Dividend valuation models

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1. Overview

When an investor buys a share of common stock, it is reasonable to expect that what an investor is willing to pay for the share reflects what he expects to receive from it. What he expects to receive are future cash flows in the form of dividends and the value of the stock when it is sold.

The value of a share of stock should be equal to the present value of all the future cash flows you expect to receive from that share. Since common stock never matures, today's value is the present value of an infinite stream of cash flows. And also, common stock dividends are not fixed, as in the case of preferred stock. Not knowing the amount of the dividends -- or even if there will be future dividends -- makes it difficult to determine the value of common stock.

2. The basic model

The basic premise of stock valuation is that in a market with rational markets, the value of the stock today is the present value of all future cash flows that will accrue to that investor in the stock. In other words, you get (in a present value sense) what you pay for. Using time value of money principles, we can determine the price of a stock today based on the discounted value of future cash flows. We refer to this price as the **intrinsic value** of the stock because it is the value of the stock that is perceived based on all available information. Is it always right on target? No, but it's close.

If dividends are constant forever, the value of a share of stock is the present value of the dividends per share per period, in perpetuity. Let D_1 represent the constant dividend per share of common stock expected next period and each period thereafter, forever, P_0 represent the price of a share of stock today, and r the required rate of return on common stock.[1] The current price of a share of common stock, P_0, is:

$$P_0 = D_1 \div r.$$

The required rate of return is the compensation for the time value of money tied up in their investment and the uncertainty of the future cash flows from these investments. The greater the uncertainty, the greater the required rate of return. If the current dividend is $2 per share and the required rate of return is 10 percent, the value of a share of stock is $20. Therefore, if you pay $20 per share and dividends remain constant at $2 per share, you will earn a 10 percent return per year on your investment every year.

If dividends grow at a constant rate, the value of a share of stock is the present value of a *growing* cash flow. Let D_0 indicate *this* period's dividend. If dividends grow at a constant rate, g, forever, the present value of the common stock is the present value of all *future* dividends, which – in the unique case of dividends growing at the constant rate g – becomes what is commonly referred to as the **dividend valuation model** (DVM):

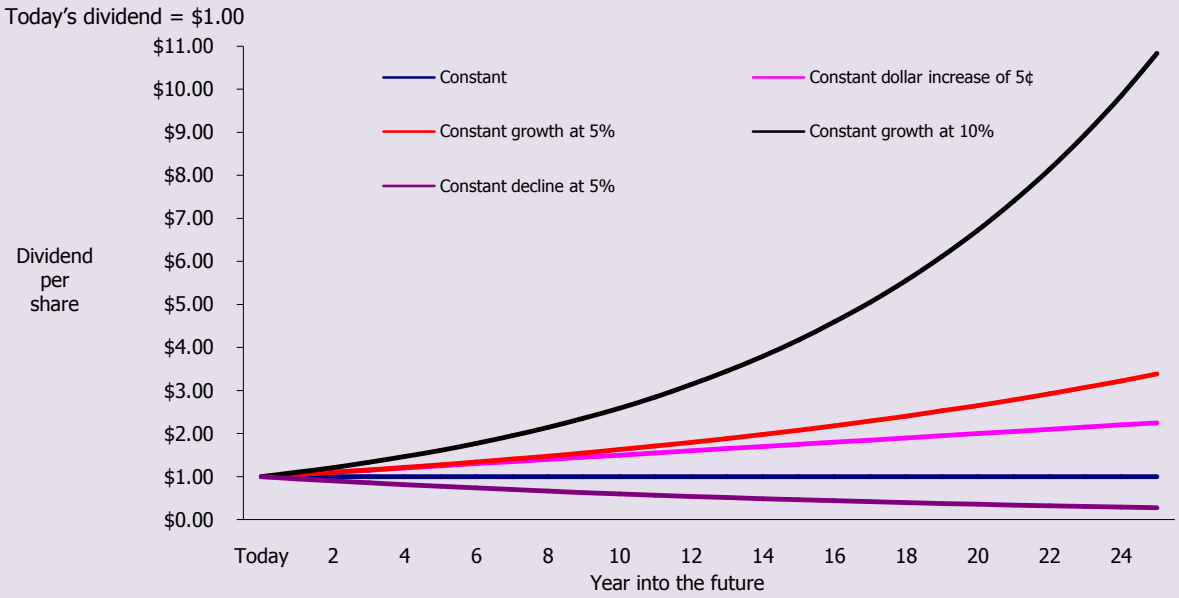
$$P_0 = \frac{D_0(1+g)}{r-g} = \frac{D_1}{r-g}$$

This model is also referred to as the **Gordon model**.[2] This model is a one of a general class of models referred to as the **dividend discount model** (**DDM**).

[1] The **required rate of return** is the return demanded by the shareholders to compensate them for the time value of money and risk associated with the stock's future cash flows.

[2] The model was first proposed by Myron J. Gordon, *The Investment Financing, and Valuation of the Corporation*, [Homewood: Irwin], 1962.

EXHIBIT 1 EXAMPLES OF DIFFERENT PATTERNS OF DIVIDEND GROWTH



If dividends are expected to be $2 in the *next* period and grow at a rate of 6 percent per year, forever, the value of a share of stock is:

$$\text{Value per share} = \$2 \div (0.10\text{-}0.06) = \$50.$$

Because we expect dividends to grow each period, we also are expecting the price of the stock to grow through time as well. In fact, the price is expected to grow at the same rate as the dividends: 6 percent per period.

We can also use the DVM to calculate the current price of a stock whether dividend grow at a constant rate, dividends do not grow (that is, g = 0 percent), or dividends actually decline at a constant rate (that is, g is negative).

EXAMPLES

Example 1
Suppose dividends on a stock today are $5 per share and dividends are expected to grow at a rate of 5% per year, ad infinitum. If the required rate of return is 8%, what is the value of a share of stock?

Solution

$$P_0 = \frac{D_0(1+g)}{r-g} = \frac{\$5(1+0.05)}{0.08-0.05} = \$175$$

Example 2
Suppose dividends on a stock today are $1.20 per share and dividends are expected to decrease each year at a rate of 2% per year, forever. If the required rate of return is 10%, what is the value of a share of stock?

Solution

$$P_0 = \frac{D_0(1+g)}{r-g} = \frac{\$1.20(1-0.02)}{0.10-\text{–}0.02} = \frac{\$1.176}{0.12} = \$9.80$$

Example 3
Suppose dividends on a stock today are $1 per shares and dividends are expected to remain the same,

forever. If the required rate of return is 8%, what is the value of a share of stock?

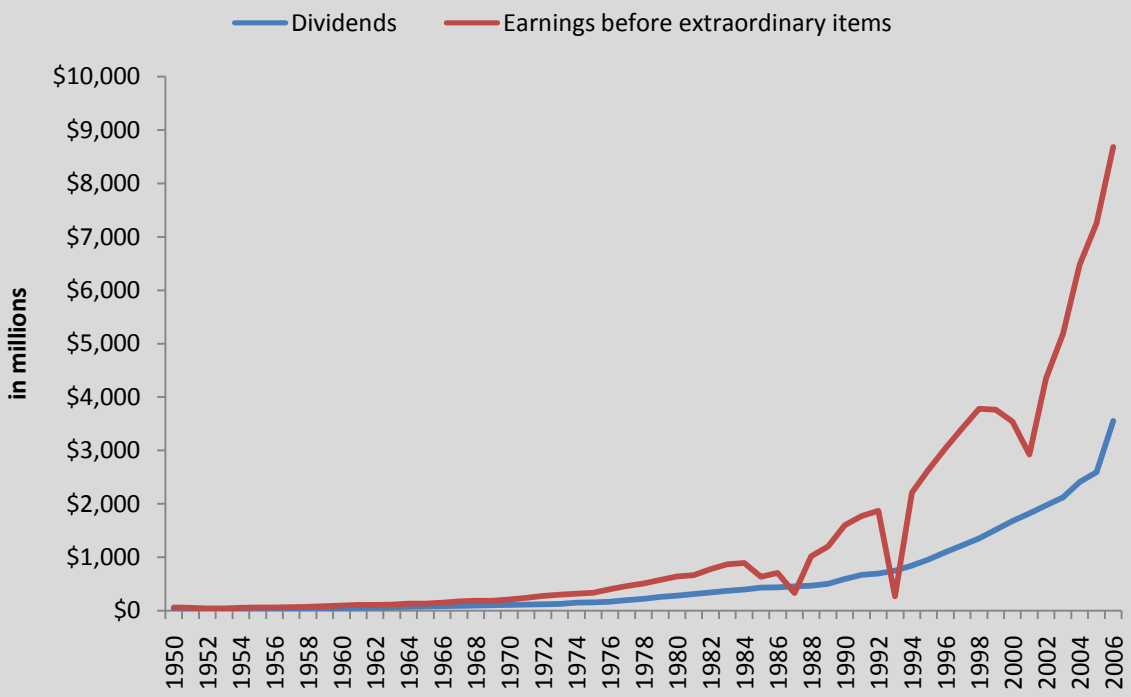
Solution

$$P_0 = \frac{D_0(1+g)}{r-g} = \frac{\$1}{0.08} = \$12.50$$

Do companies actually have dividends that tend to grow at a constant rate? Many mature companies will have dividends that grow at a constant rate through time. For example, Proctor and Gamble's dividends grow at a relatively constant rate, as shown in Exhibit 2. Companies that pay dividends generally do so such that the dividend does not decrease. This is because shareholders who receive dividend from a company expect the company to either maintain the current dividend per share or increase it. Companies that cut dividends generally experience a drop in stock price. Because of this, companies are hesitant to decrease dividends and will only raise them if they are confident that they can maintain the higher level of dividends.

Companies that pay dividends generally have policies with respect to the dividend per share: maintaining a constant or increasing dividend per share. Companies do not peg the dividend to earnings because this would result in dividends changing each period, with occasional decreases. As you can see with respect to Proctor & Gamble, the dividends and earnings follow a similar growth path, but earnings are more volatile.

EXHIBIT 2 **PROCTOR & GAMBLE'S DIVIDENDS, 1950-2006**



3. Non-constant growth in dividends

Let's look at another situation, one in which growth is expected to *change* as time goes on. This is a common scenario because companies experience a life-cycle phenomena with rapid growth in the developing stage, slowing growth in the maturing stage, and possibly declining growth in the final stage of its existence. Further, companies may experience changes in their growth due to acquisitions and divestitures.

A. Two-stage dividend growth

Consider a share of common stock whose dividend is currently $2.00 per share and is expected to grow at a rate of 10 percent per year for two years and afterward at a rate of 4 percent per year. Assume a required rate of return of 6 percent. To tackle this problem, identify the cash flows for the first stage, calculate the price at the end of the first stage, and then assemble the pieces:

$$P_0 = \underbrace{\left[\frac{\$2(1+0.10)}{(1+0.06)^1} + \frac{\$2(1+0.10)^2}{(1+0.06)^2}\right]}_{\text{Present value of dividends}} + \underbrace{\frac{P_2}{(1+0.06)^2}}_{\substack{\text{Present value of price} \\ \text{at end of two years}}}$$

$$P_0 = \frac{\$2.20}{1.06} + \frac{\$2.42}{1.1236} + \frac{P_2}{(1+0.06)^2}$$

$$\text{where } P_2 = \frac{\$2.42(1.04)}{0.06\text{-}0.04} = \$125.84$$

$$P_0 = \frac{\$2.20}{1.06} + \frac{\$2.42}{1.1236} + \frac{\$125.84}{1.1236}$$

$$P_0 = \$2.0755 + 2.1538 + 112.00 = \underline{\underline{\$116.23}}$$

This is a **two-stage growth model**. You can see that it is similar to the dividend valuation model, but with a twist: the DVM is used to determine the price beyond which there is constant growth, but the dividends during the first growth period are discounted using basic cash flow discounting. You can see by the math that we could alter the calculations slightly to allow for, say, a three-stage growth model.

B. Three-stage dividend growth

Companies often experience a life-cycle with at least three stages that are defined with respect to growth: an early, development stage with high growth, a maturing phase with moderate growth, and a declining phase with little, no, or negative growth. This pattern encourages the use of a three-stage model for the valuation of a company's equity.

Consider the valuation of a stock that has a current dividend of $1.00 per share. Dividends are expected to grow at a rate of 15 percent for the next five years. Following that, the dividends are expected to grow at a rate of 10% for five years. After ten years, the dividends are expected to grow at a rate of 5% per year, forever. If the required rate of return is 10%, what is the value of a share of this stock?

We can break the calculation in to six steps:

❶ Calculate the dividends for years 1 through 11:[3]

Year	Dividend growth rate	Dividend
1	15%	$ 1.150
2	15%	$ 1.323
3	15%	$ 1.521
4	15%	$ 1.749
5	15%	$ 2.011
6	10%	$ 2.212
7	10%	$ 2.434
8	10%	$ 2.677
9	10%	$ 2.945
10	10%	$ 3.239
11	5%	$ 3.401

❷ Calculate the present value of each of these dividends for years 1 through 10:

Year	Dividend	Present value
1	$ 1.150	$1.045455
2	$ 1.323	$1.092975
3	$ 1.521	$1.142656
4	$ 1.749	$1.194595
5	$ 2.011	$1.248895
6	$ 2.212	$1.248895
7	$ 2.434	$1.248895
8	$ 2.677	$1.248895
9	$ 2.945	$1.248895
10	$ 3.239	$1.248895

❸ Calculate the present value of the dividends beyond year 10:

$$P_{10} = \frac{\$3.401}{(0.10 - 0.05)} = \$68.0225$$

❹ Calculate the present value of the price at year 10:

$$PV_{P_{10}} = \frac{\$68.0225}{(1 + 0.10)^{10}} = \$26.22562$$

❺ Calculate the sum of the present value of the dividends:

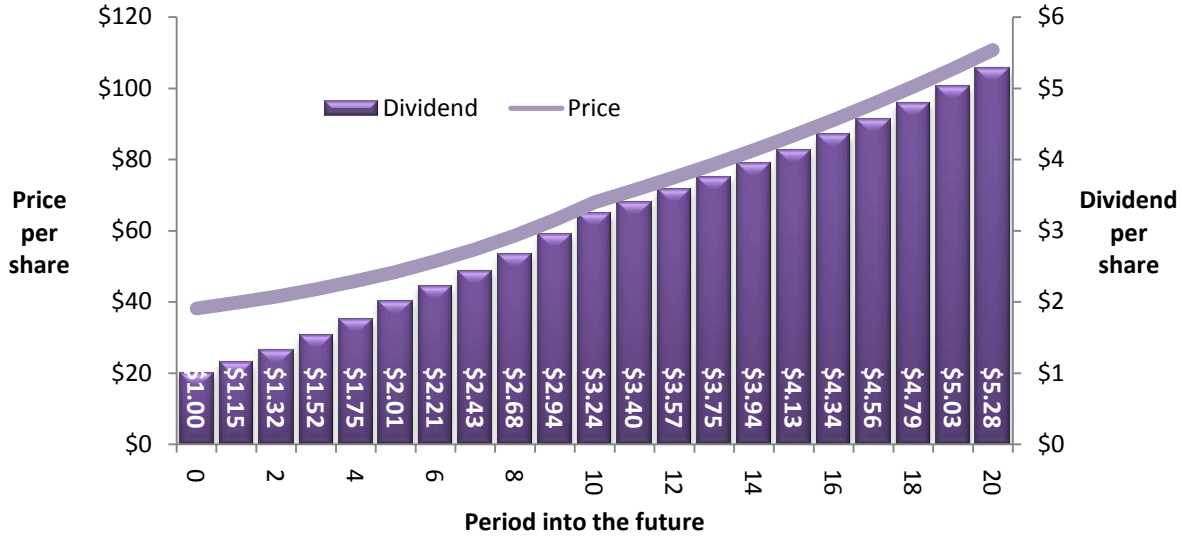
$$PV_{\text{dividends in year 1-10}} = \sum_{t=1}^{10} \frac{D_t}{(1 + 0.10)^t} = \$11.96905$$

❻ Calculate the price today as the sum of the present value of dividends in years 1-10 and the price at the end of year 10:

$$P_0 = \$26.22562 + 11.9690 = \$38.19582$$

[3] We need year 11's dividend because when we calculate the price of the stock at the end of the first two growth periods, we need to have the *next* year's dividend.

Graphical representation



C. The H-model

The single growth dividend model assumes a constant rate of growth, forever, whereas the two and three stage growth models assume that growth changes abruptly from one rate to another. Another form of the dividend valuation model is the H-model.[4] The **H-model** is used to value a stock when it is assumed that the dividend growth will change from one growth rate to another in a linear manner.

Let g_a represent the abnormal growth and let g_n represent the normal growth. Using the H-model, the growth rate will change from g_a to g_n over a period of years, 2H. If the period in which this transition takes place is 2H, the half-life of this transition is H. The formula is as follows:

$$V_0 = \underbrace{\frac{D_0 \times (1+g_n)}{r-g_n}}_{\text{Value from normal growth}} + \underbrace{\frac{D_0 \times H \times (g_a - g_n)}{(r-g_n)}}_{\text{Added value from abnormal growth}}$$

Consider a share of common stock whose dividend is currently \$2.00 per share and is expected to grow at a rate of 10 percent per year and after four years is at a rate of 4 percent per year. Assuming a required rate of return of 6 percent, the value of a share of stock is:

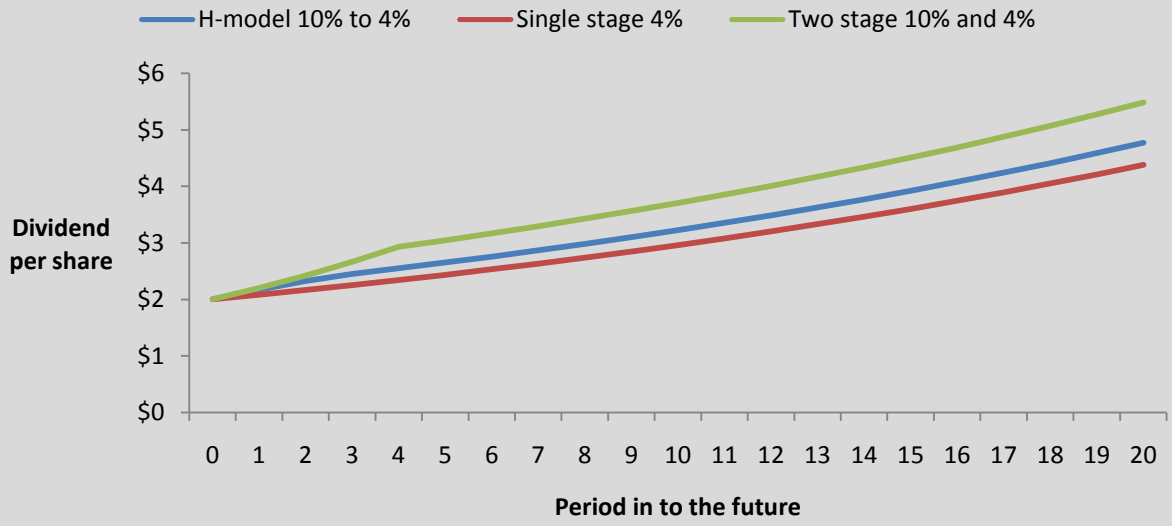
$$V_0 = \frac{\$2 \times (1+0.04)}{0.06-0.04} + \frac{\$2 \times 2 \times (0.1-0.04)}{(0.06-0.04)} = \$104 + 12 = \$\ 116$$

Therefore, \$104 of the current value is due to the underlying, normal growth, whereas \$12 is due to the abnormal growth.

[4] R. Fuller and C. C. Hsia, "A Simplified Common Stock Valuation Model," *Financial Analysts Journal* (September/October 1984) pp. 49-56.

Comparing the constant growth, two-stage growth, and H-model, we can see that these models produce different projections of dividends, which then translate in to different projections for the value of a share.

EXHIBIT 3 **COMPARISON OF DIVIDEND GROWTH WITH THE DIFFERENT MODELS**



4. The uses of the dividend valuation models

The dividend valuation model provides a device in which we can relate the value of a stock to fundamental characteristics of the company. One use is to associate the company's stock's price-to-earnings ratio to fundamental factor. The **price-earnings ratio** also known as the **price-to-earnings ratio** or **PE ratio**, is the ratio of the price per share to the earnings per share of a stock. We can relate this ratio to the company's dividend payout, expected growth, and the required rate of return. Let:

P_0 = today's price,
E_0 = current earnings per share,
D_0 = current dividend per share,
g = expected growth rate
r = required rate of return.

If we take the DVM and divide both sides by earnings per share, we arrive at an equation for the price-earnings ratio in terms of dividend payout, required rate of return, and growth:

$$\frac{P_0}{E_0} = \frac{D_0/E_0\,(1+g)}{r-g} = \frac{\text{Dividend payout ratio}\quad 1+g}{r-g}$$

This tells us that the PE ratio is

- directly related to the dividend payout [⇧ dividend payout → ⇧PE];
- inversely related to the required rate of return [⇧ r → ⇩PE]; and
- directly related to the rate of growth [⇧growth → ⇧PE].

We can also rearrange the DVM to solve for the required rate of return:

$$P_0 = \frac{D_1}{r - g} \rightarrow r = \frac{D_1}{P_0} + g$$

This tells us that the required rate of return is comprised of the **dividend yield** (that is, D_1/P_0) and the rate of growth (also referred to as the **capital yield**).

We can also use the dividend valuation model to relate the **price-book value ratio** (i.e., the ratio of the price per share to the book value per share) to factors such as the dividend payout ratio and the return on equity. First, we start with the DVM and make a substitution for the dividend payout ratio:

$$P_0 = \frac{D_0(1+g)}{r-g} = \frac{\left[\left(D_0 / E_0\right)E_0\right](1+g)}{r-g} \qquad \text{because } \left(D_0 / E_0\right)E_0 = D_0$$

Let B_0 indicate the current book value per share and let ROE_0 indicate the current return on book equity, calculated as the ratio of earnings to the book value of equity.

We know that $E_0 = B_0 \quad ROE_0$ because $ROE_0 = \frac{E_0}{B_0}$. Therefore,

$$P_0 = \frac{B_0 \quad ROE_0 \left(D_0 / E_0\right) 1+g}{r-g}$$

We can then relate the price of a stock to book value, the return on equity, the dividend payout, the required rate of return, and the growth rate:

Increase B_0	→	Increase P_0
Increase ROE_0	→	Increase P_0
Increase D_0/E_0	→	Increase P_0
Increase g	→	Increase P_0
Increase r	→	Decrease P_0

We can also relate the price-to-book ratio to the return on equity, the dividend payout, the required rate of return, and the growth rate:

$$\frac{P_0}{B_0} = \frac{ROE_0 \left(D_0 / E_0\right) 1+g}{r-g}$$

Increase ROE_0	→	Increase P_0/B_0
Increase D_0/E_0	→	Increase P_0/B_0
Increase g	→	Increase P_0/B_0
Increase r	→	Decrease P_0/B_0

In other words, we can use the dividend valuation model, along with our knowledge of financial relations (i.e., financial statements and financial ratios), to relate the stock's price and price multiples to fundamental factors.

5. Stock valuation and market efficiency

The theories of stock valuation are an expression of the belief that what rational investors will pay for a stock is related to what they expect to get from the stock in the future, in terms of cash flows, and the uncertainty related to these cash flows. Does this really work? Is the stock price really related to what we view to be a stock's intrinsic value?

Basically, yes. But in reality, stock valuation is not as simple as it looks from the models we've discussed:

- How do you deal with dividends that do not grow at a constant rate?
- What if the firm does not pay dividends now?

The DVM doesn't apply in the case when dividends do not grow at a constant rate (or at least in stages) or in the case when the company does not pay dividends. In those cases, we need to resort to other models, such as the valuing free cash flows or valuing residual income.

Valuation is the process of determining what something is worth at a point in time. When we value investments, we want to estimate the future cash flows from these investments and then discount these to the present. This process is based on the reasoning that no one will pay more today for an investment than what they could expect to get from that investment on a time and risk adjusted basis.

If a market is efficient, this means that the price today reflects all available information. This information concerns future cash flows and their risk. The price that is determined at any point in time is affected by the *marginal* investor – the one willing to pay the most for that stock. As information reaches the market that affects future cash flows or the discount rate that applies to these cash flows, the price of a stock will change. Will it change immediately to the "correct" valuation? For the most part. The more complex the information and valuation of the information, the more time it takes for the market to digest the information and the stock to be properly valued. For well-known companies, a given piece of material information will be reflected in the stock's price within fifteen minutes – too late for the individual investor to react to it.

The implication of efficient markets is that technical analysis will not be profitable. It also means that fundamental analysis, while valuable in terms of evaluating future cash flows, assessing risk, and assisting in the proper selection of investments for a portfolio, will not produce abnormal returns – it will simply produce returns commensurate with the risk assumed. We can see this with mutual funds. We assume that the fund managers have adequate access to all publicly available fundamental information. However, these fund managers cannot outperform random stock picks. Even the most sophisticated fundamental analysis cannot generate abnormal returns.

6. Summary

- The dividend valuation model is a useful model when valuing a company that pays a dividend and for which the dividend pattern is estimable.
- The basic premise behind the dividend valuation model is that the value of a stock today is the present value of all future dividends. This model can be applied whether or not the company currently pays a dividend, but if the company does not currently pay a dividend the analyst should be able to estimate the future dividends in amount and timing.
- The dividend valuation model can be modified for patterns including a constant dividend, a constantly growing dividend, and a dividend that grows at different rate depending on the period in the future.
- The dividend valuation model can be related to fundamental factors that drive the value of a company's equity, including the return on equity and the dividend payout.

7. Index

8. Further readings

- Dividend Discount Model, by John Del Vecchio for the Motley Fool
- Dividend Discount Models, by Aswath Damodoran, New York University