Optional Bonus Problems (Worth 1 Homework Point Each):

Turn in your solution to any of the problems below by 12/06/2013. To receive credit for a problem, your work must be completely flawless.

In other words, *all or nothing*.

Please work individually on these, and do not seek help from any sources other than me.

(1) Let G be a group of permutations on the set $\{1, 2, \ldots, n\}$, i.e., let $G \leq S_n$. Suppose that $\sigma \in G$ and $i, j \in \{1, 2, \ldots, n\}$ with $\sigma(i) = j$. Prove that if N is a normal subgroup of G, then

$$\sigma\left(\mathrm{orb}_N(i)\right) = \mathrm{orb}_N(j).$$

(2) Using the previous exercise, argue that the orbits under N break apart the orbits under G into blocks of *equal size*. Give an example showing that this need not happen if N is not normal in G.