Turn these problems in with the assigned problems from the text:

(1) Let n be a positive integer. For $a, b \in \mathbb{Z}$ we write $a \sim b$ if $a - b$ is divisible by n. This is an equivalence relation. Let $\bar{a}$ denote the equivalence class of a.

Set $\mathbb{Z}/n\mathbb{Z} = \{\bar{0}, \bar{1}, \cdots, \overline{n-1}\}$. We saw in class that $\mathbb{Z}/n\mathbb{Z}$ can be made into a group by defining

$$\bar{a} + \bar{b} = \overline{a+b},$$

and that this operation is well-defined.

Show that we can also multiply equivalence classes; that is, show that the operation

$$\bar{a}\bar{b} = \overline{ab}$$

is well-defined. Is $\mathbb{Z}/n\mathbb{Z}$ a group under this operation of multiplication? Why or why not?