Turn these problems in with the assigned problems from the text:

(1) Let $D : P_2 \to P_2$ be the differential operator; that is, the linear transformation defined by

$$D(ax^2 + bx + c) = 2ax + b.$$

Let $\alpha = \{x^2, x, 1\}$ be the standard basis of P_2. Consider also the basis $\beta = \{x^2 + x + 1, x^2 + x, x^2\}$.

- Find the matrix $[D]_\alpha^\alpha$.
- Find the matrix $[D]_\alpha^\beta$.
- Find the matrix $[D]_\beta^\alpha$.

(2) TRUE OR FALSE? For each of the statements below, decide whether the statement is true or false. If it is true, explain why. If it is false, show this by means of an example. In either case your answers should be brief.

T or F: If the rows of a matrix A form a basis of $RS(A)$, then the columns of A form a basis of $CS(A)$.

T or F: A set of linearly independent vectors form a basis for the space that they span.

T or F: If A is an $m \times n$ matrix and $n > m$, then the equation $A\vec{x} = \vec{b}$ always has a solution.

T or F: There is no linear transformation $T : \mathbb{R}^2 \to \mathbb{R}^2$ with

$$T\left(\begin{bmatrix}1\\1\end{bmatrix}\right) = \begin{bmatrix}1\\2\end{bmatrix} \text{ and } T\left(\begin{bmatrix}-1\\-1\end{bmatrix}\right) = \begin{bmatrix}2\\1\end{bmatrix}.$$

(Optional) Bonus Problems: For each problem that you solve correctly I will increase your homework score by one point. All or nothing for these – no partial credit.

(1) Let A be a matrix. One way to find a basis for $CS(A)$ is to transpose the matrix, perform row operations to get the matrix into row echelon form, and then transpose again. The nonzero columns of this matrix form a basis of $CS(A)$.

Here is another way, and your job for this bonus problem is to explain why this method I'm about to describe will work. Take the matrix A and perform row operations on it until it is in row echelon form. Notice that I did *not* say to take the transpose. Once you are looking at the row echelon form of A, notice which columns correspond to 'dependent variables' and which columns correspond to 'free variables' (in the sense of Section 1.1). Then the columns of the original matrix A corresponding to the dependent variables form a basis of $CS(A)$.

Example: If $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$ then after putting A in row echelon form we have

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & -3 & -6 \end{bmatrix}.$$

The columns corresponding to dependent variables are columns 1 and 2 (column 3 is 'free'). So columns 1 and 2 of A,

$$\begin{bmatrix} 1 \\ 4 \end{bmatrix}, \begin{bmatrix} 2 \\ 5 \end{bmatrix}$$

form a basis for $CS(A)$.

Feel free to ask for a hint on this.