

# Capital budgeting techniques

A reading prepared by  
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## OUTLINE

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## 1. Introduction

The value of a firm today is the present value of all its future cash flows. These future cash flows come from assets already in place and from future investment opportunities. These future cash flows are discounted at a rate that represents investors' assessments of the uncertainty that they will flow in the amounts and when expected:

$$\text{Value of the firm} = \sum_{t=1}^{\infty} \frac{CF_t}{(1+r)^t}$$

where  $CF_t$  is the cash flow in period  $t$  and  $r$  is the required rate of return. The objective of the financial manager is to maximize the value of the firm. In a corporation, the shareholders are the residual owners of the firm, so decisions that maximize the value of the firm also maximize shareholders' wealth.

The financial manager makes decisions regarding long-lived assets; this process is referred to as capital budgeting. The capital budgeting decisions for a project requires analysis of:

- its future cash flows,
- the degree of uncertainty associated with these future cash flows, and
- the value of these future cash flows considering their uncertainty.

We looked at how to estimate cash flows in a previous reading where we were concerned with a project's incremental cash flows, comprising changes in operating cash flows (change in revenues, expenses, and taxes), and changes in investment cash flows (the firm's incremental cash flows from the acquisition and disposition of the project's assets).

And we know the concept behind uncertainty: the more uncertain a future cash flow, the less it is worth today. The degree of uncertainty, or risk, is reflected in a project's cost of capital. The cost of capital is what the firm must pay for the funds to finance its investment. The cost of capital may be an explicit cost (for example, the interest paid on debt) or an implicit cost (for example, the expected price appreciation of its shares of common stock).

In this reading, we focus on evaluating the future cash flows. Given estimates of incremental cash flows for a project and given a cost of capital that reflects the project's risk, we look at alternative techniques that are used to select projects.

For now all we need to understand about a project's risk is that we can incorporate risk in either of two ways: (1) we can discount future cash flows using a higher discount rate, the greater the cash flow's risk, or (2) we can require a higher annual return on a project, the greater the risk of its cash flows.

## 2. Evaluation techniques

Look at the incremental cash flows for Project X and Project Y shown in Exhibit 1. Can you tell by looking at the cash flows for Investment A whether or not it enhances wealth? Or, can you tell by just looking at Investments A and B which one is better? Perhaps with some projects you may think you can pick out which one is better simply by gut feeling or eyeballing the cash flows. But why do it that way when there are precise methods to evaluate investments by their cash flows?

We must first determine the cash flows from each investment and then assess the uncertainty of all the cash flows in order to evaluate investment projects and select the investments that maximize wealth.

We look at six techniques that are commonly used by firms to evaluating investments in long-term assets:

1. Payback period,
2. Discounted payback period,
3. Net present value,
4. Profitability index,
5. Internal rate of return, and
6. Modified internal rate of return.

**Exhibit 1: Estimated cash flows for Investments X and Y**

Year	End of period cash flows	
	Project X	Project Y
2006	-\$1,000,000	-\$1,000,000
2007	\$0	\$325,000
2008	\$200,000	\$325,000
2009	\$300,000	\$325,000
2010	\$900,000	\$325,000

We are interested in how well each technique discriminates among the different projects, steering us toward the projects that maximize owners' wealth.

An evaluation technique should:

- Consider all the future incremental cash flows from the project;
- Consider the time value of money;
- Consider the uncertainty associated with future cash flows, and
- Have an objective criterion by which to select a project.

Projects selected using a technique that satisfies all four criteria will, under most general conditions, maximize owners' wealth.

In addition to judging whether each technique satisfies these criteria, we will also look at which ones can be used in special situations, such as when a dollar limit is placed on the capital budget.

### A. Payback period

The payback period for a project is the time from the initial cash outflow to invest in it until the time when its cash inflows add up to the initial cash outflow. In other words, how long it takes to get your

money back. The payback period is also referred to as the payoff period or the capital recovery period. If you invest \$10,000 today and are promised \$5,000 one year from today and \$5,000 two years from today, the payback period is two years -- it takes two years to get your \$10,000 investment back.

Suppose you are considering Investments X and Y, each requiring an investment of \$1,000,000 today (we're considering today to be the last day of the year 2006) and promising cash flows at the end of each of the following years through 2010. How long does it take to get your \$1,000,000 investment back? The payback period for Project X is four years:

<u>Year</u>	<u>Project X</u>	<u>Accumulated cash flows</u>
2006	-\$1,000,000	
2007	\$0	-\$1,000,000
2008	200,000	-800,000
2009	300,000	-500,000
2010	900,000	+400,000

By the end of 2009, the full \$1,000,000 is not paid back, but by 2010 the accumulated cash flow hits (and exceeds) \$1,000,000. Therefore, the payback period for Project X is four years.

The payback period for Project Y is four years. It is not until the end of 2010 that the \$1,000,000 original investment (and more) is paid back.

We have assumed that the cash flows are received at the end of the year. So we always arrive at a payback period in terms of a whole number of years. If we assume that the cash flows are received, say, uniformly, such as monthly or weekly, throughout the year, we arrive at a payback period in terms of years and fractions of years.<sup>1</sup>

For example, assuming we receive cash flows uniformly throughout the year, the payback period for Project X is 3 years and 6.6 months (assuming \$75,000 cash flow per month). Our assumption of end-of-period cash flows may be unrealistic, but it is convenient to use this assumption to demonstrate how to use the various evaluation techniques. We will continue to use this end-of-period assumption throughout the coverage of capital budgeting techniques.

Is Project X or Y more attractive? A shorter payback period is better than a longer payback period. Yet there is no clear-cut rule for how short is better. If we assume that all cash flows occur at the end of the year, Project X provides the same payback as Project Y. Therefore, we do not know in this particular case whether quicker is better.

In addition to having no well-defined decision criteria, payback period analysis favors investments with "front-loaded" cash flows: an investment looks better in terms of the payback period the sooner its cash flows are received no matter what its later cash flows look like. Payback period analysis is a type of "break-even" measure. It tends to provide a measure of the economic life of the investment in terms of its payback period. The more likely the life exceeds the payback period, the more attractive the investment. The economic life beyond the payback period is referred to as the post-payback duration. If post-payback duration is zero, the investment is worthless, no matter how short the payback. This is because the sum of the future cash flows is no greater than the initial investment outlay. And since these future cash flows are really worth less today than in the future, a zero post-payback duration means that the present value of the future cash flows is less than the project's initial investment.

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<sup>1</sup> But then we would have a challenge applying the methods that apply the time value of money, so for simplicity sake we assume end-of-period cash flows in illustrating the capital budgeting techniques.

The payback method should only be used as a coarse initial screen of investment projects. But it can be a useful indicator of some things. Because a dollar of cash flow in the early years is worth more than a dollar of cash flow in later years, the payback period method provides a simple, yet crude measure of the liquidity of the investment.

The payback period also offers some indication on the risk of the investment. In industries where equipment becomes obsolete rapidly or where there are very competitive conditions, investments with earlier payback are more valuable. That's because cash flows farther into the future are more uncertain and therefore have lower present value. In the personal computer industry, for example, the fierce competition and rapidly changing technology requires investment in projects that have a payback of less than one year since there is no expectation of project benefits beyond one year.

Because the payback method doesn't tell us the particular payback period that maximizes wealth, we cannot use it as the primary screening device for investment in long-lived assets.

## B. Discounted payback period

The discounted payback period is the time needed to pay back the original investment in terms of discounted future cash flows.

Each cash flow is discounted back to the beginning of the investment at a rate that reflects both the time value of money and the uncertainty of the future cash flows. This rate is the cost of capital -- the return required by the suppliers of capital (creditors and owners) to compensate them for time value of money and the risk associated with the investment. The more uncertain the future cash flows, the greater the cost of capital.

### The cost of capital, the required rate of return, and the discount rate

We discount an uncertain future cash flow to the present at some rate that reflects the degree of uncertainty associated with this future cash flow. The more uncertain, the less the cash flow is worth today -- this means that a higher discount rate is used to translate it into a value today.

This discount rate is a rate that reflects the opportunity cost of funds. In the case of a corporation, we consider the opportunity cost of funds for the suppliers of capital (the creditors and owners). We refer to this opportunity cost as the cost of capital.

The cost of capital comprises the required rate of return (RRR) (that is, the return suppliers of capital demand on their investment) and the cost of raising new capital if the firm cannot generate the needed capital internally (that is, from retaining earnings). The cost of capital and the required rate of return are the same concept, but from different perspective. Therefore, we will use the terms interchangeably in our study of capital budgeting.

### Calculating the discounted payback period

Returning to Projects X and Y, suppose that each has a cost of capital of 10 percent. The first step in determining the discounted payback period is to discount each year's cash flow to the beginning of the investment (the end of the year 2006) at the cost of capital:

Year	Project X		Project Y	
	Cash flows	Accumulated discounted cash flows	Cash flows	Accumulated discounted cash flows
2006	-\$1,000,000.00	-\$1,000,000.00	-\$1,000,000.00	-\$1,000,000.00
2007	\$0.00	-\$1,000,000.00	\$295,454.55	-\$704,545.45
2008	\$165,289.26	-\$834,710.74	\$268,595.04	-\$435,950.41
2009	\$225,394.44	-\$609,316.30	\$244,177.31	-\$191,773.10
2010	\$614,712.11	\$5,395.81	\$221,979.37	\$30,206.27

How long does it take for each investment's discounted cash flows to pay back its \$1,000,000 investment? The discounted payback period for both X and Y is four years.

### Discounted payback decision rule

It appears that the shorter the payback period, the better, whether using discounted or non-discounted cash flows. But how short is better? We don't know. All we know is that an investment "breaks-even" in terms of discounted cash flows at the discounted payback period -- the point in time when the accumulated discounted cash flows equal the amount of the investment.

Using the length of the payback as a basis for selecting investments, Projects X and Y cannot be distinguished. But we've ignored some valuable cash flows for both investments, those beyond what is necessary for recovering the initial cash outflow.

## C. Net present value

If offered an investment that costs \$5,000 today and promises to pay you \$7,000 two years from today and if your opportunity cost for projects of similar risk is 10 percent, would you make this investment? To determine whether or not this is a good investment you need to compare your \$5,000 investment with the \$7,000 cash flow you expect in two years. Because you determine that a discount rate of 10 percent reflects the degree of uncertainty associated with the \$7,000 expected in two years, today it is worth:

$$\text{Present value of } \$7,000 \text{ to be received in 2 years} = \frac{\$7,000}{(1 + 0.10)^2} = \$5,785.12.$$

By investing \$5,000, today you are getting in return, a promise of a cash flow in the future that is worth \$5,785.12 today. You increase your wealth by \$785.12 when you make this investment.

Another way of stating this is that the present value of the \$7,000 cash inflow is \$5,785.12, which is more than the \$5,000, today's cash outflow to make the investment. When we subtract today's cash outflow to make an investment from the present value of the cash inflow from the investment, the difference is the increase or decrease in our wealth referred to as the net present value.

The net present value (NPV) is the present value of all expected cash flows.

$$\text{Net present value} = \text{Present value of all expected cash flows.}$$

The word "net" in this term indicates that all cash flows -- both positive and negative -- are considered. Often the changes in operating cash flows are inflows and the investment cash flows are outflows. Therefore we tend to refer to the net present value as the difference between the present value of the cash inflows and the present value of the cash outflows.

We can represent the net present value using summation notation, where t indicates any particular period,  $CF_t$  represents the cash flow at the end of period t, i represents the cost of capital, and N the number of periods comprising the economic life of the investment:

$$NPV = \text{present value of cash inflows} - \text{present value of cash outflows} = \sum_{t=1}^N \frac{CF_t}{(1+r)^t}$$

Cash inflows are positive values of  $CF_t$  and cash outflows are negative values of  $CF_t$ . For any given period t, we collect all the cash flows (positive and negative) and net them together. To make things a bit easier to track, let's just refer to cash flows as inflows or outflows, and not specifically identify them as operating or investment cash flows.

Take another look at Projects X. Using a 10 percent cost of capital, the present values of inflows are:

Project X		
Year	Cash flow	Discounted cash flow
2006	-\$1,000,000	-\$1,000,000.00
2007	\$0	\$0.00
2008	200,000	165,289.26
2009	300,000	225,394.44
2010	900,000	614,712.11
L		NPV = +\$5,395.81

This NPV tells us that if we invest in X, we expect to increase the value of the firm by \$5,395.81. Calculated in a similar manner, the net present value of Project Y is \$30,206.27. We can use a financial calculator to solve for the NPV as well, inputting the cash flows in order, making sure that the \$0 cash flow for year 2007 is included in the list of cash flows.

TI-83/84	HP10B
{0,200000,300000,900000} STO listname	1000000 +/- CF <sub>i</sub>
NPV(10,-1000000,listname)	0 CF <sub>i</sub>
	200000 CF <sub>i</sub>
	300000 CF <sub>i</sub>
	900000 CF <sub>i</sub>
	10 i/YR
	NPV

We can also use Microsoft Excel to solve for the net present value. The Excel spreadsheet entries for the data would be:

	A	B
1	Year	Project X
2	2006	-\$1,000,000
3	2007	\$0
4	2008	\$200,000
5	2009	\$300,000
6	2010	\$900,000

and the net present value requires the use of the NPV function:

$$=NPV(.1,B3:B6)+B2$$

## Net Present Value Decision Rule

A positive net present value means that the investment increases the value of the firm -- the return is more than sufficient to compensate for the required return of the investment. A negative net present value means that the investment decreases the value of the firm -- the return is less than the cost of capital. A zero net present value means that the return just equals the return required by owners to

compensate them for the degree of uncertainty of the investment's future cash flows and the time value of money. Therefore,

if...	this means that...	and you...
NPV > \$0	the investment is expected to increase shareholder wealth	should accept the project.
NPV < \$0	the investment is expected to decrease shareholder wealth	should reject the project.
NPV = \$0	the investment is expected not to change shareholder wealth	should be indifferent between accepting or rejecting the project

Project X is expected to increase the value of the firm by \$5,395.81, whereas Project Y is expected to increase add \$30,206.27 in value. If these are independent investments, both should be taken on because both increase the value of the firm. If X and Y are mutually exclusive, such that the only choice is either X or Y, then Y is preferred since it has the greater NPV. Projects are said to be mutually exclusive if accepting one precludes the acceptance of the other.

## D. Profitability index

The profitability index uses some of the same information we used for the net present value, but it is stated in terms of an index. Whereas the net present value is:

$$NPV = \text{present value of cash inflows} - \text{present value of cash outflows} = \sum_{t=1}^N \frac{CF_t}{(1+r)^t}$$

The profitability index, PI is:

$$PI = \frac{\text{present value of cash inflows}}{\text{present value of cash outflows}} = \frac{\sum_{t=1}^N \frac{CIF_t}{(1+r)^t}}{\sum_{t=1}^N \frac{COF_t}{(1+r)^t}}$$

where CIF and COF are cash inflows and cash outflows, respectively.

Year	Cash flow	Project X	
			Discounted cash flow
2007	\$0		\$0.00
2008	200,000		165,289.26
2009	300,000		225,394.44
2010	900,000		<u>614,712.11</u>
			$\sum_{t=1}^N \frac{CIF_t}{(1+r)^t} = +\$1,005,395.81$

Therefore, the profitability index is:

$$PI_X = \frac{\$1,005,395.81}{\$1,000,000} = 1.0054$$

The index value is greater than one, which means that the investment produces more in terms of benefits than costs.

The decision rule for the profitability index is therefore depends on the PI relative to 1.0:

if...	this means that...	and you...
PI > 1.0	the investment is expected to increase shareholder wealth	should accept the project.
PI < 1.0	the investment is expected to decrease shareholder wealth	should reject the project.
PI = 1.0	the investment is expected not to change shareholder wealth	should be indifferent between accepting or rejecting the project

There is no direct solution for PI on your calculator; what you need to do is calculate the present value of all the cash inflows and then divide this value by the present value of the cash outflows. In the case of Project X, there is only one cash out flow and it is already in present value terms (i.e., it occurs at the end of 2006).

TI-83/84	HP10B
{0,200000,300000,900000} STO listname NPV(10,0,listname) ENTER ÷ 1000000 ENTER	0 +/- CF <sub>1</sub> 0 CF <sub>1</sub> 200000 CF <sub>1</sub> 300000 CF <sub>1</sub> 900000 CF <sub>1</sub> 10 i/YR NPV ÷ 1000000

## E. Internal rate of return

Suppose you are offered an investment opportunity that requires you to put up \$50,000 and has expected cash inflows of \$28,809.52 after one year and \$28,809.52 after two years. We can evaluate this opportunity using a time line, as shown in Exhibit 1.

Exhibit 4 Time line of investment opportunity		
0	1	2
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-\$50,000	\$28,809.52	\$28,909.52

The return on this investment is the discount rate that causes the present values of the \$28,809.52 cash inflows to equal the present value of the \$50,000 cash outflow, calculated as:

$$50,000 = \frac{28,809.52}{(1 + \text{IRR})^1} + \frac{28,809.52}{(1 + \text{IRR})^2}$$

Another way to look at this is to consider the investment's cash flows discounted at the IRR of 10 percent. The NPV of this project if the discount rate is 10 percent (the IRR in this example), is zero:

$$50,000 = \frac{28,809.52}{(1 + 0.10)^1} + \frac{28,809.52}{(1 + 0.10)^2}$$

An investment's internal rate of return (IRR) is the discount rate that makes the present value of all expected future cash flows equal to zero. We can represent the IRR as the rate that solves:

$$0 = \sum_{t=1}^N \frac{CF_t}{(1 + \text{IRR})^t}$$

The IRR for X is the discount rate that solves:

$$-\$1,000,000 = \frac{\$0}{(1 + \text{IRR})^1} + \frac{\$200,000}{(1 + \text{IRR})^2} + \frac{\$300,000}{(1 + \text{IRR})^3} + \frac{\$900,000}{(1 + \text{IRR})^4}$$

Using a calculator or a computer, we get the more precise answer of 10.172 percent per year.

TI-83/84	HP10B
{0,200000,300000,900000} STO <i>listname</i> IRR(-1000000, <i>listname</i> )	-1000000 +/- CF <sub>i</sub> 0 CF <sub>i</sub> 200000 +/- CF <sub>i</sub> 300000 +/- CF <sub>i</sub> 900000 +/- CF <sub>i</sub> IRR

Looking back at the investment profiles of Projects X and Y, you'll notice that each profile crosses the horizontal axis (where NPV = \$0) at the discount rate that corresponds to the investment's internal rate of return. This is no coincidence: by definition, the IRR is the discount rate that causes the project's NPV to equal zero.

### Internal rate of return decision rule

The internal rate of return is a yield -- what we earn, on average, per year. How do we use it to decide which investment, if any, to choose? Let's revisit Investments A and B and the IRRs we just calculated for each. If, for similar risk investments, owners earn 10 percent per year, then both A and B are attractive. They both yield more than the rate owners require for the level of risk of these two investments:

<u>Investment</u>	<u>IRR</u>	<u>Cost of capital</u>
X	10.172%	10%
Y	11.388%	10%

The decision rule for the internal rate of return is to invest in a project if it provides a return greater than the cost of capital. The cost of capital, in the context of the IRR, is a hurdle rate -- the minimum acceptable rate of return. For independent projects and situations in which there is no capital rationing, then

if...	this means that...	and you...
IRR > cost of capital	the investment is expected to increase shareholder wealth	should accept the project.
IRR < cost of capital	the investment is expected to decrease shareholder wealth	should reject the project.
IRR = cost of capital	the investment is expected not to change shareholder wealth	should be indifferent between accepting or rejecting the project

### The IRR and mutually exclusive projects

What if we were forced to choose between projects X and Y because they are mutually exclusive? Project Y has a higher IRR than Project X -- so at first glance we might want to accept Project Y. What about the NPV of X and Y? What does the NPV tell us to do? If we use the higher IRR, it tells us to go with Y. If we use the higher NPV if the cost of capital is 5 percent, we go with X. Which is correct? Choosing the project with the higher net present value is consistent with maximizing owners' wealth. Why? Because if the cost of capital is 10 percent, we would calculate different NPVs and come to a different conclusion, as you can see from the investment profiles in Exhibit 3.

When evaluating mutually exclusive projects, the one with the highest IRR may not be the one with the best NPV. The IRR may give a different decision than NPV when evaluating mutually exclusive projects because of the reinvestment assumption:

- NPV assumes cash flows reinvested at the cost of capital.
- IRR assumes cash flows reinvested at the internal rate of return.

This reinvestment assumption may cause different decisions in choosing among mutually exclusive projects when:

- the timing of the cash flows is different among the projects,
- there are scale differences (that is, very different cash flow amounts), or
- the projects have different useful lives.

With respect to the role of the timing of cash flows in choosing between two projects: Project Y's cash flows are received sooner than X's. Part of the return on either is from the reinvestment of its cash inflows. And in the case of Y, there is more return from the reinvestment of cash inflows. The question is "What do you do with the cash inflows when you get them?" We generally assume that if you receive cash inflows, you'll reinvest those cash flows in other assets.

With respect to the reinvestment rate assumption in choosing between these projects: Suppose we can reasonably expect to earn only the cost of capital on our investments. Then for projects with an IRR above the cost of capital we would be overstating the return on the investment using the IRR.

**Bottom line:** If we evaluate projects on the basis of their IRR, it is possible that we may select one that does not maximize value.

With respect to the NPV method: if the best we can do is reinvest cash flows at the cost of capital, the NPV assumes reinvestment at the more reasonable rate (the cost of capital). If the reinvestment rate is assumed to be the project's cost of capital, we would evaluate projects on the basis of the NPV and select the one that maximizes owners' wealth.

## The IRR and capital rationing

What if there is capital rationing? Suppose Investments A and B are **independent projects**. Projects are independent if that the acceptance of one does not prevent the acceptance of the other. And suppose the capital budget is limited to \$1,000,000. We are therefore forced to choose between A or B. If we select the one with the highest IRR, we choose A. But A is expected to increase wealth less than B. Ranking investments on the basis of their IRRs may not maximize wealth.

We saw this dilemma in the previous reading pertaining to projects X and Y when we looked at their investment profiles. The discount rate at which X's NPV is \$0.00 is X's IRR = 10.172 percent, where X's profile crosses the horizontal axis. Likewise, the discount rate at which Y's NPV is \$0.00 is B's IRR = 11.388 percent. The discount rate at which X's and Y's profiles cross is the cross-over rate, 7.495 percent. For discount rates less than 7.495 percent, X has the higher NPV. For discount rates greater than 7.495 percent, Y has the higher NPV. If Y is chosen because it has a higher IRR and if Y's cost of capital is less than 7.495 percent, we have not chosen the project that produces the greatest value.

The source of the problem in the case of capital rationing is that the IRR is a percentage, not a dollar amount. Because of this, we cannot determine how to distribute the capital budget to maximize wealth because the investment or group of investments producing the highest yield does not mean they are the ones that produce the greatest wealth.

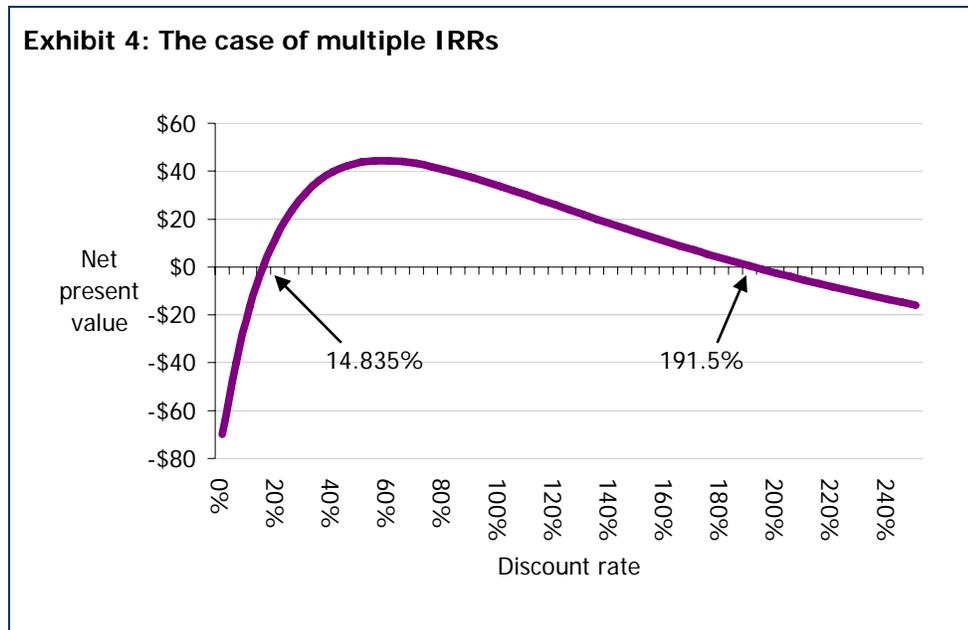
## Multiple internal rates of return

The typical project usually involves only one large negative cash flow initially, followed by a series of future positive flows. But that's not always the case. Suppose you are involved in a project that uses environmentally sensitive chemicals. It may cost you a great deal to dispose of them. And that will mean a negative cash flow at the end of the project.

Suppose we are considering a project that has cash flows as follows:

Period	End of period cash flow
0	-\$100
1	+260
2	+260
3	-490

What is this project's IRR? One possible solution is IRR = 14.835 percent, yet another possible solution is IRR = 191.5 percent.



We can see this graphically in Exhibit 4, where the NPV of these cash flows are shown for discount rates from 0 percent to 250 percent.

Remember that the IRR is the discount rate that causes the NPV to be zero. In terms of this graph, this means that the IRR is the discount rate where the NPV is

\$0, the point at which the present value changes sign -- from positive to negative or from negative to positive. In the case of this project, the present value changes from negative to positive at 14.835 percent and from positive to negative at 250 percent.

**Bottom line:** We can't use the internal rate of return method if the sign of the cash flows change more than once during the project's life.

## F. Modified internal rate of return

The internal rate of return method assumes that cash flows are reinvested at the investment's internal rate of return. Consider Project X. The IRR is 10.17188 percent. If we take each of the cash inflows from Project X and reinvest them at 10.17188 percent, we will have \$1,472,272.53 at the end of 2010:

Number of periods earning a return	Future value of cash flow reinvested at 10.17188%
3	\$0.00
2	242,756.88
1	330,515.65
0	<u>900,000.00</u>
	\$1,473,272.53

The \$1,473,272.53 is referred to as the project's **terminal value**.<sup>2</sup> The terminal value is how much the company has from this investment if all proceeds are reinvested at the IRR. So what is the return on this project? Using the terminal value as the future value and the investment as the present value,

$$\begin{aligned} \text{FV} &= \$1,473,272.53 \\ \text{PV} &= \$1,000,000.00 \\ \text{N} &= 4 \text{ years} \end{aligned}$$

$$i = \sqrt[4]{\frac{\$1,473,272.53}{\$1,000,000.00}} = 10.17188\%$$

In other words, by investing \$1,000,000 at the end of 2006 and receiving \$1,473,272.53 produces an average annual return of 10.1718 percent, which is the project's internal rate of return.

The modified internal rate of return is the return on the project assuming reinvestment of the cash flows at a specified rate. Consider Project X if the reinvestment rate is 5 percent:

Number of periods earning a return	Future value of cash flow reinvested at 5%
3	\$0.00
2	220,500.00
1	315,000.00
0	<u>900,000.00</u>
	\$1,435,500.00

The modified internal rate of return is 9.4588 percent:

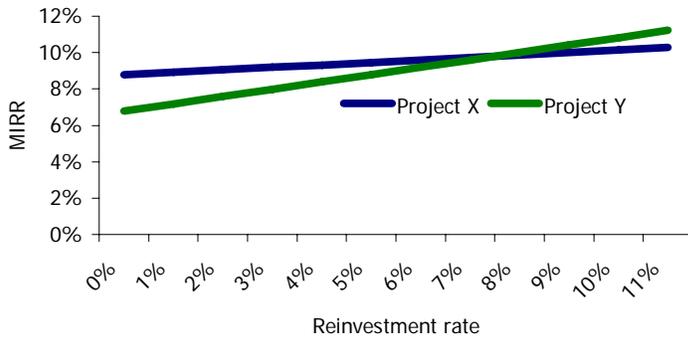
$$\begin{aligned} \text{FV} &= \$1,435,500 \\ \text{PV} &= \$1,000,000 \\ \text{N} &= 4 \text{ years} \end{aligned}$$

$$i = \sqrt[4]{\frac{\$1,435,500}{\$1,000,000.00}} = 9.4588\%$$

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<sup>2</sup> For example, the 2008 cash flow of \$200,000 is reinvested at 10.17188 percent for two periods (that is, for 2009 and 2010), or  $\$200,000 (1 + 0.1017188)^2 = \$242,756.88$ .

**Exhibit 5 MIRR for Project X and Project Y**



The MIRR is therefore a function of both the reinvestment rate and the pattern of cash flows, with higher the reinvestment rates leading to greater MIRRs. You can see this in Exhibit 5, where the MIRR of both Project X and Project Y is plotted for different reinvestment rates. Project Y's MIRR is more sensitive to the reinvestment rate because more of its cash flows are received sooner, relative to Project X's cash flows.

If we wish to represent this technique in a formula,

$$MIRR = \sqrt[N]{\frac{\sum_{t=1}^N CIF_t (1+i)^{N-t}}{\sum_{t=1}^N \frac{COF_t}{(1+i)^t}}}$$

where the  $CIF_t$  are the cash inflows and the  $COF_t$  are the cash outflows. In the previous example, the present value of the cash outflows is equal to the \$1,000,000 initial cash outlay, whereas the future value of the cash inflows is \$1,435,500.

If...	this means that...	and you...
MIRR > cost of capital	the investment is expected to return more than required	should accept the project.
MIRR < cost of capital	the investment is expected to return less than required	should reject the project.
MIRR = cost of capital	the investment is expected to return what is required	are indifferent between accepting or rejecting the project

## G. Scale differences

Scale differences -- differences in the amount of the cash flows -- between projects can lead to conflicting investment decisions among the discounted cash flow techniques. Consider two projects, Project Big and Project Little, that each have a cost of capital of 5 percent per year with the following cash flows:

End of period	Project Big	Project Little
0	-\$1,000,000	-\$1.00
1	+ 400,000	+ 0.40
2	+ 400,000	+ 0.40
3	+ 400,000	+ 0.50

Applying the discounted cash flow techniques to each project,

Technique	Project Big	Project Little
NPV	\$89,299	\$0.1757
PI	1.0893	1.1757
IRR	9.7010%	13.7789%
MIRR	8.0368%	10.8203%

### Mutually exclusive projects

If Big and Little are mutually exclusive projects, which project should a firm prefer? If the firm goes strictly by the PI, IRR, or MIRR criteria, it would choose Project Little. But is this the better project? Project Big provides more value -- \$89,299 versus \$0.18. The techniques that ignore the scale of the investment -- PI, IRR, and MIRR -- may lead to an incorrect decision.

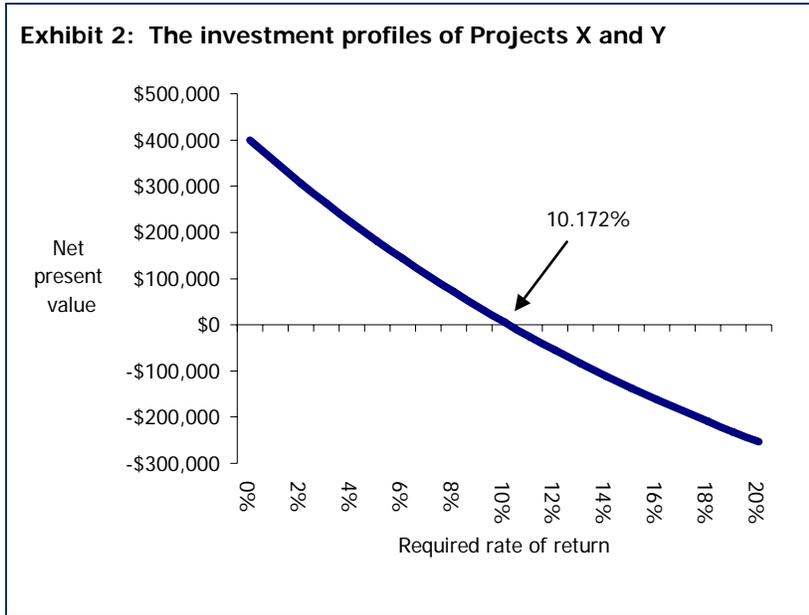
### Capital rationing

If the firm is subject to capital rationing -- say a limit of \$1,000,000 -- and Big and Little are independent projects, which project should the firm choose? The firm can only choose one -- spend \$1 or \$1,000,000, but not \$1,000,001. If you go strictly by the PI, IRR, or MIRR criteria, the firm would choose Project Little. But is this the better project? Again, the techniques that ignore the scale of the investment -- PI, IRR, and MIRR -- leading to an incorrect decision.

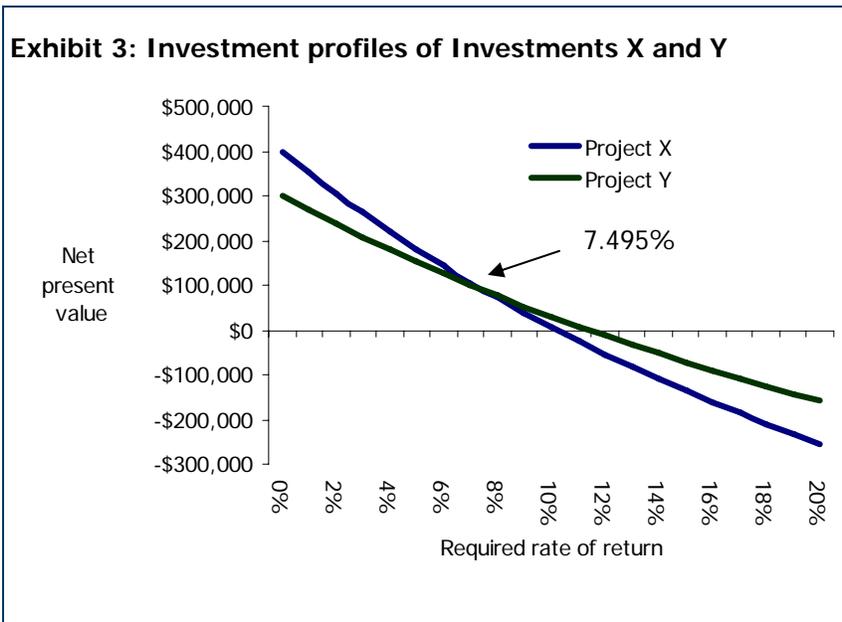
## H. The investment profile

We may want to see how sensitive is our decision to accept a project to changes in our cost of capital. We can see this sensitivity in how a project's net present value changes as the discount rate changes by looking at a project's investment profile, also referred to as the net present value profile. The investment profile is a graphical depiction of the relation between the net present value of a project and the discount rate: the profile shows the net present value of a project for each discount rate, within some range.

The net present value profile for the two projects is shown in Exhibit 2 for discount rates from 0 percent to 20 percent. To help you get the idea behind this graph, we've identified the NPV's of this project for discount rates of 5 percent and 10 percent. You should be able to see that the NPV is positive for discount rates from 0 percent to 10.172 percent, and negative for discount rates higher than 10.172 percent. The 10.172 percent is the internal rate of return; that is, the discount rate at which the net present value is equal to \$0. Therefore, Project X increases owners' wealth if the cost of capital on this project is less than 10.172 percent and decreases owners' wealth if the cost of capital on this project is greater than 10.172 percent.



Let's impose X's NPV profile on the NPV profile of Project Y, as shown in the graph in Exhibit 3. If X and Y are mutually exclusive projects -- we invest in only one or neither project -- this graph clearly shows that the project we invest in depends on the discount rate. For higher discount rates, B's NPV falls faster than A's. This is because most of B's present value is attributed to the large cash flows four and five years into the future. The present value of the more distant cash flows is more sensitive to changes in the discount rate than is the present value of cash flows nearer the present.



If the discount rate is less than 7.495 percent, X adds more values than Y. If the discount rate is more than 7.495 percent but less than 11.338 percent, Y increases wealth more than X. If the discount rate is greater than 11.338 percent, we should invest in neither project because both would decrease wealth.

The 7.495 percent is the cross-over discount rate which produces identical NPV's for the two projects. If the discount rate is 7.495 percent, the net present value of both investments

is \$88,660.<sup>3</sup>

<sup>3</sup> The precise cross-over rate is 7.49475 percent, at which the NPV for both projects is \$88,659.

### Example 1: The investment profile

#### Problem

Consider a project that has the following expected cash flows:

End of year	Cash flow
2005	-\$1,000,000
2006	800,000
2007	400,000
2008	70,000
2009	30,000

Draw this project's investment profile for discount rates from 0 percent to 20 percent.

#### Solution

Step 1: Calculate the NPV if the discount rate = 0%. You calculate this by simply adding up all cash flows (both positive and negative). In this example, this is \$300,000.

Step 2: Calculate the IRR. In this case, this is 19.95%

Step 3: Calculate the NPV for some discount rate between 0% and the IRR.

Step 4: Mark the result from Steps 1, 2 and 3 on the graph and connect the points.



### Solving for the cross-over rate

For Projects X and Y, the cross-over rate is the rate that causes the net present value of the two investments to be equal. Basically, this boils down to a simple approach: calculate the differences in the cash flows and then solve for the internal rate of return of these differences.

Year	Project X	Project Y	Difference
2006	-\$1,000,000	-\$1,000,000	\$0
2007	\$0	\$325,000	-\$325,000
2008	\$200,000	\$325,000	-\$125,000
2009	\$300,000	\$325,000	-\$25,000
2010	\$900,000	\$325,000	\$575,000

The internal rate of return of these differences is the cross-over rate. Does it matter which project's cash flows you deduct from the other? Not at all – just be consistent each period.

**Bottom line:** The cross-over rate is the decision point between two mutually exclusive projects.

TI-83/84	HP10B
{-325000,-125000,-25000,575000} STO listname IRR(0,listname)	0 +/- CF <sub>i</sub> 325000 +/- CF <sub>i</sub> 125000 +/- CF <sub>i</sub> 25000 +/- CF <sub>i</sub> 575000 CF <sub>i</sub> IRR

### Example Cross-over rates

#### Problem

Consider two projects, P & Q, with the following sets of cash flows:

End of period	P	Q
0	-\$10	-\$20
1	4.2	0
2	4.2	0
3	4.2	26

What is the cross-over rate for these two projects' investment profiles?

#### Solution

End of period	P	Q	Difference
0	-\$10	-\$20	+\$10
1	4.2	0	+4.2
2	4.2	0	+4.2
3	4.2	26	-21.8

Cross-over rate is the IRR of the differences, or **7.52 percent**

## 3. Comparing techniques

If we are dealing with mutually exclusive projects, the NPV method leads us to invest in projects that maximize wealth, that is, capital budgeting decisions consistent with owners' wealth maximization. If we are dealing with a limit on the capital budget, the NPV and PI methods lead us to invest in the set of projects that maximize wealth.

The advantages and disadvantages of each of the techniques for evaluating investments are summarized in Table 1. We see in this table that the discounted cash flow techniques are preferred to the non-discounted cash flow techniques. The discounted cash flow techniques -- NPV, PI, IRR, MIRR -- are preferable since they consider (1) all cash flows, (2) the time value of money, and (3) the risk of future cash flows. The discounted cash flow techniques are also useful because we can apply objective decision criteria -- criteria we can actually use that tells us when a project increases wealth and when it does not.

We also see in this table that not all of the discounted cash flow techniques are right for every situation. There are questions we need to ask when evaluating an investment and the answers will determine which technique is the one to use for that investment:

- Are the projects mutually exclusive or independent?
- Are the projects subject to capital rationing?
- Are the projects of the same risk?
- Are the projects of the same scale of investment?

Here are some simple rules:

1. If projects are independent and not subject to capital rationing, we can evaluate them and determine the ones that maximize wealth based on any of the discounted cash flow techniques.
2. If the projects are mutually exclusive, have the same investment outlay, and have the same risk, we must use only the NPV or the MIRR techniques to determine the projects that maximize wealth.
3. If projects are mutually exclusive and are of different risks or are of different scales, NPV is preferred over MIRR.

If the capital budget is limited, we can use either the NPV or the PI. We must be careful, however, not to select projects simply on the basis of their NPV or PI (that is, ranking on NPV and selecting the highest NPV projects), but rather how we can maximize the NPV of the total capital budget. In other words, which set of capital projects will maximize owners' wealth?

### Try it! Capital budgeting techniques

Suppose an investment requires an initial outlay of \$5 million and has expected cash flows of \$1 million, \$3.5 million and \$2 million for the first three years, respectively. What is this project's:

1. Payback period?
2. Discounted payback period using a 10 percent required rate of return?
3. Net present value using a 10 percent required rate of return?
4. Internal rate of return?
5. Modified internal rate of return using 5 percent reinvestment rate?

## 4. Capital budgeting techniques in practice

Among the evaluation techniques in this chapter, the one we can be sure about is the net present value method. NPV will steer us toward the project that maximizes wealth in the most general circumstances. But what evaluation technique do financial decision makers really use?

We learn about what goes on in practice by anecdotal evidence and through surveys. We see that:

- there is an increased use of more sophisticated capital budgeting techniques;
- most financial managers use more than one technique to evaluate the same projects, with a discounted cash flow technique (NPV, IRR, PI) used as a primary method and payback period used as a secondary method; and
- the most commonly used is the internal rate of return method, though the net present value method is gaining acceptance.
- IRR is popular most likely because it is a measure of yield and therefore easy to understand. Moreover, since NPV is expressed in dollars -- the expected increment in the value of the firm and financial managers are accustomed to dealing with yields, they may be more comfortable dealing with the IRR than the NPV.

The popularity of the IRR method is troublesome since it may lead to decisions about projects that are not in the best interest of owners in certain circumstances. However, the NPV method is becoming more widely accepted and, in time, may replace the IRR as the more popular method.

And is the use of payback period troublesome? Not necessarily. The payback period is generally used as a screening device, eliminating those projects that cannot even break-even.

Further, the payback period can be viewed as a measure of a yield. If the future cash flows are the same amount each period and if these future cash flows can be assumed to be received each period forever -- essentially, a perpetuity -- then  $1/\text{payback period}$  is a rough guide to a yield on the investment. Suppose you invest \$100 today and expect \$20 each period, forever. The payback period is 5 years. The inverse,  $1/5 = 20$  percent per year, is the yield on the investment.

Now let's turn this relation around and create a payback period rule. Suppose we want a 10 percent per year return on our investment. This means that the payback period should be less than or equal to 10 years. So while the payback period may seem to be a rough guide, there is some rationale behind it.

Use of the simpler techniques, such as payback period, does not mean that a firm has unsophisticated capital budgeting. Remember that evaluating the cash flows is only one aspect of the process:

- cash flows must first be estimated,
- cash flows are evaluated using NPV, PI, IRR, MIRR or a payback method; and
- project risk must be assessed to determine the cost of capital.

## 5. Summary

The payback period and the discounted payback period methods give us an idea of the time it takes to recover the initial investment in a project. Both of these methods are disappointing because they do not necessarily consider all cash flows from a project. Further, there is no objective criteria that we can use to judge a project, except for the simple criterion that the project must pay back.

The net present value method and the profitability index consider all of the cash flows from a project and involve discounting, which incorporates the time value of money and risk. The net present value method produces an amount that is the expected added value from investing in a project. The profitability index, on the other hand, produces an indexed value that is useful in ranking projects.

The internal rate of return is the yield on the investment. It is the discount rate that causes the net present value to be equal to zero. IRR is hazardous to use when selecting among mutually exclusive projects or when there is a limit on capital spending.

The modified internal rate of return is a yield on the investment, assuming that cash inflows are reinvested at some rate other than the internal rate of return. This method overcomes the problems associated with unrealistic reinvestment rate assumptions inherent with the internal rate of return method. However, MIRR is hazardous to use when selecting among mutually exclusive projects or when there is a limit on capital spending.

Each technique we look at offers some advantages and disadvantages. The discounted flow techniques -- NPV, PI, IRR, and MIRR -- are superior to the non-discounted cash flow techniques -- the payback period and the discounted payback period.

To evaluate mutually exclusive projects or projects subject to capital rationing, we have to be careful about the technique we use. The net present value method is consistent with owners' wealth maximization whether we have mutually exclusive projects or capital rationing.

Looking at capital budgeting in practice, we see that firms do use the discounted cash flow techniques, with IRR the most widely used. Over time, however, we see a growing use of the net present value technique.

## 6. Try it! Solutions

### Capital budgeting techniques

1. Payback period?
  - The sum of the cash flows at the end of two years is \$4.5 million
  - The sum at the end of three years is \$6.5 million
  - Payback = **Three years**
2. Discounted payback period using a 10 percent required rate of return?
  - The sum of the discounted cash flows at the end of three years is:<sup>4</sup>  
 $\$0.9091 + 2.8926 + 1.5026 = \$5.3043$
  - Discounted payback period = **Three years**.
3. Net present value using a 10 percent required rate of return?
  - Present value of inflows = \$5.3043 million (we know this from the discounted payback period calculation).
  - Present value of outflows = \$5 million
  - NPV =  $\$5.3043 - 5 = \mathbf{\$0.3043 \text{ million}}$
4. Internal rate of return?
  - We know that the IRR must be greater than 10 percent because the NPV is positive when the discount rate is 10 percent.
  - IRR = **13.13 percent**
5. Modified internal rate of return using a 5 percent reinvestment rate?
  - Terminal value =  $\$1 (1.05)^2 + \$3.5 (1.05) + \$2 = \$1.1025 + 3.675 + 2 = \$6.7775$  million
  - TV = FV = \$6.7775; N = 3; PV = \$5; Solve for i
  - MIRR = **10.6708 percent**

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<sup>4</sup> Why not check for discounted payback after two years? Because if it does not payback in two years using undiscounted cash flows, it does not payback in terms of discounted cash flows.