Name:
Pledge:

Math 434 HW 2

Due Wednesday 3/2 at beginning of class.

(1) Let V be a vector space with basis $v_1, v_2, \ldots, v_n$ and let $c \in F$ be a scalar. Show that the list

$$v_1 + cv_2, v_2, \ldots, v_n$$

is also a basis of V. (More generally, it should be clear to you that replacing any basis vector v_i in the original list with $v_i + cv_j$, for $i \neq j$, will still result in a basis of V.)

(2) Let $T\colon V \to W$ be a linear map between finite dimensional vector spaces. Show that there exists a subspace U of V so that by restricting the domain of T to U, one gets an isomorphism from U to $\operatorname{range} T$. (Hint: study the proof of Theorem 3.22.)

(3) Let $T\colon V \to W$ be a linear map between finite dimensional vector spaces. Show that there exists bases of V and W so that, with respect to these bases,

$$\mathcal{M}(T)_{j,k} = \begin{cases} 1, & \text{if } (j,k) = (i,i) \text{ and } i \leq \dim \operatorname{range} T \\ 0, & \text{otherwise.} \end{cases}$$

In other words, show there exist bases of V and W so that $\mathcal{M}(T)$ looks like

$$\begin{pmatrix} 1 & 0 & 0 & & 0 \\ 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & & 0 \\ & \vdots & & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & \end{pmatrix}$$

where the number of 1s down the main diagonal is equal to the dimension of $\operatorname{range} T$. (Hint: study the proof of Theorem 3.22.)

(4) Consider the linear map $S\colon F^4 \to F^4$ defined by:

$$S(a_1, a_2, a_3, a_4) = (a_1 + a_2 + a_3 + a_4) \cdot (1, 1, 1, 1).$$

(a) Write down the matrix for S, with respect to the standard basis of both the domain and codomain.
(b) Construct bases of the domain and codomain so that the matrix $\mathcal{M}(S)$ has the form of problem 3 (and write down this matrix). One bonus point for you if you are able to use the *same* basis for both domain and codomain.