Homework Assignment 1

Due Wednesday 9/10/2014 at start of class.

Assigned:
Section 2.3: 3, 6, 11, 17, 18, 27
Section 2.4: 4, 5, 6, 7, 22, 24, 32
Section 2.5: 6, 15, 18, 25, 29, 32, 44

Collected:

(1) Solve the linear system. If there is a unique solution, find it. If there is no solution, show why. If there are many solutions, try to describe them all (if this is too difficult, give at least 3 distinct solutions).

(a)

$$\begin{aligned} x - y + z &= 0 \\ 2x - 3y + 4z &= -2 \\ -2x - y + z &= 7 \end{aligned}$$

(b)

$$\begin{aligned} x_1 + x_2 - x_3 + 2x_4 &= 1 \\ x_1 + x_2 + x_4 &= 2 \\ x_1 + 2x_2 - 4x_3 &= 1 \\ 2x_1 + x_2 + 2x_3 + 5x_4 &= 1 \end{aligned}$$

(c)

$$\begin{aligned} 2x + 3y - z &= 3 \\ -x - y + 3z &= 0 \\ x + 2y + 2z &= 3 \\ y + 5z &= 3 \end{aligned}$$

Give a quick verbal description of the geometric picture behind each system of equations (i.e., the row picture). Was the solution to parts (b) and (c) expected or unexpected?

(2) Find a quadratic function

$$y = ax^2 + bx + c$$

that passes through the points $(1, 4)$, $(-1, -2)$, $(5, 64)$.

(3) Which elimination matrices E_{21} and E_{31} produce zeros in the $(2,1)$ and $(3,1)$ positions of $E_{21}A$ and $E_{31}A$?

$$A = \begin{bmatrix} 2 & 1 & 0 \\ -2 & 0 & 1 \\ 8 & 5 & 3 \end{bmatrix}$$

Find the single matrix E that produces both zeros at once. Multiply EA to check that it works.

(4) Describe all vectors that are perpendicular to $\begin{bmatrix} 1 \\ 2 \\ 4 \end{bmatrix}$ and $\begin{bmatrix} 5 \\ 3 \\ 1 \end{bmatrix}$.

(5) Consider the matrix

$$A = \begin{bmatrix} 2 & 1 & 3 \\ 2 & 1 & 1 \\ 4 & 5 & 1 \end{bmatrix}.$$

Show that A^{-1} exists (by finding it!) and then use A^{-1} to quickly solve the systems:

(a) $A\vec{x} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$

(b) $A\vec{x} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$

(c) $A\vec{x} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$