1. Return and risk

When we make financial decisions, we must evaluate the benefits and the costs associated with the
decision. When we assess the benefits and costs, however, we must also understand the uncertainty
associated with the possible outcomes of the investments. For example, in evaluating a new product, the
company will use its knowledge of marketing to estimate the expected annual sales for each of the next
few years.

While we typically think of forecasts as point estimates, there really is a distribution about those
estimates. If next year’s sales are estimated to be $100 million, there is a chance that sales will be more
or less than this point estimate. Therefore, forecasts really entail looking at the entire probability
distribution; that is, all the possible sales and their associated probabilities. The purpose of this reading
is to develop a method of quantifying this probability distribution to make it easier to incorporate risk into
the decision-making process.

2. Expected return

We refer to both future benefits and future costs as expected returns. *Expected returns* are a measure
of the tendency of returns on an investment. This doesn't mean that these are the only returns possible,
just our best measure of what we expect.

Suppose we are evaluating the investment in a new product. We do not know and cannot know precisely
what the future cash flows will be. But from past experience, we can at least get an idea of possible
flows and the likelihood -- the probability -- they will occur. After consulting with colleagues in marketing
and production management, we figure out that there are two possible cash flow outcomes, success or
failure, and the probability of each outcome. Next, consulting with colleagues in production and
marketing for sales prices, sales volume, and production costs, we develop the following possible cash
flows in the first year:

<table>
<thead>
<tr>
<th>Scenario</th>
<th>Cash flow</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>Product success</td>
<td>$4,000,000</td>
<td>40%</td>
</tr>
<tr>
<td>Product flop</td>
<td>-2,000,000</td>
<td>60%</td>
</tr>
</tbody>
</table>

But what is the expected cash flow in the first year? The expected cash flow is the average of the
possible cash flows, weighted by their probabilities of occurring:

\[
\text{Expected cash flow} = 0.40 \times ($4,000,000) + 0.60 \times (-$2,000,000)
\]
The expected cash flow is $400,000.

The expected value is a guess about the future outcome. It is not necessarily the most likely outcome. The most likely outcome is the one with the highest probability. In the case of our example, the most likely outcome is $-2,000,000.

A general formula for any expected value is:

\[ \text{Expected value} = \mathbb{E}(X) = p_1x_1 + p_2x_2 + p_3x_3 + \ldots + p_Nx_N. \]

where \( \mathbb{E}(X) \) is the expected value; 
\( n \) indicates a possible outcome; 
\( N \) is the number of possible outcomes; 
\( p_n \) is the probability of the \( n \)th outcome; and 
\( x_n \) is the value of the \( n \)th outcome.

We can abbreviate this formula by using summation notation:

\[ \text{Expected value} = \mathbb{E}(X) = \sum_{n=1}^{N} p_n x_n. \]

Applying the general formula to our example,

\[ N = 2 \text{ (there are two possible outcomes)} \]
\[ p_1 = 0.40 \quad \text{Must sum to 1.0 or 100 percent} \]
\[ p_2 = 0.60 \]
\[ x_1 = $4,000,000 \]
\[ x_2 = -$2,000,000 \]

\[ \text{E(cash flow)} = \sum_{n=1}^{2} p_n x_n = p_1 x_1 + p_2 x_2 \]
\[ \text{E(cash flow)} = 0.40 ($4,000,000) + 0.60 (-$2,000,000) = $400,000. \]

Considering the possible outcomes and their likelihoods (i.e., probabilities), we expect a $400,000 cash flow.

The calculation of the expected value requires that all possible outcomes be included. Therefore, the probabilities (the \( p_i \)'s) must sum to 1.00 or 100 percent -- if not, you have left out a possible outcome:

\[ \sum_{n=1}^{N} p_n = 1.00 \text{ or 100 percent.} \]

3. **Standard deviation of the possible outcomes**

The expected return gives us an idea of the tendency of the future outcomes -- what we expect to happen, considering all the possibilities. But the expected return is a single value and does not tell us anything about the diversity of the possible outcomes. Are the possible outcomes close to the expected value? Are the possible outcomes much different than the expected value? Just how much uncertainty is
there about the future? Because we are concerned about the degree of uncertainty (risk), as well as the expected return, we need some way of quantifying the risk associated with decisions.

Suppose we are considering two products, Product A and Product B, with estimated returns under different scenarios and their associated probabilities:

<table>
<thead>
<tr>
<th>Scenario</th>
<th>Probability of possible outcome</th>
<th>Outcome</th>
<th>Probability of possible outcome</th>
<th>Outcome</th>
</tr>
</thead>
<tbody>
<tr>
<td>Success</td>
<td>25 percent</td>
<td>24 percent</td>
<td>Success</td>
<td>10 percent</td>
</tr>
<tr>
<td>Moderate success</td>
<td>50 percent</td>
<td>10 percent</td>
<td>Moderate success</td>
<td>30 percent</td>
</tr>
<tr>
<td>Failure</td>
<td>25 percent</td>
<td>-4 percent</td>
<td>Failure</td>
<td>60 percent</td>
</tr>
</tbody>
</table>

We refer to a product's set of the possible outcomes and their respective probabilities as the probability distribution for those outcomes.

We can calculate the expected cash flow for each product as follows:

Product A

<table>
<thead>
<tr>
<th>Return</th>
<th>Probability</th>
<th>( p_n )</th>
<th>( p_n x_n )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Failure</td>
<td>-4%</td>
<td>25%</td>
<td>-1%</td>
</tr>
<tr>
<td>Moderate success</td>
<td>10%</td>
<td>50%</td>
<td>5%</td>
</tr>
<tr>
<td>Success</td>
<td>24%</td>
<td>25%</td>
<td>6%</td>
</tr>
</tbody>
</table>

\[ E(x) = 10\% \]

Product B

<table>
<thead>
<tr>
<th>Return</th>
<th>Probability</th>
<th>( p_n )</th>
<th>( p_n x_n )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Failure</td>
<td>-5%</td>
<td>60%</td>
<td>-3%</td>
</tr>
<tr>
<td>Moderate success</td>
<td>30%</td>
<td>30%</td>
<td>9%</td>
</tr>
<tr>
<td>Success</td>
<td>40%</td>
<td>10%</td>
<td>4%</td>
</tr>
</tbody>
</table>

\[ E(x) = 10\% \]

Both Product A and Product B have the same expected return. Let's now see if there is any difference in the possible outcomes for the two products.

The possible returns for Product A range from -4 percent to 24 percent, whereas the possible returns for Product B range from -5 percent to 40 percent. The range is the span of possible outcomes. For Product A the span is 4 – 24 percent = 28 percent; for Product B the span is 45 percent. A wider span indicates more risk, so Product B has more risk than Product A.

If we represent graphically the possible cash flow outcomes for Products A and B, with their corresponding probabilities, as in Exhibit 1, we see there is more dispersion of possible outcomes with Product B -- they are more spread out -- than those of Product A.
But the range by itself doesn't tell us much about the possible cash flows at these extremes, nor within the extremes. And the range does not tell us anything about the probabilities at or within the extremes.

A measure of risk that does tell us something about how much to expect and the probability that it will happen is the standard deviation. The **standard deviation** is a measure of dispersion that considers the values and probabilities for each possible outcome. The larger the standard deviation, the greater the deviation of possible outcomes from the expected value. The standard deviation considers the deviation, or distance, of each possible outcome from the expected value and the probability associated with it.

The standard deviation of possible returns, represented by \( \sigma(x) \), is calculated in six steps:

1. **Step 1**: Calculate the expected value.
2. **Step 2**: Calculate the deviation of each possible outcome from the expected value.
3. **Step 3**: Square each deviation.
4. **Step 4**: Weight each squared deviation, multiplying it by the probability of the outcome.
5. **Step 5**: Sum these weighted squared deviations.
6. **Step 6**: Take the square root of the sum of the squared deviations.

Let's calculate the standard deviation of the expected cash flows for Product A:

<table>
<thead>
<tr>
<th>Return</th>
<th>Probability</th>
<th>( p_n )</th>
<th>( x_n - \bar{x} )</th>
<th>( (x_n - \bar{x})^2 )</th>
<th>( p_n (x_n - \bar{x})^2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Failure</td>
<td>-4%</td>
<td>25%</td>
<td>-1%</td>
<td>-0.14</td>
<td>0.0196</td>
</tr>
<tr>
<td>Moderate</td>
<td>10%</td>
<td>50%</td>
<td>5%</td>
<td>0.00</td>
<td>0.00000</td>
</tr>
<tr>
<td>Success</td>
<td>24%</td>
<td>25%</td>
<td>6%</td>
<td>0.14</td>
<td>0.0196</td>
</tr>
</tbody>
</table>

\[ \bar{x} = 10\% \]

\[ \sigma^2 = 0.000980 \]

\[ \sigma = 0.098995 \text{ or } 9.8995\% \]

We can represent these six steps in a single formula:

\[
\text{Standard deviation of possible outcomes} = \sigma(x) = \sqrt{\sum_{n=1}^{N} p_n (x_n - \bar{x})^2}
\]

For Product B,
Return Probability \( p_n \) \( x_n \) \( E(x) \) \( (x_n - E(x))^2 \) \( p_n(x_n - E(x))^2 \)

<table>
<thead>
<tr>
<th>Status</th>
<th>Probability</th>
<th>Return</th>
<th>( E(x) )</th>
<th>( (x_n - E(x))^2 )</th>
<th>( p_n(x_n - E(x))^2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Failure</td>
<td>-5%</td>
<td>60%</td>
<td>-3%</td>
<td>-0.15</td>
<td>0.0225</td>
</tr>
<tr>
<td>Moderate</td>
<td>30%</td>
<td>30%</td>
<td>9%</td>
<td>0.20</td>
<td>0.0400</td>
</tr>
<tr>
<td>Success</td>
<td>40%</td>
<td>10%</td>
<td>4%</td>
<td>0.30</td>
<td>0.0900</td>
</tr>
</tbody>
</table>

\[ E(x) = 10\% \]

\[ \sigma^2 = 0.03450 \]

\[ \sigma = 0.185742 \text{ or } 18.5742\% \]

In summary,

<table>
<thead>
<tr>
<th>Product</th>
<th>Expected return</th>
<th>Standard deviation of possible outcomes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Product A</td>
<td>10%</td>
<td>9.90%</td>
</tr>
<tr>
<td>Product B</td>
<td>10%</td>
<td>18.57%</td>
</tr>
</tbody>
</table>

Whereas the expected value of both products is the same, there is a different distribution of possible outcomes for the two products. When we calculate the standard deviation around the expected value, we see that Product B has a larger standard deviation. The larger standard deviation for Product B tells us that Product B has more risk than Product A since its possible outcomes are more distant more from its expected value.

### The variance and the standard deviation

The variance and the standard deviation are both measures of dispersion. In fact, they are related: the standard deviation is the square root of the variance. So why do we go beyond the calculation of the variance to get the standard deviation? For two reasons.

First, the variance is in terms of squared units of measure (say, squared dollars or squared returns), whereas the standard deviation is in terms of the original unit of measure. It gets tough trying to interpret squared dollars or squared returns.

Second, if the probability distribution is approximately normally distributed (that is, bell-shaped, with certain other characteristics), we can use the standard deviation to compactly describe the probability distribution; not so with the variance. There are uses for the variance in statistical analysis, but for purposes of describing and comparing probability distributions, we focus on the expected value and the standard deviation.

If we are comparing investments with different expected values, we can restate the risk as the coefficient of variation, which is the ratio of the standard deviation to the expected value, and make comparisons. The coefficient of variation is not useful in a comparison, however, in cases in which the expected return is zero or negative.

### Try it! Standard deviation of a probability distribution

Consider the following probability distribution

<table>
<thead>
<tr>
<th>Scenario</th>
<th>Probability</th>
<th>Net income</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>20 percent</td>
<td>$1 million</td>
</tr>
<tr>
<td>B</td>
<td>30 percent</td>
<td>$2 million</td>
</tr>
<tr>
<td>C</td>
<td>40 percent</td>
<td>$3 million</td>
</tr>
<tr>
<td>D</td>
<td>10 percent</td>
<td>$4 million</td>
</tr>
</tbody>
</table>

What is the expected net income and standard deviation of the net income given this information?
4. **Summary**

Evaluating risk and its effect on financial decisions is challenging. The purpose of demonstrating one method of evaluating risk in investment decision-making is to illustrate the relation between risk and return. The expected return on an investment is a point estimate. The risk is related to the possible deviation of the possible returns from what is expected. If we can quantify the possible outcomes from and investment and their associated probabilities, we can estimate a measure of risk, the standard deviation. We can then use the expected return and standard deviation in our decision-making.

5. **Solutions to Try it!**

**Standard deviation of a probability distribution**

<table>
<thead>
<tr>
<th>Scenario</th>
<th>Probability</th>
<th>Net income</th>
<th>P x in millions</th>
<th>(x- ( \bar{x} ))^2</th>
<th>p(x- ( \bar{x} ))^2</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>20 percent</td>
<td>$1 million</td>
<td>$0.2</td>
<td>1.96</td>
<td>0.392</td>
</tr>
<tr>
<td>B</td>
<td>30 percent</td>
<td>$2 million</td>
<td>0.6</td>
<td>0.16</td>
<td>0.048</td>
</tr>
<tr>
<td>C</td>
<td>40 percent</td>
<td>$3 million</td>
<td>1.2</td>
<td>0.36</td>
<td>0.144</td>
</tr>
<tr>
<td>D</td>
<td>10 percent</td>
<td>$4 million</td>
<td>0.4</td>
<td>2.56</td>
<td>0.256</td>
</tr>
</tbody>
</table>

Expected value = $2.4 million

Standard deviation = \( \sqrt{0.84} \) = $0.9165 million

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