Math 238 Course Review (The BIG Ideas):

Chap. 1

- Elimination. Know about row echelon form and reduced row echelon form. Know how to solve *any* system of linear equations.
- Inverse of a square matrix. We know a fast way to find it using row operations. There is also an "adjoint method" using determinants. Know how the inverse of a matrix relates to solving equations.
- Determinants. Know how to compute determinants. There is a quick way using row operations, there is the "cofactor expansion". Properties like $\det(AB) = \det(A)\det(B)$.
- Determinants can be used to solve a system of equations (for example, with Cramer's Rule). They can be used to tell if a (square) system of equations has a solution. They can be used to tell if a matrix is invertible. Know about these relationships.

Chap. 2

- Vector spaces. Know what a vector space is, and examples of them.
- Subspaces. Know what a subspace is. Be able to check if a subset of a vector space is actually a subspace. Know what the subspace *spanned* by a set of vectors is.
- Independence. Know what it means for a set of vectors to be linearly independent, and how to check this. Know what a basis of vector space is, and how to check if a set of vectors is a basis for a particular vector space. Know what the "dimension" of a vector space is.
- The most useful subspaces are attached to a particular matrix. Know about the nullspace, row space, and column space of a matrix. Know how to compute bases for them, and know how their dimensions relate. Know what the *rank* of a matrix is.

Chap. 5

- Linear transformations. Know what a linear transformation is and how to check if a particular map is one. Know examples of them.
- Know what the kernel and range of a linear transformation are. Know basic facts about these subspaces.
- Matrices for linear transformations. Know how to compute the matrix of a linear transformation with respect to a basis of the domain and a basis of the codomain. Know the major theorems

related to this and how to use them.

$$[T(\vec{v})]_\beta = [T]_\alpha^\beta[\vec{v}]_\alpha \qquad [S]_\beta^\gamma[T]_\alpha^\beta = [ST]_\alpha^\gamma$$

- Know about *change of basis*, and how the above formulas specialize in this case.
- Know about eigenvalues and eigenvectors. Their definitions, how to find them, how to compute bases for eigenspaces. Know how to diagonalize a matrix (if possible).

Chap. 3

- Know what a differential equation is, and the different types. Know what an initial value problem is.
- Separable DEs. Solve these by integration.
- Exact DEs.
- Linear DEs. We only did first order linear DEs in chapter 3. You can use an integrating factor to solve them.
- Bernoulli-type DEs.
- Basic applications of first order DEs. Mixing, cooling, growth/decay.

Chap. 6

- Know what a system of first order linear DEs is. Know how to pass back and forth between the different notations (separate equations, matrix notation, etc.) Know that the solutions of a *homogeneous* system of n first order DEs is a vector space of dimension n. Know about "fundamental solutions".
- Know how to solve a homogeneous system using diagonalization (if possible). Recall there are different things we do depending on if eigenvalues are real or complex.
- Know that even if a matrix is not diagonalizable, it can still be brought into Jordan Canonical Form (a particular upper triangular form) and so systems can still be solved in this way.
- Know how to solve a non-homogeneous system of (first order) linear DEs. Know that the general solution looks like $Y_H + Y_P$.
- We can convert a high order linear DE into a first order system, although we would rather use the techniques of chapter 4.

Chap. 4

- Know what an nth order linear differential equation is. Know that the solutions of an nth order *homogeneous* linear DE form a vector space of dimension n. Know about "fundamental solutions".
- Know how to solve a homogeneous nth order linear DE using the roots of the characteristic polynomial $p(\lambda)$. Recall there are

different things that we do if the roots are real or complex, and the multiplicity of the root also matters.

- Know how to solve a non-homogeneous nth order linear DE. In particular, be able to find a particular solution y_P with the "method of undetermined coefficients". Know that the general solution looks like $y_H + y_P$.