Turn these problems in with the assigned problems from the text:

(1) Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be the linear transformation that rotates all vectors in the plane counterclockwise by θ radians. Let α be the standard basis of $\mathbb{R}^2$. Draw me a picture to convince me that

$$[T]_\alpha^\alpha = \begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix}.$$

(In your picture you may assume that $0 < \theta < \frac{\pi}{2}$.)

(2) Use the previous problem to derive the following famous trigonometric identies:

- $\sin(\alpha + \beta) = \sin(\alpha)\cos(\beta) + \cos(\alpha)\sin(\beta)$
- $\cos(\alpha + \beta) = \cos(\alpha)\cos(\beta) - \sin(\alpha)\sin(\beta)$.

(Hint: Rotate by α, then by β.)

(3) Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be defined by

$$T(\vec{x}) = A\vec{x},$$

where $A = \begin{bmatrix} 3 & -4 \\ 2 & -3 \end{bmatrix}$.

- Find the two eigenvalues of A, and find an eigenvector for each one.
- Since the two eigenvectors you got above are independent, they must be a basis for $\mathbb{R}^2$. Call this basis of eigenvectors β. Compute:

$$[T]_\beta^\beta.$$

(There is a quick way to do this.)

(Optional) Bonus Problems: For each problem that you solve correctly I will increase your homework score by one point. All or nothing for these – no partial credit.

Let A be the matrix

$$A = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}.$$

1 point: Find the eigenvalues of A.

1 point: Find an eigenvector corresponding to each of the eigenvalues of A.

1 point: Find a formula for the nth Fibonacci number by writing $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$ as a linear combination of these eigenvectors, and then computing $A^n \begin{bmatrix} 1 \\ 0 \end{bmatrix}$.