Math 424 Assignment key

The following tables list the loss functions, the distribution of observations $z_i, i = 1, 2, 3, 4$ given the states of nature $\theta_i, i = 1, 2, 3$ and 4 strategies $s_i, i = 1, 2, 3, 4$.

Loss Table

	a_1	a_2	a_3	a_4
θ_1	0	2	4	8
θ_2	6	4	2	5
θ_3	3	2	1	0

Distribution of observations given state of nature

	z_1	z_2	z_3	z_4
θ_1(0.2)	0	0.4	0.6	0
θ_2(0.3)	0.4	0.2	0.2	0.2
θ_3(0.5)	0.1	0.2	0.3	0.4

Suppose the prior distributions of θ is given by $P(\theta = \theta_1) = 0.2, P(\theta = \theta_2) = 0.3, P(\theta = \theta_3) = 0.5$.

1). Find the conditional distribution of θ given $z = z_1$. Based on this distribution, which action should you take?

$p(\theta = \theta_1|z_1) = \frac{p(z_1|\theta_1)p(\theta_1)}{p(z_1|\theta_1)p(\theta_1)+p(z_1|\theta_2)p(\theta_2)+p(z_1|\theta_3)p(\theta_3)} = \frac{0*0.2}{p(z_1)} = 0.$

$p(\theta = \theta_2|z_1) = \frac{p(z_1|\theta_2)p(\theta_2)}{p(z_1|\theta_1)p(\theta_1)+p(z_1|\theta_2)p(\theta_2)+p(z_1|\theta_3)p(\theta_3)} = \frac{0.4*0.3}{0*0.2+0.4*0.3+0.1*0.5} = 12/17.$

and $p(\theta = \theta_3|z_1) = 5/17$.

The average loss of each action:

a_1 : 0*0+6*12/17+3*5/17=87/17,

a_2 : 2*0+4*12/17+2*5/17=58/17,

a_3 : 4*0+2*12/17+1*5/17=29/17,

a_4 : 8*0+5*12/17+0*5/17=60/17.

The best action to take is a_3 if z_1 is observed.

2). Find the best action to take given $z = z_i, i = 2, 3, 4$.

Similarly, we can find the best action a_3 if z_2 is observed, the best action is a_3 if z_3 is observed, and the best action is a_4 if z_4 is observed.

3). Based on the results in 1) and 2), find the Bayes strategy and the associated Bayes risk.

Therefore, the best strategy (the Bayes strategy) is $s = (a_3, a_3, a_3, a_4)$.

The expected loss of s given $\theta = \theta_1$ is:

0.4*4+0.6*4=4,

The expected loss of s given $\theta = \theta_2$ is

0.4*2+0.2*2+0.2*2+0.2*5=2.6,
The expected loss of s given $\theta = \theta_3$ is
0.1*1+0.2*1+0.3*1+0.4*0=0.6,
The mean risk (Bayes risk) is
4*0.2+2.6*0.3+0.6*0.5=1.88.

Another way to compute the Bayes risk.
The margin distribution of z is
$p(z = z_1) = 0.17, p(z = z_2) = 0.24, p(z = z_3) = 0.33, p(z = z_4) = 0.26.$
Use the law of total probability $p(z_1) = \sum_i p(z_1|\theta_i)p(\theta_i)$
29/17*0.17+2.25*0.24+25/11*0.33+15/13*0.26=1.88.