

Optional Bonus Problems (Worth 1 Homework Point Each):

Turn in your solution to any of the problems below by 10/21/2013. To receive credit for a problem, your work must be completely flawless.

In other words, *all or nothing*.

Please work individually on these, and do not seek help from any sources other than me.

- (1) Suppose that G is a subgroup of S_n . For $i, j \in \{1, 2, \dots, n\}$ write $i \sim j$ if and only if there exists some $\sigma \in G$ so that $\sigma(i) = j$. Prove that “ \sim ” is an equivalence relation on $\{1, 2, \dots, n\}$.
- (2) Let G be a finite group that possesses an automorphism σ with the property that $\sigma(g) = g$ if and only if $g = e$. Furthermore, assume that σ^2 is the identity map from G to G . Prove that G is abelian. (*Hint: Argue that every element g of G can be written in the form $g = x^{-1}\sigma(x)$, for some $x \in G$. Apply σ to this factored expression. Now use exercise 10 in Chapter 6.*)