Optional Bonus Problems (Worth 2 Points Each):

(1) Suppose that G is a subgroup of S_n. For $i, j \in \{1, 2, \cdots, n\}$ write $i \sim j$ if and only if there exists some $\sigma \in G$ so that $\sigma(i) = j$. Prove that "$\sim$" is an equivalence relation on $\{1, 2, \cdots, n\}$.

(2) Let G be a finite group that possesses an automorphism σ with the property that $\sigma(g) = g$ if and only if $g = e$. Furthermore, assume that σ^2 is the identity map from G to G. Prove that G is abelian. (*Hint: Argue that every element g of G can be written in the form $g = x^{-1}\sigma(x)$, for some $x \in G$. Apply σ to this factored expression. Now use exercise* 10 *in Chapter* 6.