Time value of money formulas
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1. Time value of a lump-sum

A. Discrete compounding

\[ FV = PV (1 + i)^n \]

where \( FV \) is the future value, \( PV \) is the present value, \( i \) is the rate of interest, and \( n \) is the number of compounding periods.

\[ PV = \frac{1}{(1 + i)^n} \]

\[ i = \left( \frac{FV}{PV} \right)^{\frac{1}{n}} - 1 \]

\[ n = \frac{\ln(FV - \ln PV)}{\ln (1+i)} \]

B. Continuous compounding

\[ FV = PV e^{\text{APR} \times x} \]

where \( x \) is the number of years, \( \text{APR} \) is the annual percentage rate, \( e \) is Euler’s e.

\[ PV = \frac{FV}{e^{\text{APR} \times x}} \]

2. Time value of annuities

A. Ordinary annuity

\[ FV = \sum_{t=1}^{N} CF(1+i)^{N-t} = CF \sum_{t=1}^{N} (1+i)^{N-t} \]

\[ PV = \sum_{t=1}^{N} \frac{CF}{(1+i)^t} = CF \sum_{t=1}^{N} \frac{1}{(1+i)^t} = CF \left( 1 - \frac{(1+i)^N}{i} \right) \]

where \( \sum_{t=1}^{N} \frac{1}{(1+i)^t} \) is the annuity discount factor; and
\[
\sum_{t=1}^{N} (1+i)^{N-t} \quad \text{is the annuity compound factor.}
\]

B. **Annuity due**

\[
FV = CF(1 + i)^1 + CF(1 + i)^2 + \ldots + CF(1 + i)^{N+1} = CF \left( \sum_{t=0}^{N} (1+i)^{t+1} \right)
\]

\[
PV = \sum_{t=1}^{N} \frac{CF}{(1+i)^{t-1}} = CF \sum_{t=1}^{N} \frac{1}{(1+i)^{t-1}}
\]

C. **Perpetuity**

\[
PV = \frac{CF}{i}
\]

3. **Determining annual interest rates**

A. **Discrete compounding**

\[
APR = i \times n
\]

\[
EAR = (1 + i)^n - 1 = (1 + \frac{APR}{n})^n - 1
\]

B. **Continuous compounding**

\[
EAR = e^{APR} - 1
\]