Name:
Pledge:

Math 434 HW 3

Due Friday 4/15 at beginning of class.

(1) Suppose that $S, T \in \mathcal{L}(V)$, where V is a finite dimensional vector space over $\mathbb{C}$. If $ST = TS$, show that S and T have a common eigenvector.

(2) Find an orthonormal basis (with respect to usual dot product) for the subspace of $\mathbb{R}^5$ spanned by:

$$(1, 1, 0, -1, 0), (-1, 1, 1, 1, 1), (1, -1, 1, 0, 1), (1, 1, -1, -1, 0).$$

Find the vector in this subspace that is closest to $(1, 2, 3, 4, 5)$, in the sense of usual euclidean distance.

(3) Let V be a finite dimensional vector space. An operator $T \in \mathcal{L}(V)$ is said to be *nilpotent* if $T^n = 0$ for some positive integer n. Must a nilpotent operator have an eigenvalue? Describe the eigenvalues of T.

(4) Consider $T \in \mathcal{L}(\mathbb{R}^2)$ defined by

$$T(x, y) = (x - y, 2y - x).$$

Find the eigenvalues of T.

Bonus! (1 point each)

(1) Figure out how the operator in problem 4 above relates to the Fibonacci numbers. Use the eigenvectors of that operator to find a formula for the Fibonacci numbers. See book or ask me for hints.

(2) Let V be a finite dimensional vector space. Consider the space

$$V' = \mathcal{L}(V, F).$$

Let $v_1, v_2, \ldots, v_n$ be a basis for V. For $i = 1, 2, \ldots, n$ define $v_i' \in V'$ by

$$v_i'(v_j) = \begin{cases} 1 & \text{if } j = i \\ 0 & \text{if } j \neq i. \end{cases}$$

Show that $v_1', v_2', \ldots, v_n'$ is a basis of V'.

(3) Let V be finite dimensional and let T be an operator on V. Define $T' \in \mathcal{L}(V')$ by

$$T'(f) = fT.$$

(You should mentally check this defines an operator on V'. Ask me if you do not understand the notation.) Let $v_1, v_2, \ldots, v_n$ be a basis for V. Describe the relationship of the two matrices

$$\mathcal{M}(T)_{v_1,v_2,\ldots,v_n} \quad \text{and} \quad \mathcal{M}(T')_{v_1',v_2',\ldots,v_n'}$$

to each other.