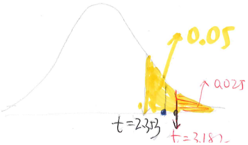
One sample t test

Now assume σ unknown.
Conditions: Simple random sample
$n > 30$ or population distribution is normal.
$H_0 : \mu = \mu_0$
$H_1 : \mu > \mu_0$ (right-tailed)
$H_1 : \mu < \mu_0$ (left-tailed)
$H_1 : \mu \neq \mu_0$ (two-tailed)
test statistic $t = \frac{\bar{x}-\mu_0}{s/\sqrt{n}} = t^*$.
P-value = $P(t > t^*)$ if $H_1 : \mu > \mu_0$
P-value = $P(t < t^*)$ if $H_1 : \mu < \mu_0$
P-value = $2P(t > |t^*|)$ if $H_1 : \mu \neq \mu_0$
Note the t chart only gives positive t values. $P(t < -2) = P(t > 2)$ as the t density curve is symmetric around 0

t Table

cum. prob	$t_{.50}$	$t_{.75}$	$t_{.80}$	$t_{.85}$	$t_{.90}$	$t_{.95}$	$t_{.975}$	$t_{.99}$	$t_{.995}$	$t_{.999}$	$t_{.9995}$
one-tail	**0.50**	**0.25**	**0.20**	**0.15**	**0.10**	**0.05**	**0.025**	**0.01**	**0.005**	**0.001**	**0.0005**
two-tails	**1.00**	**0.50**	**0.40**	**0.30**	**0.20**	**0.10**	**0.05**	**0.02**	**0.01**	**0.002**	**0.001**
df											
1	0.000	1.000	1.376	1.963	3.078	6.314	12.71	31.82	63.66	318.31	636.62
2	0.000	0.816	1.061	1.386	1.886	2.920	4.303	6.965	9.925	22.327	31.599
3	0.000	0.765	0.978	1.250	1.638	2.353	3.182	4.541	5.841	10.215	12.924
4	0.000	0.741	0.941	1.190	1.533	2.132	2.776	3.747	4.604	7.173	8.610
5	0.000	0.727	0.920	1.156	1.476	2.015	2.571	3.365	4.032	5.893	6.869
6	0.000	0.718	0.906	1.134	1.440	1.943	2.447	3.143	3.707	5.208	5.959
7	0.000	0.711	0.896	1.119	1.415	1.895	2.365	2.998	3.499	4.785	5.408
8	0.000	0.706	0.889	1.108	1.397	1.860	2.306	2.896	3.355	4.501	5.041
9	0.000	0.703	0.883	1.100	1.383	1.833	2.262	2.821	3.250	4.297	4.781
10	0.000	0.700	0.879	1.093	1.372	1.812	2.228	2.764	3.169	4.144	4.587
11	0.000	0.697	0.876	1.088	1.363	1.796	2.201	2.718	3.106	4.025	4.437
12	0.000	0.695	0.873	1.083	1.356	1.782	2.179	2.681	3.055	3.930	4.318
13	0.000	0.694	0.870	1.079	1.350	1.771	2.160	2.650	3.012	3.852	4.221
14	0.000	0.692	0.868	1.076	1.345	1.761	2.145	2.624	2.977	3.787	4.140
15	0.000	0.691	0.866	1.074	1.341	1.753	2.131	2.602	2.947	3.733	4.073
16	0.000	0.690	0.865	1.071	1.337	1.746	2.120	2.583	2.921	3.686	4.015
17	0.000	0.689	0.863	1.069	1.333	1.740	2.110	2.567	2.898	3.646	3.965
18	0.000	0.688	0.862	1.067	1.330	1.734	2.101	2.552	2.878	3.610	3.922
19	0.000	0.688	0.861	1.066	1.328	1.729	2.093	2.539	2.861	3.579	3.883
20	0.000	0.687	0.860	1.064	1.325	1.725	2.086	2.528	2.845	3.552	3.850
21	0.000	0.686	0.859	1.063	1.323	1.721	2.080	2.518	2.831	3.527	3.819
22	0.000	0.686	0.858	1.061	1.321	1.717	2.074	2.508	2.819	3.505	3.792
23	0.000	0.685	0.858	1.060	1.319	1.714	2.069	2.500	2.807	3.485	3.768
24	0.000	0.685	0.857	1.059	1.318	1.711	2.064	2.492	2.797	3.467	3.745
25	0.000	0.684	0.856	1.058	1.316	1.708	2.060	2.485	2.787	3.450	3.725
26	0.000	0.684	0.856	1.058	1.315	1.706	2.056	2.479	2.779	3.435	3.707
27	0.000	0.684	0.855	1.057	1.314	1.703	2.052	2.473	2.771	3.421	3.690
28	0.000	0.683	0.855	1.056	1.313	1.701	2.048	2.467	2.763	3.408	3.674
29	0.000	0.683	0.854	1.055	1.311	1.699	2.045	2.462	2.756	3.396	3.659
30	0.000	0.683	0.854	1.055	1.310	1.697	2.042	2.457	2.750	3.385	3.646
40	0.000	0.681	0.851	1.050	1.303	1.684	2.021	2.423	2.704	3.307	3.551
60	0.000	0.679	0.848	1.045	1.296	1.671	2.000	2.390	2.660	3.232	3.460
80	0.000	0.678	0.846	1.043	1.292	1.664	1.990	2.374	2.639	3.195	3.416
100	0.000	0.677	0.845	1.042	1.290	1.660	1.984	2.364	2.626	3.174	3.390
1000	0.000	0.675	0.842	1.037	1.282	1.646	1.962	2.330	2.581	3.098	3.300
z	0.000	0.674	0.842	1.036	1.282	1.645	1.960	2.326	2.576	3.090	3.291
	0%	50%	60%	70%	80%	90%	95%	98%	99%	99.8%	99.9%
	Confidence Level										

t-table.xls 7/14/2007



Example 8.16

A SRS of n=76 subjects were on a low fat diet. Sample mean weight loss was $\bar{x} = 2.2$ kg with $s = 6.1$ kg. Can we conclude the mean weight loss μ of the population is greater than 0? Use $\alpha = 0.05$.

$H_0 : \mu = 0$

$H_1 : \mu > 0$

Test statistic $t = \frac{\bar{x} - \mu_0}{s/\sqrt{n}} = \frac{2.2 - 0}{6.1/\sqrt{76}} = 3.144$.

P-value = $P(t > 3.144)$ which is between 0.001 and 0.005.

P-value < 0.05. Reject H_0. We conclude that the mean weight loss is greater than 0.

Example 8.17

Compare a generic drug to the brand-name.
$n = 7$ subjects applied the ointment. The amounts absorbed (mg):
2.6, 3.2, 2.1, 3.0, 3.1, 2.9, 3.7.
Does the mean amount absorbed differs from 3.5 mg? Use $\alpha = 0.01$.
Make a dot plot or stem-and-leaf plot of data: No outliers, no strong skewness, so we can assume data come from a normal distribution.

Example 8.17: Solutions

$H_0 : \mu = 3.5$
$H_1 : \mu \neq 3.5$
$\bar{x} = 2.9429, s = 0.4995$ (computed from data)
$t = \frac{\bar{x}-\mu_0}{s/\sqrt{n}} = \frac{2.9429-3.5}{0.4995/\sqrt{7}} = -2.951.$
P-value $= 2P(t > 2.951)$ which is between 0.02 and 0.05.
P-value > 0.01. Do not reject H_0. There is not enough evidence that the mean amount absorbed differs from 3.5 mg.

How to find the P-value for t test: Find the degrees of freedom then move along that line to the right until you find two values than bracket your t value. Then move to the top to read the range of the P-value. Go to the one tail line (the 2nd line from the top) for one-tailed test and go to the two tail line (3rd line from top) for two-tailed test. If t value is off the chart, then the P-value is less than the value at the rightmost edge.

exercise

Seven subjects who identified themselves as Buddhist reported hours per week watching TV of:
2,1,1,3,2,3,2.
Assume the TV watching time of a budddist is approximately normally distributed.
1). Estimate the sample mean $\bar{x}$ and sample standard deviation s.
2). Construct a 90% confidence interval for μ, the mean TV watching time per week for all Buddhists.
3). Test if the mean TV watching time per week for Buddhist is smaller than the that for the general public which is 4 hours using $\alpha = 0.05$.

$\bar{x} = 2, s = 0.816.$
CI: $2 \pm 1.943 * 0.816/\sqrt{7} = (1.40, 2.60)$ hours.

$H_0 : \mu = 4$
$H_1 : \mu < 4$
$t = \frac{2-4}{0.816/\sqrt{7}} = -6.48$
the P-value < 0.0005. Reject H_0.

check your understanding exercise

A random sample of dioxin emissions is taken from an incinerator for 4 days this year. The observations were:
2, 3, 3, and 2 grams of dioxin per day. Assume the emissions are approximately normally distributed.
1). Construct a 90% confidence interval for the mean emission of dioxin per day.
2). The researcher suspects the incinerator is emitting a higher level of dioxin this year. Test if the mean emission this year is higher than last year's mean emission which was 1.8 grams. Use $\alpha = 0.05$.

CI: $\bar{x} \pm t\frac{s}{\sqrt{n}} = 2.5 \pm 2.353 * 0.577/\sqrt{4} = (1.82, 3.18)$ grams.

$H_0 : \mu = 1.8$
$H_1 : \mu > 1.8$
$t = \frac{2.5-1.8}{0.577/\sqrt{4}} = 2.43.$
P-value is between 0.025 and 0.05.
Reject H_0.

Example with Solutions

Prob 18, p. 378. An economist wants to find if the mean FICO credit score of people is lower than the 720 cutoff of 720. She finds the mean FICO credit score of a SRS of 100 people had a mean of 703 with a standard deviation of 92. Can she conclude the population mean FICO score is lower than 720? Use $\alpha = 0.05$.

$\bar{x} = 703, n = 100, s = 92.$

$H_0 : \mu = 720$

$H_1 : \mu < 720$

$t = \frac{703-720}{92/\sqrt{100}} = -1.85$

P-value $= P(t < -1.85)$ which is between 0.025 and 0.05.

Reject H_0. She has sufficient edivence to conclude the population mean score is lower than 720.

Check your understanding exercise

The mean annual tuition and fees in the 2009-1010 academic year for a sample of 14 private colleges in CA was \$30,500 with a standard deviation \$4,500. A dotplot shows it is reasonable to assume the population distribution is normal. Can you conclude the mean tuition and fees for private colleges in CA differs from \$30,000?

a. State H_0, H_1.

b. Compute the test statistic.

c. Find the p-value. What is your conclusion using $\alpha = 0.01$ level of significance?

Solutions

$H_0 : \mu = 30,000$
$H_1 : \mu \neq 30,000$
The t statistic is:
$t = \frac{30500 - 30000}{\frac{4500}{\sqrt{14}}} = 0.416.$
P-value = $2P(t > 0.416)$ which is between 0.50 and 1.00.
Do not reject H_0. There is not sufficient evidence in the data that the mean tuition differs from \$30,000.