Types of risk

1. Distinguish between sales risk and operating risk. Can firm have a high degree of sales risk and a low degree of operating risk? Explain.

   Sales risk is the uncertainty regarding the number of units sold and the price per unit. This risk is affected by economic and market conditions. Operating risk is the uncertainty in operating earnings arising from the mix of variable and fixed operating costs. A firm can have a great deal of sales risk (e.g., a very competitive industry) and yet have low operating risk because of their operating cost structure.

2. Consider two bonds. Bond A has a face value of $1,000 and a coupon rate of 10%. Bond B has a face value of $1,000 and a coupon rate of 5%. Both bonds have the same maturity. Which bond has the greater interest rate risk?

   Bond B because it has the lower coupon rate.

3. Consider two bonds. Bond C has a face value of $1,000 and five years remaining to maturity. Bond D has a face value of $1,000 and ten years remaining to maturity. Both bonds have the same coupon rate of 10%. Which bond has the greater interest rate risk?

   Bond D because it has the longer maturity.

4. Consider the Gum Company. Gum sells packs of gum for $0.50 each. It costs $0.20 per pack to manufacture and distribute the gum. Gum has fixed operating costs of $5,000 and fixed financing costs of $3,000.

   a. What is Gum's degree of operating leverage at 50,000 packs produced and sold?

      \[
      \text{DOL} = \frac{50,000 \times (0.50 - 0.20)}{50,000 \times (0.50 - 0.20) - 5,000} = 1.5
      \]

   b. What is Gum's degree of financial leverage at 50,000 packs produced and sold?

      \[
      \text{DFL} = \frac{(50,000 \times (0.50 - 0.20) - 5,000)}{(50,000 \times (0.50 - 0.20) - 5,000 - 3,000)} = 1.42857
      \]

   c. What is Gum's degree of total leverage at 50,000 packs produced and sold?

      \[
      \text{DTL} = 1.5 \times 1.42857 = 2.142857
      \]
Risk measurement

1. For each of the following probability distributions, calculate the expected value and standard deviation:

a. | Outcome | Probability | Outcome value | px | x-E(x) | (x-E(x))^2 | p(x-E(x))^2 |
---|---|---|---|---|---|---|
| Good | 30% | $40 | $12 | $16 | $256 | 77 |
| Normal | 50% | $20 | $10 | -$4 | $16 | 8 |
| Bad | 20% | $10 | $2 | -$14 | $196 | 39 |
| 100% | E(x) = $24 | variance = 124 | standard deviation = $11 |

b. | Outcome | Probability | Outcome value | px | x-E(x) | (x-E(x))^2 | p(x-E(x))^2 |
---|---|---|---|---|---|---|---|
| Pessimistic | 10% | $1,000,000 | $100,000 | -$3,700,000 | $13,690,000,000,000 | 1,369,000,000,000 |
| Moderate | 40% | $4,000,000 | $1,600,000 | -$700,000 | $490,000,000,000,000 | 196,000,000,000,000 |
| Optimistic | 50% | $6,000,000 | $3,000,000 | $1,300,000 | $1,690,000,000,000,000 | 845,000,000,000,000 |
| 100% | E(x) = $4,700,000 | variance = 2,410,000,000,000 | standard deviation = $1,552,417 |

c. | Outcome | Probability | Outcome value | px | x-E(x) | (x-E(x))^2 | p(x-E(x))^2 |
---|---|---|---|---|---|---|---|
| One | 10% | 60% | 0.060000 | 0.320000 | 0.102400 | 0.010240 |
| Two | 50% | 40% | 0.200000 | 0.120000 | 0.014400 | 0.007200 |
| Three | 30% | 20% | 0.060000 | -0.080000 | 0.006400 | 0.001920 |
| Four | 10% | -40% | -0.040000 | -0.680000 | 0.462400 | 0.046240 |
| E(x) = 0.280000 | variance = 0.065600 | standard deviation = 25.61% |

d. | Outcome | Probability | Outcome value | px | x-E(x) | (x-E(x))^2 | p(x-E(x))^2 |
---|---|---|---|---|---|---|---|
| A | 10% | $1,000 | $100 | -$2,000 | 4,000,000,000 | 400,000 |
| B | 20% | $2,000 | $400 | -$1,000 | 1,000,000,000 | 200,000 |
| C | 40% | $3,000 | $1,200 | $0 | 0 | 0 |
| D | 20% | $4,000 | $800 | $1,000 | 1,000,000,000 | 200,000 |
| E | 10% | $5,000 | $500 | $2,000 | 4,000,000,000 | 400,000 |
| E(x) = 3,000 | variance = 1,200,000 | standard deviation = $1,095 |

2. There is a 50% probability that the Plum Company’s sales will be $10 million next year, a 20% probability that they will be $5 million, and a 30% probability that they will be $3 million.

a. What are the expected sales of Plum Company next year?

   Expected value = $6.9 million

b. What is the standard deviation of Plum’s next year’s sales?

Solutions to risk and return practice problems
Variance = 4.8050 + 0.7220 + 4.5630 = 10.09

Standard deviation = square root of 10.09 = $3.1765 million

3. You want to win the Lottery? Good Luck! The odds of winning a $6 million lottery jackpot in Florida are 1 in 14,000,000. What is the expected value of a $1 lottery ticket investment?

\[
E(\text{Lottery ticket cash flow}) = [0.0000000714 ($6,000,000)] + [0.9999999286 ($0)]
\]

\[
= \begin{array}{c@{\quad}c}
\text{winning} & \text{losing} \\
$0.43 & $0
\end{array}
\]

= $0.43.

Spending $1 for a ticket with an expected value of 43¢ means that you expect to lose 57¢

4. Consider the following investments:

<table>
<thead>
<tr>
<th>Investment</th>
<th>Expected return</th>
<th>Standard deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>5%</td>
<td>10%</td>
</tr>
<tr>
<td>B</td>
<td>7%</td>
<td>11%</td>
</tr>
<tr>
<td>C</td>
<td>6%</td>
<td>12%</td>
</tr>
<tr>
<td>D</td>
<td>6%</td>
<td>10%</td>
</tr>
</tbody>
</table>

Which investment would you prefer between the following pairs?

a. A and D? **D**
b. B and C? **B**
c. C and D? **D**

Risk, return and diversification

1. The covariance of the returns on the two securities, A and B, is -0.0005. The standard deviation of A’s returns is 4% and the standard deviation of B’s returns is 6%. What is the correlation between the returns of A and B?

\[
\text{Correlation} = \frac{-0.0005}{(0.04)(0.06)} = -0.2083
\]

2. Company X has a beta of 1.45. The expected risk-free rate of interest is 2.5% and the expected return on the market as a whole is 10%. Using the CAPM, what is ABC’s expected return?

\[
r = 2.5\% + 1.45(10\% - 2.5\%) = 13.375\%
\]

3. Consider a portfolio comprised of four securities in the following proportions and with the indicated security beta.

<table>
<thead>
<tr>
<th>Security</th>
<th>Amount invested</th>
<th>Beta</th>
<th>Expected return</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>$1.5 million</td>
<td>1.0</td>
<td>12.0%</td>
</tr>
<tr>
<td>B</td>
<td>$1.0 million</td>
<td>1.5</td>
<td>13.5%</td>
</tr>
<tr>
<td>C</td>
<td>$2 million</td>
<td>0.8</td>
<td>9.0%</td>
</tr>
</tbody>
</table>

a. What is the portfolio's beta?

\[
\beta_p = (1.5/4.5) 1.0 + (1/4.5) 1.5 + (2/4.5) 0.8
\]
\[ \beta_p = 0.3333 + 0.3333 + 0.3556 = 1.0222 \]

b. What is the portfolio's expected return?

\[ E(p) = (1.5/4.5)0.12 + (1/4.5) 0.135 + (2/4.5) 0.09 \]

\[ E(p) = 0.04 + 0.03 + 0.04 = 0.11 \text{ or } 11\% \]

4. ABC Company has a beta of 1.2. The expected risk-free rate of interest is 4% and the expected premium for the market as a whole is 5%. What is the expected return for ABC Company stock?

\[ r = 4\% + 1.2(5\%) = 10\% \]

5. Consider Securities D and E with the following estimates:

\[ E(R_D) = 8\% \quad \sigma_D = 12\% \quad E(R_E) = 13\% \quad \sigma_E = 20\% \]

Now consider the portfolios that can be formed with D and E, assuming that the investment is equal between D and E (that is, each has a weight of 50%). What is the portfolio's standard deviation if the correlation between D and E for each of the following?

(a) $r_{ij} = 1.0$

\[ \sigma_p = \sqrt{\sum_{i=1}^{N} X_i^2 \sigma_i^2 + \sum_{i=1}^{N} \sum_{j=1, j\neq i}^{N} X_i X_j \sigma_i \sigma_j r_{ij}} \]

\[ \sigma_p = \sqrt{(0.5^2 \times 0.12^2) + (0.5^2 \times 0.2^2) + 2 \times (0.5 \times 0.5 \times 0.12 \times 0.2) \times 1.0} \]

\[ \sigma_p = \sqrt{0.0036 + 0.0100 + 0.012} = \sqrt{0.0256} = 0.16 \]

(b) $r_{ij} = 0.3$

\[ \sigma_p = \sqrt{(0.5^2 \times 0.12^2) + (0.5^2 \times 0.2^2) + 2 \times (0.5 \times 0.5 \times 0.12 \times 0.2) \times 0.3} \]

\[ \sigma_p = \sqrt{0.0036 + 0.0100 + 0.0036} = \sqrt{0.0172} = 0.131149 \]

(c) $r_{ij} = 0.0$

\[ \sigma_p = \sqrt{(0.5^2 \times 0.12^2) + (0.5^2 \times 0.2^2) + 2 \times (0.5 \times 0.5 \times 0.12 \times 0.2) \times 0.0} \]

\[ \sigma_p = \sqrt{0.0036 + 0.0100 + 0.0} = \sqrt{0.0136} = 0.116619 \]

(d) $r_{ij} = -1.0$

\[ \sigma_p = \sqrt{(0.5^2 \times 0.12^2) + (0.5^2 \times 0.2^2) + 2 \times (0.5 \times 0.5 \times 0.12 \times 0.2) \times (-1.0)} \]

\[ \sigma_p = \sqrt{0.0036 + 0.0100 - 0.012} = \sqrt{0.0016} = 0.04 \]

6. Consider Securities X and Y with the following estimates:

\[ E(R_X) = 5\% \quad \sigma_X = 10\% \quad E(R_Y) = 15\% \quad \sigma_Y = 25\% \]
If the portfolio is comprise of 40% X and 60% Y and if the correlation between the returns on X and Y is -0.25, what is the portfolio’s expected return and risk?

Expected return = 0.4(0.05) + 0.6(0.15) = 0.02 + 0.09 = \textbf{0.11 or 11\%}

Variance = (0.4)(0.4)(0.10)(0.10) + (0.6)(0.6)(.25)(.25)+(2)(0.4)(0.6)(0.1)(0.25)(-0.25)

Variance = 0.0016 + 0.0225 - 0.0030 = 0.0211

Standard deviation = \textbf{14.5268\%}