Homework Assignment 4

Due Wednesday 10/08/2014 at start of class.

Assigned:
Section 3.2: 1–3, 7, 9, 11, 12, 15, 17, 18, 26, 30, 33, 36, 37
Section 3.3: 1, 2, 4, 6, 8, 10, 28
Section 3.4: 1, 2, 4, 5, 13, 16, 17, 19, 27, 31
Section 3.5 1, 2, 5, 7, 8, 10, 11, 13, 16, 17, 20, 21, 22, 26, 38, 39, 41
Section 3.6 To appear.

Collected:

(1) Write down all possible shapes for a 3×4 matrix in *reduced* row echelon form.

For example, here is one shape

$$\begin{bmatrix} 1 & * & 0 & * \\ 0 & 0 & 1 & * \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

where the symbol $*$ denotes an arbitrary real number.

(2) Find the *complete solution* to $A\vec{x} = \vec{b}$, for the following:

(a) $A = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 0 & 1 \\ 0 & 2 & 0 & 1 \end{bmatrix}$, $\vec{b} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$

(b) $A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 1 & 2 & 3 & 4 \end{bmatrix}$, $\vec{b} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

(c) $A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 1 & 2 & 3 & 4 \end{bmatrix}$, $\vec{b} = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$

(d) $A = \begin{bmatrix} 1 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}$, $\vec{b} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$

(3) For each matrix A in the previous problem, find a basis for the column space of A. Please explain how you are doing this.

(4) Consider the matrix

$$A = \begin{bmatrix} 1 & 1 \\ -1 & 1 \\ 1 & 1 \\ 1 & 1 \\ -1 & -1 \end{bmatrix}.$$

(a) Give a basis for $C(A)$. What is the dimension of $C(A)$?

(b) Suppose $(b_1, b_2, b_3, b_4, b_5)$ is in $C(A)$. Give equations that must be satisfied by the b_i's.

(c) Using these equations, find a matrix B with the property that $C(A) = N(B)$.

(d) Give a basis for $R(A)$ (the row space of A). What is the dimension of $R(A)$?

(5) For the given vector spaces, construct a basis and state the dimension.

(a) The space of all vectors (a_1, a_2, a_3, a_4) in $\mathbb{R}^4$ with the property that $a_1 + a_2 + a_3 + a_4 = 0$.

(b) The space of all 2×2 matrices with trace zero. (The trace of a matrix is the sum of its diagonal entries.)

(c) The subspace of $\mathbb{R}^4$ spanned by the following six vectors:

$$\begin{bmatrix} 1 \\ 0 \\ 0 \\ -1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ -1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ -1 \\ 0 \\ -1 \end{bmatrix}.$$

Bonus! For each problem that you successfully complete below, I will add one point to this homework's score. The standard by which I will grade these is higher than usual. Your solution must be completely clear and error-free or you will not receive any credit.

(1) We showed in class that if $\vec{x_p}$ is a particular solution of $A\vec{x} = \vec{b}$ and $\vec{x_n}$ is an an element of $N(A)$, then $\vec{x_p} + \vec{x_n}$ is another solution of $A\vec{x} = \vec{b}$. Explain why this indeed gives *all* of the solutions of $A\vec{x} = \vec{b}$. That is, show that any solution of $A\vec{x} = \vec{b}$ can be written in the form $\vec{x_p} + \vec{x_n}$ ($\vec{x_p}$ is fixed, $\vec{x_n}$ depends on the given solution.)

(2) (a) Let A be a matrix. As we saw in class, one way to get a basis for $C(A)$ is to transpose the matrix, perform row operations to get into row echelon form U (or R), then transpose again to get U^T. The nonzero columns of the resulting matrix U^T form a basis of $C(A)$. Explain why this works.

(b) Again start with a matrix A. Another way to get a basis for $C(A)$ is to perform row operations until it is in row echelon form U (or R). Note that I did *not* say to take the transpose. Observe which columns of U contain the pivots. Then those columns of the original matrix A will be a basis for $C(A)$. Explain why this works.