Turn these problems in with the assigned problems from the text:

(1) A *permutation matrix* P is a square matrix of zeros and ones with the following property: every row contains a single one and every column contains a single one.
Here are some examples of permutation matrices:

$$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \quad \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}, \quad \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

(These are not the only examples for matrices of these sizes, think of more...)

Show that any permutation matrix will have determinant ± 1.

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(2) The matrix $A = \begin{bmatrix} 2 & -2 \\ 2 & -2 \end{bmatrix}$ is a "vector" in the space $M_{2\times 2}(\mathbb{R})$ of all 2×2 matrices with real entries.

Write down the zero vector in this space, the vector $\frac{1}{2}A$, and the vector $-A$.

What matrices are in the smallest subspace containing A?

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(3) Describe a subspace of $M_{2\times 2}(\mathbb{R})$ that contains $A = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$ but not $B = \begin{bmatrix} 0 & 0 \\ 0 & -1 \end{bmatrix}$.

If a subspace contains A and B, must it contain I?

Describe a subspace of $M_{2\times 2}(\mathbb{R})$ that contains no nonzero diagonal matrices.

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(4) Consider the following subset of $\mathbb{R}^n$:

$$\left\{ \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_n \end{bmatrix} \mid a_1 + a_2 + \cdots + a_n = 0 \right\}.$$

Is this a subspace of $\mathbb{R}^n$? Explain why or why not.

(Optional) Bonus Problems: For each problem that you solve correctly I will increase your homework score by one point. All or nothing for these – no partial credit.

(1) This question is also about permutation matrices. It is not difficult to count permutation matrices. There are two permutation matrices of size 2×2, there are six permutation matrices of size 3×3, and in general there are $n! = n(n-1)\cdots 2 \cdot 1$ permutation matrices of size $n \times n$. (Convince yourself of this fact, and feel free to ask me about it.)

In the problem above you were asked to show that any permutation matrix will have determinant either 1 or -1. Your job for this bonus problem is to tell me exactly how many permutation matrices of size $n \times n$ have determinant equal to 1. You must also give a good explanation for your answer.