Name:
Pledge:

Math 434 HW 1

Due Friday 2/5 at beginning of class.

(1) If U_1, U_2, W are subspaces of V, does $U_1 + W = U_2 + W$ imply that $U_1 = U_2$? Does $U_1 \oplus W = U_2 \oplus W$ imply that $U_1 = U_2$? Give proofs or specific counterexamples please.

(2) Prove that the intersection of *any* collection $\{U_\alpha \mid \alpha \in J\}$ of subspaces of V is a subspace of V.

(3) Consider the vector space $\mathbf{F}^5$. Suppose

$$U = \{(x, y, x - y, 3y, -x) \in \mathbf{F}^5 \mid x, y \in \mathbf{F}\}.$$

(a) Find a subspace W of $\mathbf{F}^5$ so that $\mathbf{F}^5 = U \oplus W$.

(b) Find three subspaces W_1, W_2, W_3, none of which are $\{0\}$, so that $\mathbf{F}^5 = U \oplus W_1 \oplus W_2 \oplus W_3$.

(4) Let $V = \mathbf{F}^{\mathbb{Z}}$.

(a) Find a sequence of subspaces of V:

$$W_1 \subset W_2 \subset \cdots$$

where all the containments are proper.

(b) Find a sequence of subspaces of V:

$$U_1 \supset U_2 \supset \cdots$$

where all the containments are proper.