**Math 435**
**Homework Assignment 4**

**Due Monday 11/2/2015**

(1) (3 pts) Let $A, B$ denote subsets of a space $X$. Prove the following:

- If $A \subset B$, then $\overline{A} \subset \overline{B}$;
- $\overline{A \cup B} = \overline{A} \cup \overline{B}$;
- $\overline{\cup_\alpha A_\alpha} \supset \cup_\alpha \overline{A_\alpha}$; give an example of proper containment.

(2) (2 pts) Show that a space $X$ is Hausdorff if and only if the *diagonal*

$$\Delta = \{(x, x) \in X \times X \,|\, x \in X\}$$

is closed in $X \times X$.

(3) (2 pts) Let $Y$ be an ordered set in the order topology, and let $f, g \colon X \to Y$ be continuous.

- Show that the set $\{x \in X \,|\, f(x) \leq g(x)\}$ is closed in $X$.
- Show that the function $h \colon X \to Y$, $h(x) = \min\{f(x), g(x)\}$ is continuous. (Hint: use the pasting lemma.)

(4) Let $f \colon S^1 \to \mathbb{R}$ be continuous. Show that there exists $x \in S^1$ such that $f(x) = f(-x)$.

(5) Consider a figure eight, i.e., a curve homeomorphic to the one given by the equation

$$((x-1)^2 + y^2 - 1)((x+1)^2 + y^2 - 1) = 0.$$

Show that this figure is not homeomorphic to the circle.

(6) Describe all continuous functions $f \colon \mathbb{R} \to \mathbb{Q}$.

**Bonus Problems**

To appear later.