ or $n > 30$.
3. We know population standard deviation σ (this assumption is unrealistic and will be relaxed later).

Confidence intervals for μ

A **point estimate** is a single number that is used to estimate an unknown parameter.

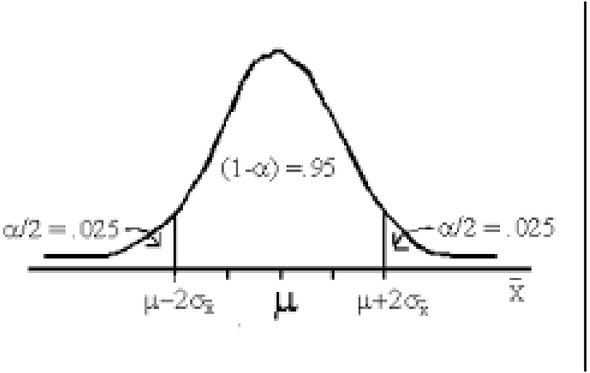
A level $(1-\alpha)100\%$ confidence interval for a parameter has
1. an interval of the form
point estimate $\pm$ margin of error
2. a confidence level $(1-\alpha)100\%$ which gives the success rate for the method.

Confidence interval

$$\bar{x} \pm z \frac{\sigma}{\sqrt{n}}$$

where $z = z_{\alpha/2}$.
Common critical values:
90% : $z_{0.05} = 1.645$
95% : $z_{0.025} = 1.96$
99% : $z_{0.005} = 2.576$
Example 7.3.

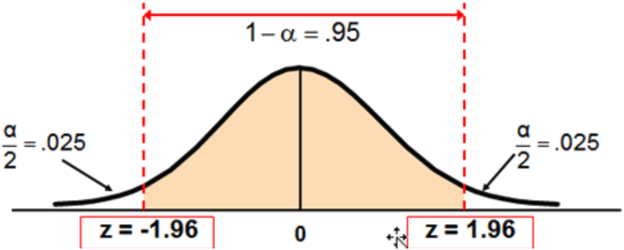
The sampling distribution of $\bar{x}$

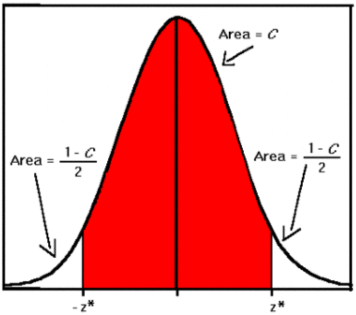


Note here $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$

The empirical rule says 95% of the $\bar{x}$ s are within 2 (1.96 to be exact) $\sigma_{\bar{x}}$ of μ, so a margin of error $2\sigma_{\bar{x}}$ can guarantee a 95% success rate.

Find the z critical value





use p=$\frac{1-ConfidenceLevel}{2}$ or p=$\frac{1+ConfidenceLevel}{2}$ in the z chart to find z.
e.g., for 92% confidence interval, use p=$\frac{1+0.92}{2} = 0.96$ to find $z = 1.75$.
for 88% confidence interval, use p=$\frac{1+0.88}{2} = 0.94$ to find $z = 1.55$.

example

Example 7.5. A simple random sample of 6 cereal boxes produces $\bar{x} = 20.25$ ounces. The fill weights are normally distributed with $\sigma = 0.2$. Get a 90% CI for the mean fill weight.

$z_{0.05} = 1.645$, the CI is

$\bar{x} \pm 1.645 \frac{\sigma}{\sqrt{n}} = 20.25 \pm 1.645 \frac{0.2}{\sqrt{6}} = 20.25 \pm 0.1343 = (20.12, 20.38)$ oz.

Exercises

A random sample of $n = 360$ three month boys produce a sample mean weight $\bar{x} = 25.5$ pounds. Suppose the population standard deviation is given as $\sigma = 5.3$.
Get a 95% CI for the population mean weight.

Solutions

$z_{0.025} = 1.96$,
$25.5 \pm 1.96 * \frac{5.3}{\sqrt{360}} = 25.5 \pm 0.55 = (24.95, 26.05)$ lb.

Exercise

Section 7.1 prob. 43.
The mean math SAT score of a random sample of 100 freshmen is 458. Assume $\sigma = 116$. Find a 99% CI for the population mean.

$458 \pm 2.576 \frac{116}{\sqrt{100}} = 458 \pm 29.88 = (458.12, 487.88)$.
For confidence level=0.99, use $p = 0.995$ in the z chart to find $z = 2.57$ or $z = 2.58$, a more exact value is $z = 2.576$.

Margin of error

Note margin of error (denoted by m) in the confidence interval is

$$m = z\frac{\sigma}{\sqrt{n}}$$

sample size needed to achieve a certain margin of error:

$$n = (\frac{z\sigma}{m})^2$$

Example 7.7

The weight of mice fed a certain diet is normally distributed with $\sigma = 3$ grams. How many mice must be weighted so than a 95% CI will have a margin of error of 0.5 grams?

$0.5 = 1.96\frac{3}{\sqrt{n}}$

or

$$n = (\frac{1.96 * 3}{0.5})^2 = 138.30 \approx 139$$

We always round up n when computing the needed sample size.

Exercise 51

In a random sample of 100 electronic components, the mean life time was 125 hours. Assume that the components lifetimes are normally distributed with population standard deviation $\sigma = 20$ hours.
a). Construct a 98% CI for the mean battery life.
b). Find the sample size needed so that a 99% CI will have a margin of error of 3.

Solutions

a). $z_{0.01} = 2.33, \bar{x} \pm z\frac{\sigma}{\sqrt{n}}$
$= 125 \pm 2.33\frac{20}{\sqrt{100}} = 125 \pm 4.66 = (120.34, 129.66).$
b)$z_{0.005} = 2.576, n = (\frac{z\sigma}{m})^2 = (\frac{2.576*20}{3})^2 = 295.$