Math 220 Test 2

1. `Note I have included the answers to both versions in this solution.`

   Which of the following z scores bound the middle 92% of the area under the standard normal curve?

   a). (-1.41,1.41); b) (-1.75,1.75); c) (-1.28,1.28); d) (-2.33,2,33)

   answer: b). Use p=0.96 in the z chart to find z.

   If the area is 96%, then use p=0.98 in the z chart to find z and the answer is (-2.05, 2.05).

2. For a normal distribution with mean 20 and standard deviation 5, we can conclude that approximately 68% of the values fall between

   a). (15,25); b) (10,30); (c) (5,35).

   The emiprical rules says about 68% of the values fall within 1 standard deviation of the mean, so this interval is (20-5, 20+5)=(15,25).

   If $\sigma = 8$, then the interval is (12, 28).

3. The next three questions refer to the information given below.

   Assume the commuting times of American workers follow a normal distribution with mean mu=23 minutes and standard deviation sigma=5 minutes. What proportion of American workers have a commuting time less than 30 minutes?

   a). 0.9192; b) 0.0808; c) 0.6654; d) 0.3346

   answer: a).

   $P(x < 30) = P(z < \frac{30-23}{5}) = P(z < 1.4) = 0.9192.$

4. Find the the 86th percentile of the commuting times.

   a) 27.3; b) 17.6; c) 28.4; d) 29.2

   Answer: c). use p=0.86 to find z=1.08 in the z chart and $x = \mu + z\sigma = 23 + 1.08 * 5 = 28.4$.

   For the 74th percentile, the answer is 26.2. Use p=0.74 to find z=0.64, and $x = 23 + 0.64 * 5 = 26.2$.

5. A random sample of 16 American workers is taken. What is the probability that the sample mean commuting time is greater than 25 minutes?

   a). 0.0548; b) 0.9452; c) 0.3446; d) 0.6554

   answer: a). $P(\bar{x} > 25) = P(z > \frac{25-23}{5/\sqrt{16}}) = p(z > 1.6) = 0.0548$. (need to substract the proportion on the z chart from 1)

6. A gallup poll estimates the proportion of people who believe the economy is getting better to be 0.25. What sample size is needed so that the 95% CI will have a margin of error 0.02?

   a) 2202; b) 1801; c) 1798; d) 2125.

   The answer is b).

   $n = 0.25 * 0.75 * (\frac{1.96}{0.02})^2 = 1801.$

   If the margin of error is 0.03, then $n = 0.25 * 0.75 * (\frac{1.96}{0.03})^2 = 801.$

   Note we always round up the numbers when we compute the needed sample size.

7. The next three questions refer to the data given below.

   Hosmer Winery has a machine for dispensing their wine into bottles. Four random observations taken yesterday were 745, 740, 750, 745 ml.

   Compute the sample mean and the sample standard deviation.

   $\bar{x} = 745, s^2 = \frac{(745-745)^2+(740-745)^2+(750-745)^2+(745-745)^2}{4-1} = 16.67, s = 4.08.$

8. Construct a 95% confidence interval for the mean amount dispensed. Round your number to 2 decimal places.

   d.f.=3, the t critical value is t=3.182.

   $\bar{x} \pm t\frac{s}{\sqrt{n}} = 745 \pm 3.182 * \frac{4.08}{\sqrt{4}} = 745 \pm 6.49 = (738.51, 751.49)$ ml.

9. When the machine is in statistical control, the amount dispensed has a mean of 752 ml. Perform a hypothesis test to examine if the machine is in statistical control. Use $\alpha = 0.05$ level of significance. In the space below, write down the null and alternative hypothesis H_0 and H_1, compute the test statistic value and find the p-value for the test. You can use mu to denote the population mean.

$H_0 : \mu = 752$

$H_1 : \mu \neq 752$

$t = \frac{\bar{x}-\mu_0}{s/\sqrt{n}} = \frac{745-752}{4.08/\sqrt{4}} = -3.43$

The p-value is between 0.02 and 0.05. (The exact p-value is 0.04 which can be found by software).

Reject H_0. There is sufficient evidence the machine is not in statistical control.

When changing 752 to 750, then

$H_0 : \mu = 750$

$H_1 : \mu \neq 750$

$t = \frac{\bar{x}-\mu_0}{s/\sqrt{n}} = \frac{745-750}{4.08/\sqrt{4}} = -2.45$

The p-value is between 0.05 and 0.10. (The exact p-value is 0.09 which can be found by software).

Do not reject H_0. There is no sufficient evidence the machine is not in statistical control.

10. The next two questions refer to the data given below.

The National Health Interview Survey, which included a questionnaire administered during in-person interviews with 21,781 adults, found that 20.6 percent of them were smokers in 2008. (New York Times, Nov 18, 2009). Round your numbers to 3 decimal places.

Find a 95% confidence interval for the proportion of American adults who smoked in 2008.

$\hat{p} = 0.206$, the CI is

$\hat{p} \pm z\sqrt{\hat{p}(1-\hat{p})/n} = 0.206 \pm 1.96\sqrt{0.206 * 0.794/21781} = 0.206 \pm 0.005 = (0.201, 0.211)$

Find a 90% confidence interval for the proportion of American adults who smoked in 2008.

$\hat{p} \pm z\sqrt{\hat{p}(1-\hat{p})/n} = 0.206 \pm 1.645\sqrt{0.206 * 0.794/21781} = 0.208 \pm 0.0045 = (0.2015, 0.2105)$

11. A group of 5 patients with high cholesterol levels were given a new drug to reduce cholesterol levels. Cholesterol levels, in milligrams per deciliter, were measured before and after the treatment for each individual. If we perform a paired t test to examine if the new drug helped reduce cholesterol levels, the degrees of freedom for your t test should be

    5-1=4.

    For a group of 6 patients,the d.f. is 6-1=5.