

## Test 3 Study Aid

Here are some topics that may appear on Test 3. The test will cover material that we have learned in the course up until now, with a focus on Chapters 7, 9, and 10. I do not claim completeness here; in fact, this document is deliberately vague to encourage you to study seriously for this test. You should be carefully reading the book, consulting your notes, and most of all **working problems!**

### Chap. 7 (Cosets, Lagrange's Theorem):

- Given a subgroup  $H$  of a group  $G$ , you should know what the left (or right) cosets of  $H$  in  $G$  are. That is, you should know the definition as well as how to work them out for concrete examples. You should know some of the terminology that comes with cosets. Do you know what the *index* of  $H$  in  $G$  is?
- You should be very familiar with the properties of cosets listed in the first lemma of chapter 7. Realize that we have used these properties many times, and not just in chapter 7.
- Know what Lagrange's theorem says. Can you convey the idea of its proof with a picture? Know why the converse of Lagrange's theorem is false. Lagrange's theorem is the most important theorem about finite groups. Enough said.
- Do you know what the set  $HK$  is? Can you calculate its size? Why would you ever want to do that?
- Given a permutation group  $G$  on a set  $S$ , and a point  $s \in S$ , you should know what the stabilizer of  $s$  in  $G$  is. You should also know what the orbit of  $s$  under  $G$  is. In fact, try to write down their definitions right now without cracking open the book. There is a nice theorem relating the sizes of these two sets to the order of  $G$ . We used it to calculate the order of some groups of rotations. What was the name of that theorem again? Bonus points if you can remember what all this has to do with cosets.

### Chap. 9 (Normal Subgroups, Factor Groups):

- Know what a normal subgroup of a group  $G$  is. How would you try to show that a particular subgroup is normal in  $G$ ? Why are normal subgroups important? What do they allow you to do? Try to answer this question before reading anymore.

- What is a factor group? What condition needs to be met in order to have a factor group? Why is that condition necessary?
- Know the standard notation and terminology associated with factor groups:  $G/H$ , “ $G \bmod H$ ”, etc. What do the elements of  $G/H$  look like? What is the group operation?
- We covered a few interesting applications of factor groups in this chapter. What information does the factor group  $G/Z(G)$  hold? Using factor groups we proved a partial converse to Lagrange’s theorem (for abelian groups). What am I talking about?

**Chap. 10 (Homomorphisms):**

- Know what a group homomorphism is. Know what the kernel of a homomorphism is. Know examples of these concepts.
- Make sure you know the properties in theorems 10.1 and 10.2, and make sure you can prove them.
- Given a homomorphism  $\phi: G \rightarrow \overline{G}$ , you know and are able to prove that  $\ker \phi \trianglelefteq G$ . (Right?) You also know that the image of  $G$  under  $\phi$ , written  $\phi(G)$ , is a subgroup of  $\overline{G}$ . The First Isomorphism Theorem states that ... is isomorphic to ... ? What is the isomorphism? Describe what is going on here with a picture.

**Some Advice:**

- Bottom line: cosets are an important and extremely useful idea. Chapters 7, 9, and 10 are really all about cosets.
- You should at this point in the course have completed many exercises. Work and rework them. The exercises are the best way for you to measure what you do and do not understand.