Practice problems

1. A study examined the exposure of carbon monoxide on a group of workers. The table belows reports the numbers of workers with various symptoms along with the shift(morning, afternoon, evening) they worked.

	Influenza	Headache	Weakness	Short of Breath
Morning	16	24	11	7
Afternoon	13	33	16	9
Evening	18	6	5	9

Conduct a chi-square test to examine whether symptoms were dependent on the shift the workers worked.

1). Write down H_0, H_1.

2). The chi-square test statitics is comptued as $\chi^2 = 17.57$.
Find the d.f. for this test.
Find the p-value for this test.
What is your conclusion using level of significance $\alpha = 0.05$?

Solutions: d.f.=6, p-value is between 0.005 and 0.01. Reject H_0.

2. The following table presents the number years of study in English writing and the SAT writing score for some students.

Years of Study	0.5	1.0	2.0	3.0	4.0
SAT score	418	427	455	459	498

1). Obtain the least squares regression line for predicting SAT score from years of study.

2). If the years of study of two students differ by 2 years, by how much would you predict their SAT score differ?

3). Predict the SAT score for a student with 2.5 years of study.

Solutions: $\hat{y} = 406.506 + 21.378x$

Differ by 21.378*2=42.756 points or about 43 points.

$\hat{y} = 460$.

3 . A group of 8 individuals with high cholesterol levels were given a new drug. Cholesteral levels, in milligrams per deciliter, were measured before and after treatment for each individual.

individual	1	2	3	4	5	6	7	8
before	283	299	274	284	248	275	293	277
after	215	206	187	212	178	212	192	196

1). Get a 90% CI for the mean reduction in cholesteral level.

2). A doctor claimed the men reduction is more than 80 mpd. Does the CI contradict his claim?

Solutions: This is a matched pair design. Take difference of each pair of values first.

CI is (70.4, 88.3).

Does not contradict as the CI include values higher than 80.

4. A poll asked a random sample of 1325 adults in the US how much confidence they had in banks. a total of 149 said that they had a lot of confidence. An economist claims that less than 15% of US adults had a great deal of confidence in banks.

1). State H_0, H_1.

2). Compute the test statistic.

3). Using $\alpha = 0.05$, can you conclude the economist's claim is true?

4). Using $\alpha = 0.01$, can you conclude the economist's claim is true?

5). Get a

Solutions: $H_0 : p = 0.15, H_1 : p < 0.15$

$\hat{p} = 0.112, z = -3.83$, p-value ≈ 0.001. 3). yes. 4) yes.

5). CI is: $0.112 \pm 0.014 = (0.098, 0.126)$.

5. A machine that fills beverage cans is supposed to put 12 ounces of beverage in each can. Following are the amounts measured in a simple random sample of 8 cans:

11.96, 12.10, 12.04, 12.13, 11.98, 12.05, 11.91, 12.03.

A dot plot shows it is reasonable to assume the data come from a normal distribution.

1).Perform a hypothesis test to determine if the mean volume differs from 12 ounces. Use $\alpha = 0.05$.

2). Get a 95% CI for the mean volume.

Solutions: $H_0 : \mu = 12, H_1 : \mu \neq 12, \bar{x} = 12.025$,

$s = 0.0727, t = 0.97$, p-value is between 0.30 and 0.40.

CI (11.96, 12.09).

6. It is reported that the mean monthly rent for a one-bedroom apartment in Manhattan is \$2631. Assume the standard deviation is \$500. A real estate firm samples 100 apartments.

1). What i the probability that the sample mean rent is greater than 2700?

2). What the probability that the sample mean rent is between 2500 and 2700?

3). Find the 60th percentile of the sample mean.

4). Can you tell whether it would be unusual if the sample mean were greater than 2800?

5). Do you think it would be unusual for an individual apartment to have a rent greather than 2800? Explain.

$\mu = 2631, \sigma = 500, n = 100.$

$\mu_{\bar{x}} = 2631, \sigma_{\bar{x}} = 500/\sqrt{100} = 50.$

1). $P(\bar{x} > 2700) = P(z > 1.38) = P(z < -1.38) = 0.0838..$

2). $P(2500 \le x \le 2600) = P(-2.62 \le z \le -0.62) = 0.2676 - 0.0044 = 0.2632.$

3). $p = 0.60, z = 0.25, \bar{x} = 2631 + 0.25 * 50 = 2643.5.$

4). Yes. $P(\bar{x} > 2800) = P(z > 3.38) = 0.0004$. That is, only about 0.04 percent of the samples means are greater than 2800.

5). No. $P(x < 2800) = P(z > 0.34) = 0.3669$. That is, about 37 percent of the individual rents are greater than 2800. (here we assume the individuals rents follow a normal distribution).

7. According to thepoultry.com, the weights of broilers are approximately normally distributed with mean 1387 grams and standard deviation 161 grams.

a). What percentage of broilers weigh between 1100 and 1200 grams?

b). What is the probability that a randomly selected broilers weigh more than 1500 grams?

c). Is it unusual for a broiler to weigh more than 1550 grams?

d). Find the 90th percentile of the broiler weights.

$\mu = 1387, \sigma = 161$

a). $P(1100 \le x \le 1200) = P(-1.79 \le 1.16) = 0.0855.$

b). $P(x > 1500) = P(z > 0.70) = 0.2420.$

c). No. Because $P(x > 1550) = P(z > 1.01) = 0.1562$. that is, about 16 percent of broilers weigh more than 1550 grams.

d). z=1.28, x=1387+1.28*161=1593.08 grams.

8. A fast food chain has 600 outlets in USA. The following table presents the number of restaurants classified by location and city population size.

	Region			
City population	NE	SE	SW	NW
Under 50,000	30	35	15	5
50,000 to 500,000	60	90	70	30
Over 500,000	150	25	30	60

a). Given that a restaurant is located in a city with population size over 500,000, what is the probability that it is in NE?

b). Given that a restaurant is located in SE, what is the probability that it is in a city with population size under 50,000?

c). What is the probability that the randomly selected restaurant is in a city with population size over 500,00 or in SE?

a). 0150/265=0.566. b) 35/150=0.2333. c)390/600=0.65.

9. Jim encounters three intersections on his way to work. Intersection 1 has green lights 50% of the times, intersection 2 has green lights 40% of the times and intersection 3 has green lights 20% of the times. What is the probability that Jim will encounter green lights all the way to work?

.50*.40*.20=0.04.

10. An article reports that 73% of Fortune 500 companies have Twitter accounts. A economist thinks the percentage is higher at tech companies. She sampled 70 tech companies and found 55 of them have Twitter accounts. Can she conclude that more than 73% of tech companies have Twitter accounts?

Also get a 95% CI for the population proportion of tech companies with Twitter accounts.

$\hat{p} = 55/70 = 0.786$,

$H_0 : p = 0.73$

$H_1 : p > 0.73$
$z = \frac{.786-0.73}{\sqrt{0.73*.27/50}} = 0.02$
p-value $= P(z > 0.02) = 0.492$.
Do not reject H_0.
CI: $0.786 \pm 1.96 * \sqrt{(}0.786 * 0.214/70) = 0.786 \pm 0.096 = (0.690, 0.882)$.