Solutions to Time value of money practice problems

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1. What is the balance in an account at the end of 10 years if $2,500 is deposited today and the account earns 4% interest, compounded annually? quarterly?

   Annual compounding:
   FV = $2,500 $(1 + 0.04)^{10}$ = $2,500 (1.4802) = **$3,700.61**
   Quarterly compounding:
   FV = $2,500 $(1 + 0.01)^{40}$ = $2,500 (1.4889) = **$3,722.16**

2. If you deposit $10 in an account that pays 5% interest, compounded annually, how much will you have at the end of 10 years? 50 years? 100 years?

   10 years: FV = $10 $(1+0.05)^{10}$ = $10 (1.6289) = **$16.29**
   50 years: FV = $10 $(1 + 0.05)^{50}$ = $10 (11.4674) = **$114.67**
   100 years: FV = $10 $(1 + 0.05)^{100}$ = $10 (131.50) = **$1,315.01**

3. How much interest on interest is earned in an account by the end of 5 years if $100,000 is deposited and interest is 4% per year, compounded continuously?

   Note: Interest on interest is the difference between the future value calculated using compounded interest and the future value calculated using simple interest, because simple interest includes only interest on the principal amount, not the interest-on-interest.

   Continuously compounded:
   FV = $100,000 $e^{0.04(5)}$ = $100,000 (1.2214) = **$122,140.28**

   Simple interest:
   FV = $100,000 + [$100,000(0.04)(5)] = $100,000 + 20,000 = **$120,000**

   Interest on interest = $122,140.28 = $120,000 = **$2,140.28**

4. How much will be in an account at the end of five years the amount deposited today is $10,000 and interest is 8% per year, compounded semi-annually?

   FV = $10,000 (1+0.04)10 = $10,000 (1.4802) = **$14,802.44**

5. Complete the following, solving for the present value, PV:

Case	Future value	Interest rate	Number of periods	Present value
A	$10,000	5%	5	**$7,835.26**
B	$563,000	4%	20	**$256,945.85**
C	$5,000	5.5%	3	**$4,258.07**

6. Suppose you want to have $0.5 million saved by the time you reach age 30 and suppose that you are 20 years old today. If you can earn 5% on your funds, how much would you have to invest today to reach your goal?

Given: FV = $500,000; i = 5%; n = 10
PV = $500,000 (1 / (1 + 0.05)^{10}) = $500,000 (0.6139) = **$306,959.63**

7. How much would I have to deposit in an account today that pays 12% interest, compounded quarterly, so that I have a balance of $20,000 in the account at the end of 10 years?

Given: FV = $20,000; i = 12%/4 = 3%; n = 10 x 4 = 40 quarters
PV = **$6,131.14**

8. Suppose I want to be able to withdraw $5,000 at the end of five years and withdraw $6,000 at the end of six years, leaving a zero balance in the account after the last withdrawal. If I can earn 5% on my balances, how much must I deposit today to satisfy my withdrawals needs?

Given: Hint -- There are two different future values. Treat as two separate present values, then combine.

FV = $5,000; n = 5, i = 5%
PV = $3,917.63

FV = $6,000; n = 6, i = 5%
PV = $4,477.29
PV of the two future values = $3,917.63 + 4,477.29 = **$8,394.92**

Or, can use the NPV function in a financial calculator:
- In the TI-83/84, the cash flows are {0,0,0,0,5000,5000}
- In the HP10B, the cash flows are 0,0,0,0,0,5000,5000

9. Consider a loan of $1 million that is paid off quarterly over a period of nine years. Calculate the dollar amount of interest and loan principle repaid corresponding to each payment if the interest rate is 6% per year, compounded quarterly.

Year	Quarter	Beginning balance	Payment	Interest	Principal repayment	Remaining principal
	0					$ 1,000,000.00
	1	$ 1,000,000.00	$ 36,152.40	$ 15,000.00	$ 21,152.40	$ 978,847.60
	2	$ 978,847.60	$ 36,152.40	$ 14,682.71	$ 21,469.68	$ 957,377.92
	3	$ 957,377.92	$ 36,152.40	$ 14,360.67	$ 21,791.73	$ 935,586.20
1	4	$ 935,586.20	$ 36,152.40	$ 14,033.79	$ 22,118.60	$ 913,467.59
	5	$ 913,467.59	$ 36,152.40	$ 13,702.01	$ 22,450.38	$ 891,017.21
	6	$ 891,017.21	$ 36,152.40	$ 13,365.26	$ 22,787.14	$ 868,230.07
	7	$ 868,230.07	$ 36,152.40	$ 13,023.45	$ 23,128.94	$ 845,101.13
2	8	$ 845,101.13	$ 36,152.40	$ 12,676.52	$ 23,475.88	$ 821,625.25
	9	$ 821,625.25	$ 36,152.40	$ 12,324.38	$ 23,828.02	$ 797,797.23
	10	$ 797,797.23	$ 36,152.40	$ 11,966.96	$ 24,185.44	$ 773,611.80
	11	$ 773,611.80	$ 36,152.40	$ 11,604.18	$ 24,548.22	$ 749,063.58
3	12	$ 749,063.58	$ 36,152.40	$ 11,235.95	$ 24,916.44	$ 724,147.14
	13	$ 724,147.14	$ 36,152.40	$ 10,862.21	$ 25,290.19	$ 698,856.95
	14	$ 698,856.95	$ 36,152.40	$ 10,482.85	$ 25,669.54	$ 673,187.41
	15	$ 673,187.41	$ 36,152.40	$ 10,097.81	$ 26,054.58	$ 647,132.82
4	16	$ 647,132.82	$ 36,152.40	$ 9,706.99	$ 26,445.40	$ 620,687.42
	17	$ 620,687.42	$ 36,152.40	$ 9,310.31	$ 26,842.08	$ 593,845.34
	18	$ 593,845.34	$ 36,152.40	$ 8,907.68	$ 27,244.72	$ 566,600.62

	19	$ 566,600.62	$ 36,152.40	$ 8,499.01	$ 27,653.39	$ 538,947.23
5	20	$ 538,947.23	$ 36,152.40	$ 8,084.21	$ 28,068.19	$ 510,879.05
	21	$ 510,879.05	$ 36,152.40	$ 7,663.19	$ 28,489.21	$ 482,389.84
	22	$ 482,389.84	$ 36,152.40	$ 7,235.85	$ 28,916.55	$ 453,473.29
	23	$ 453,473.29	$ 36,152.40	$ 6,802.10	$ 29,350.30	$ 424,122.99
6	24	$ 424,122.99	$ 36,152.40	$ 6,361.84	$ 29,790.55	$ 394,332.44
	25	$ 394,332.44	$ 36,152.40	$ 5,914.99	$ 30,237.41	$ 364,095.03
	26	$ 364,095.03	$ 36,152.40	$ 5,461.43	$ 30,690.97	$ 333,404.06
	27	$ 333,404.06	$ 36,152.40	$ 5,001.06	$ 31,151.33	$ 302,252.73
7	28	$ 302,252.73	$ 36,152.40	$ 4,533.79	$ 31,618.60	$ 270,634.12
	29	$ 270,634.12	$ 36,152.40	$ 4,059.51	$ 32,092.88	$ 238,541.24
	30	$ 238,541.24	$ 36,152.40	$ 3,578.12	$ 32,574.28	$ 205,966.96
	31	$ 205,966.96	$ 36,152.40	$ 3,089.50	$ 33,062.89	$ 172,904.07
8	32	$ 172,904.07	$ 36,152.40	$ 2,593.56	$ 33,558.83	$ 139,345.24
	33	$ 139,345.24	$ 36,152.40	$ 2,090.18	$ 34,062.22	$ 105,283.02
	34	$ 105,283.02	$ 36,152.40	$ 1,579.25	$ 34,573.15	$ 70,709.87
	35	$ 70,709.87	$ 36,152.40	$ 1,060.65	$ 35,091.75	$ 35,618.12
9	36	$ 35,618.12	$ 36,152.40	$ 534.27	$ 35,618.12	$ (0.00)

10. Suppose you deposit $100,000 in an account today that pays 6% interest, compounded annually. How long does it take before the balance in your account is $500,000?

    Given: I = 6%; PV = $100,000; FV = $500,000
    Solution: N = **28**

11. The Lucky Loan Company will lend you $10,000 today with terms that require you to pay off the loan in thirty-six monthly installments of $500 each. What is the effective annual rate of interest that the Lucky Loan Company is charging you?

    Given: PV = $10,000; N = 36; PMT = 500
    Solve for i: i = 3.6007%
    EAR = $(1 + 0.036007)^{12} - 1$ = **52.8806%**

12. How long does it take for your money to grow to ten times its original value if the interest rate of 5% per year?

    Given: PV = 1; FV = 10; I = 5%
    Solution: **48 years**

13. Under what conditions does the effective annual rate of interest (EAR) differ from the annual percentage rate (APR)?

    If interest is compounded more frequently than once a year, the EAR will be different than the APR; the EAR will be greater than the APR except in the case in which there is annual compounding (in which case the EAR will be equal to the APR)

14. As the frequency of compounding increases within the annual period, what happens to the relation between the EAR and the APR?

    The EAR will become larger than the APR as the frequency of compounding increases. The largest difference between the two is in the case in which interest is compounded continuously.

15. If interest is paid at a rate of 5% per year, compounded quarterly, what is the:

    a) annual percentage rate?

APR = **5%**

b) effective annual rate?

EAR = $(1 + 0.0125)^4 - 1$ = **5.0945%**

16. L. Shark is willing to lend you $10,000 for three months. At the end of six months, L. Shark requires you to repay the $10,000, plus 50%.

a) What is the length of the compounding period?

Six months

b) What is the rate of interest per compounding period?

50%

c) What is the annual percentage rate associated with L. Shark's lending activities?

APR = 50% x 2 = **100%**

d) What is the effective annual rate of interest associated with L. Shark's lending activities?

EAR = $(1 + 0.50)^2 - 1$ = **125%**

17. The Consistent Savings and Loan is designing a new account that pays interest quarterly. They wish to pay, effectively, 16% per year on this account. Consistent desires to advertise the annual percentage rate on this new account, instead of the effective rate, since its competitors state their interest on an annualized basis. What is the APR that corresponds to an effective rate of 16% for this new account?

EAR = 16%
APR = $1.16^{0.25}$ *this takes the fourth root of 1 + EAR*
i = 3.78%
APR = 3.78% x 4 = **15.121%**

18. Consider an annuity consisting of three cash flows of $2,000 each. Assume a 4% interest rate. What is the present value of the annuity if the first cash flow occurs:

a) today.

PV of annuity due = **$5,772.19**

b) one year from today.

PV of ordinary annuity = **$5,550.18**

c) two years from today.

PV of a deferred annuity = $5,550.18 / 1.04 = **$5,336.71**

d) three years from today.

PV of a deferred annuity = $5,550.18 / 1.04^2 = **$5,131.45**

e) four years from today.

PV of a deferred annuity = $5,550.18 / 1.04^3 = **$4,934.09**

f) five years from today.

PV of a deferred annuity = \$5,550.18 / 1.04^4 = **\$4,744.32**

19. Suppose you wish to retire forty years from today. You determine that you need \$50,000 per year once you retire, with the first retirement funds withdrawn one year from the day you retire. You estimate that you will earn 6% per year on your retirement funds and that you will need funds up to and including your 25th birthday after retirement.

a) How much must you deposit in an account today so that you have enough funds for retirement?

PV_{retire} = \$50,000 (PV annuity factor, N=25 and i=6%)
Given: PMT = \$50,000; N = 25; I = 6%; Solve for PV
PV_{retire} = \$639.167.81

Given: FV = \$639,167.81; N = 40; I = 6%; Solve for PV
PV_{today} = \$639,167.81 / $(1 + 0.06)^{40}$ = **\$62,141.29**

b) How much must you deposit each year in an account, starting one year from today, so that you have enough funds for retirement?

PV_{retire} = \$50,000 (PV annuity factor, N=25 and i=6%)
Given: PMT = \$50,000; N = 25; I = 6%; Solve for PV
PV_{retire} = \$639.167.81

Given: FV = \$639,167.81; N = 40; I = 6%; Solve for PMT
PMT = \$4,130.01

20. Using an interest rate of 5% per year, what is the value today of the following cash flows:

Years from today	Cash flow
1	£ 0
2	£ 0
3	£ 10,000
4	£ 10,000

FV = \$8,638.38 + 8,227.02 = **\$16,865.40**

Note: Cash flow list: {0,0,10000,10000}