Math 435
Homework Assignment 2

Due Monday 9/28/2015

(1) Prove that a finite set with n elements contains exactly 2^n subsets.

(2) Prove that the real line $\mathbb{R}$ and the set $2^{\mathbb{Z}_+}$ have the same cardinality. What is the cardinality of $2^{\mathbb{Z}}$?

(3) In the plane $\mathbb{R}^2$, consider a set A of non-intersecting discs. What is the cardinality of A?

(4) Let A be the set of all binary sequences (i.e., sequences whose terms are 0's or 1's) that have only a finite number of 1's. What is the cardinality of A?

(5) What is the cardinality of the set of all functions $f\colon \{0,1\} \to \mathbb{Z}_+$?

(6) Prove that the cardinality of $\mathbb{R}^n$ is continuum for all $n \in \mathbb{Z}_+$.

(7) (2 pts) Let $A_1, A_2, \ldots, A_n, \ldots$ be a countably infinite collection of sets each of which has cardinality continuum. Prove that the union $\cup_{n=1}^{\infty} A_n$ has cardinality continuum.

(8) (2 pts) We equip the space $\mathbb{R}^2$ with two metrics d_1 and d_2 as follows. For points $a = (a_1, a_2)$ and $b = (b_1, b_2)$ in $\mathbb{R}^2$:

$$d_1(a,b) = \sqrt{(b_1 - a_1)^2 + (b_2 - a_2)^2}, \quad d_2(a,b) = |b_1 - a_1| + |b_2 - a_2|.$$

Given a metric space Y and a map $f\colon \mathbb{R}^2 \to Y$, prove that the map $f\colon (\mathbb{R}^2, d_1) \to Y$ is continuous if and only if the map $f\colon (\mathbb{R}^2, d_2) \to Y$ is.