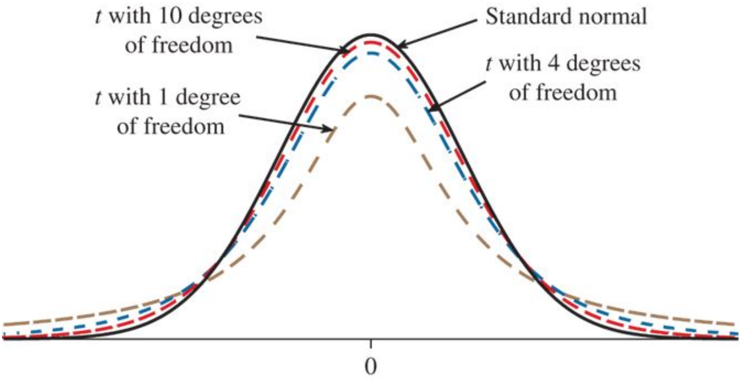
# Unknown population standard deviation

Conditions: SRS
Population distribution is normal or $n > 30$.
We do not know $\sigma$

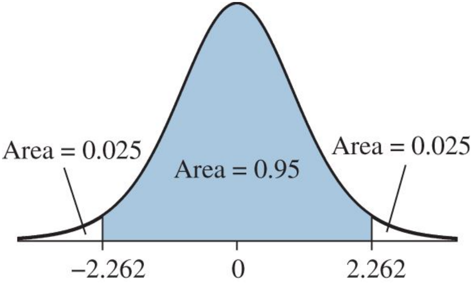
# Replace $\sigma$ with $s$

Recall $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$.
$z = \frac{\bar{x}-\mu}{\sigma/\sqrt{n}} \sim Normal(0, 1)$.
Now $t = \frac{\bar{x}-\mu}{s/\sqrt{n}}$ has a t distribution with $n-1$ degrees of freedom.

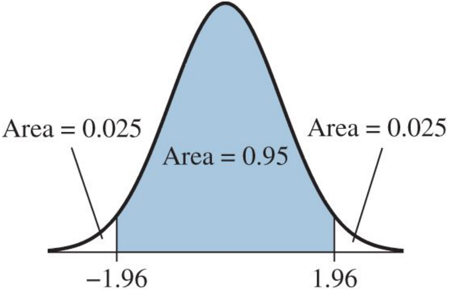
# t distribution



# t and z critical value




t critical value with d.f. = 9




z critical value

# Confidence interval

level $(1-\alpha)100\%$ confidence interval:
$\bar{x} \pm t_{\alpha/2}\frac{s}{\sqrt{n}}$.
Example 7.12.
A sample of 123 people reported the number of hours spent on internet per week, $\bar{x} = 8.20$, $s = 9.84$ hours. Get a 95% CI for $\mu$.
$\bar{x} \pm t_{\alpha/2}\frac{s}{\sqrt{n}} = 8.20 \pm 1.984\frac{9.84}{\sqrt{123}} = 8.20 \pm 1.7603 = (6.44, 9.96)$ hours.
Exercise 29, 31.

# Exercise

The healing rate of wounds made on 18 newts was measured with a sample mean of 25.67 micrometers per hour and a standard deviation $s = 8.324$. A stem-and-leaf plot of data does not show obvious departure form normality. Get a 95% CI for the population mean.

# Solution

n=18, d.f. = 17,
$25.67 \pm 2.110 \frac{8.324}{\sqrt{18}} = 25.67 \pm 4.14 = (21.53, 29.81)$ hours.

# example

Gas bubbles inside ancient amber gives the following nitrogen (percent).
63.4, 65.0, 64.4,63.3, 54.8, 64.5, 60.8, 49.1, 51.
$\bar{x} = 59.59$, $s = 6.26$.
Get a 90% CI for $\mu$, the mean nitrogen content in ancient air.

CI: $59.59 \pm 1.860 * \frac{6.26}{\sqrt{9}} = 59.59 \pm 3.88 = (55.71, 63.47)$.

# Exercise

Six measurements were made of the mineral content ( in percent) of spinach. It is reasonable to assume the population is approximately normal:

19.1, 20.8, 20.8, 21.4, 20.5, 17.4.

Get a 95% CI for the mean mineral content.

$\bar{x} = 20.0, s = 1.49,$

CI : $20.0 \pm 2.571 * 1.49/\sqrt{6} = 20 \pm 1.56 = (18.44, 21.56)$.

Note: $20.0 \pm 2.571 * 1.49/\sqrt{6} = 20.0 \pm (2.571 * 1.49/\sqrt{6})$.