

Test 2 Study Aid

Here are some topics that may appear on Test 2. The test will cover material that we have learned in the course up until now, with a focus on Chapters 4, 5, and 6. I do not claim completeness here; in fact, this document is deliberately vague to encourage you to study seriously for this test. You should be carefully reading the book, consulting your notes, and most of all **working problems!**

Chap. 4 (Cyclic Groups):

- Know the definition of a cyclic group, what a generator is. Know the correct notation for this. Know and be comfortable working with the fundamental examples.
- There is essentially only one cyclic group of each order, including infinite order. We actually have the correct language for this now: Any infinite cyclic group is *isomorphic* to...? Any finite cyclic group of order n is *isomorphic* to...? Know the basic facts about how to operate with cyclic groups, how to distinguish elements, what the subgroups are, etc.
- For any cyclic group of order n , you should be able to list out all the distinct elements, know the order of any element, know all of the subgroups and the elements inside them and how the subgroups are contained in each other. You should easily be able to tell me all of the elements of any particular order. All of these things are related, and you should know how and why they relate.

Chap. 5 (Permutation Groups):

- Know the definition of a permutation on a set A , and a permutation group on a set A .
- Permutation groups provide a rich source of examples in group theory, and you should be familiar with all of the examples of permutation groups we have discussed.
- Be comfortable rolling up your sleeves and actually calculating with permutations. Know about cycle notation and how to operate with it.
- Know why cycle notation is useful. You know a condition for two cycles to commute, it is...? You can easily tell the order of a permutation that is written in disjoint cycle notation. In fact, you can describe all of the elements in S_n of a particular order.

- Even more uses of cycle notation. It helps you count permutations. It allows you to tell if a permutation is even or odd. Know about even permutations and the Alternating groups.

Chap. 6 (Isomorphisms):

- Know the definition of a group isomorphism $\phi: G \rightarrow \bar{G}$. Be familiar with the notation and words that go along with this idea.
- Know examples of isomorphisms. Know example of groups that are isomorphic (and what the isomorphism is) and examples of groups that are not isomorphic (with reasons why not).
- Know the amazing fact that any group is isomorphic to a permutation group (Cayley's Theorem). Understand why this fact is true (how the isomorphism is constructed).
- There are two list-type theorems in this chapter stating properties of isomorphisms. Nearly all of them are just exercises in using the definitions. You should be able to prove these statements without difficulty (test yourself).
- Know what an automorphism of a group is (know the definition). Know some examples of automorphisms of some groups. Know what an inner automorphism is, and play with some examples. Know the notation $\text{Aut}(G)$ and $\text{Inn}(G)$. You should be able to show that $\text{Aut}(G)$ is a group and $\text{Inn}(G)$ is a subgroup of $\text{Aut}(G)$.
- Know what all of the automorphisms of a cyclic group are.

Some Advice:

- Notice the number of times I mention that you should know definitions and examples. You should know all of the definitions cold; this is basic and you cannot succeed in this course otherwise. At this point in this course you should have many examples in your head of each concept we have discussed. You should know diverse examples of specific groups, and you should probably have a favorite group or two.
- You should at this point in the course have completed many exercises. Work and rework them. The exercises are the best way for you to measure what you do and do not understand.